TMME40 Vibration Analysis of Structures

Assignment 1

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4 October, 2016

An aircraft undergoes a whole range of rig tests as part of the airworthiness certification. The airframe response and structural integrity is checked in different statical and dynamical tests. A typical test for the wings consists of mounting them in a hydraulic load rig and subject them to different force protocols.

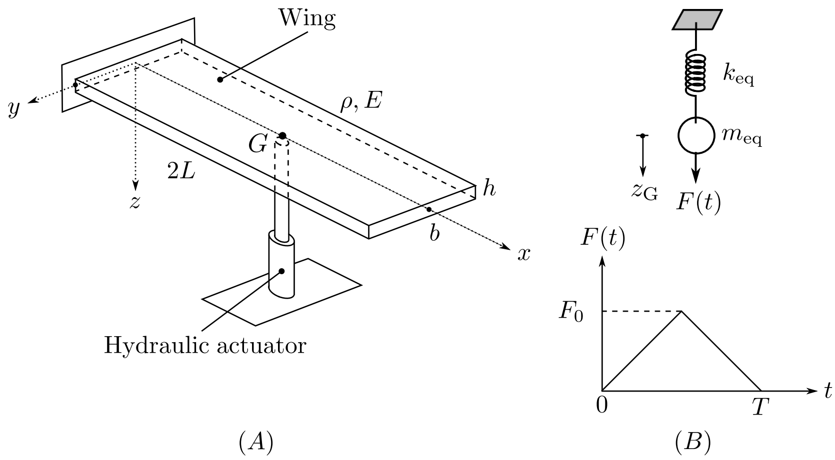

Figure 1: The model and the external load for the wing.

A model of the right wing for a sport plane is presented in Fig. 1A. The wing is approximated by a homogeneous box of length $2L$, width b, and height h. The density of the material in the box is ρ and the elastic modulus (Young's modulus) is E. The root of the wing is rigidly attached to a supporting structure and can be considered as fixed. The wing is loaded by a hydraulic actuator mounted in the mass centre G. The actuator generates a force in the form of a single sawtooth with amplitude F_0 and duration T, see Fig. 1B.

Questions (5 points)

In this assignment you will study the response of the wing by transforming the continuous model above to a single degree-of-freedom mass-spring-model. The wing can be assumed to undergo small vibrations.

Question 1

Derive the equivalent stiffness k_{eq} and equivalent mass m_{eq} at G by considering the wing to be a cantilever beam.

Question 2

Setup the governing equation for the oscillation and state the natural frequency ω_{n}. (You may use $F(t)$ for the external force in the governing equation.)

Question 3

If the wing is at rest in its static equilibrium position at $t = 0$, compute the motion of mass centre (z_{G}) as a function of time using the forcing in Fig. 1. (It is recommendable to use the symbolic math equation capability in Matlab, Maple, or a similar program for this question.)

Question 4

Check your result in Question 3 by computing the oscillation numerically, e.g., using the function `ode45` in Matlab. Data is found in the Table below.

Parameter	Value	Parameter	Value	Parameter	Value
$2L$	5 m	b	1.0 m	h	0.05 m
E	211 GPa	ρ	1500 kg/m^3		
F_0	100 kN	T	2 s		

Question 5

The model used herein is very crude. Propose a simple modification which makes it more accurate. The modified model must still be solvable as above with a modest extra effort.

Reporting

Write a short report where you present the solution to Questions 1-5. Hand-written reports are accepted if they are neatly written.

- Fill in the data sheet uploaded on the course homepage and use as cover page to your report.
- Write your name and personal identity number (or LiU-id) at the top of each page.

- Upload your report as a PDF under the relevant map on the coursepage in Lisam no later than 22 October, 2016.

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Assignment 2

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4 October, 2016

Seismic waves from earthquakes is an important factor to consider when constructing tall buildings in certain regions. These waves may cause resonance phenomena and excessive oscillations which compromises the structural integrity of the building. It is therefore essential to install vibration control devices which counters resonance problems and keep the oscillations within acceptable limits. An example of such a device is to use a pendulum suspended in the upper parts of the building. This relatively simple technique is used in the 509 meter tall Taipei 101 building in Taiwan. Between the 87th and the 92nd floors, a gigantic pendulum consisting of a steel sphere 5.5 meters across and weighing 728 tons, see left panel in Fig. 1. The ball can move up to 1.5 meters in any direction and thereby reducing the amplitude up to 40 percent.

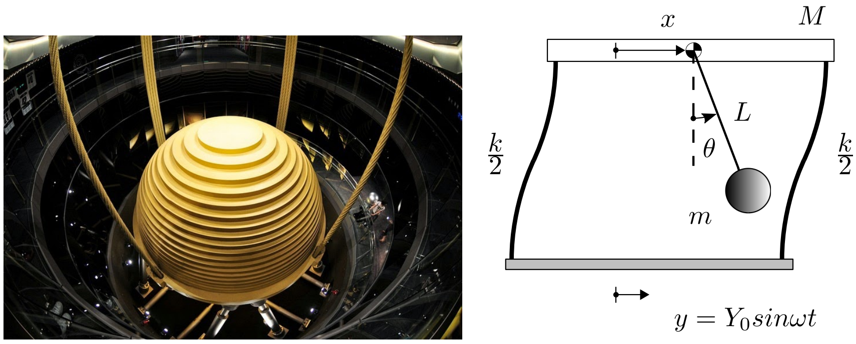

Figure 1: The pendulum vibration absorption system in the Tapei 101 building. Photo taken from `www.flickr.com/photos/rinux/2885419140` (left). An illustration of the simplified pendulum vibration absorber in a two-storey building (right).

Questions (5 points)

In this assignment, you will study the principles of a pendulum vibration absorber for a simplified system consisting of a two-storey building, see right panel in Fig. 1. The upper floor of mass M is supported by two vertical beams of the equivalent stiffness $k/2$. A pendulum of length L and mass m is suspended from the upper floor. The seismic waves causes a sinusoidal base motion with amplitude Y_0 and frequency ω. The motion of the upper floor is given by the coordinate x and the swing angle of the pendulum is θ.

Question 1

Derive the equations of motion for the system above under the assumption of small motions. You may either derive the equations using free-body diagrams together with Newton's second law, or by using Lagrange's equations[1]. Answer on matrix-vector form, i.e. $[\boldsymbol{M}]\{\ddot{\boldsymbol{x}}\} + [\boldsymbol{K}]\{\boldsymbol{x}\} = \{\boldsymbol{F}\}$, where $[\boldsymbol{M}]$ is the mass matrix, $[\boldsymbol{K}]$ is the stiffness matrix, $\{\boldsymbol{x}\} = \{x, \theta\}^T$, and $\{\boldsymbol{F}\}$ is a forcing vector.

Question 2

(a) Compute the magnification factor for the two coordinates. Introduce the natural frequencies $\omega_{\mathrm{M}}^2 = k/M$ and $\omega_{\mathrm{m}}^2 = g/L$, and express the result using the ratios $r = \omega/\omega_{\mathrm{M}}$, $p = \omega_{\mathrm{m}}/\omega_{\mathrm{M}}$ and $\mu = m/M$ together with L. (b) Set $p = 1.0$ and $\mu = 0.01$ and plot the magnification factors for $0 \leq r \leq 2$. (c) Make small variations in p and μ and describe (in words) how the response is affected?

Question 3

Compute the eigenvalues λ_k and eigenvectors $X^{(k)}$ to the dynamical matrix $(\boldsymbol{K}^{-1}\boldsymbol{M})$ using the *matrix iteration method* described in Chapter 7.5 in Rao. Present the iterations in tables similar to the ones used in Chapter 7.5. (Note: The eigenvectors are not orthogonal since the stiffness matrix is non-symmetric.) Data for Question 3 is given in Table 1. The equivalent stiffness for the vertical beams is given by $k = 12EI/L^3$.

Question 4

Derive the uncoupled equations of motion using modal analysis. Use the mass normalized eigenvectors $\boldsymbol{\phi}^{(k)}$ (which satisfy $\boldsymbol{\phi}^{(k)} \cdot \boldsymbol{M}\boldsymbol{\phi}^{(k)} = 1$) and the principal coordinates q_k. Assume the structure to be at rest in its equilibrium position at time $t = 0$.

Reporting

Write a short report where you present the solution to Questions 1-4. Handwritten reports are accepted if they are neatly written.

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[1]Hint: For Lagrange's equations, let the motion of each part k be described by the vector $\{\boldsymbol{x}_k\} = \{x_k, y_k\}^T$. The velocity for part k is then given by $\boldsymbol{v}_k = \dot{\boldsymbol{x}}_k$.

E (GPa)	I (m^4)	L (m)	M (kg)	m (kg)
208	$19.43 \cdot 10^{-6}$	4.0	$20 \cdot 10^3$	200

Table 1: Data for question 3 and 4.

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Assignment 3

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6 October, 2016

Question (5 points)

Determine the free-vibration solution of a string fixed at both ends under the initial conditions $w(x, 0) = w_0 \sin(\pi x/L)$ and $\partial w/\partial t(x, 0) = 0$, where w_0 is a constant. The string deflection is $w(x, t)$ and the distance between the fixed ends is L.

Reporting

Write a short report where you present the solution to the question above. Hand-written reports are accepted if they are neatly written.

- Fill in the data sheet uploaded on the course homepage and use as cover page to your report.
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Hints to exam assignments October 2016

Assignment 1

(Q1) Formulas for the deflection $w(x)$ of a cantilever beam loaded by the force P at the mid-point gives $k_{\mathrm{eq}} = P/w_{\mathrm{G}} = 3EI/L^3$, where $w_{\mathrm{G}} = w(L/2)$. By requiring the kinetic energy to remain the same, we compute the equivalent mass to be $m_{\mathrm{eq}} = 122m/35$.

(Q2) The equation of motion (equivalent system) reads

$$\ddot{w}_{\mathrm{G}} + \omega_{\mathrm{n}}^2 w_{\mathrm{G}} = \frac{\omega_{\mathrm{n}}^2}{k_{\mathrm{eq}}} F(t),$$

where $\omega_{\mathrm{n}}^2 = k_{\mathrm{eq}}/m_{\mathrm{eq}}$.

(Q3) The force is

$$F(t) = \begin{cases} \dfrac{2F_0 t}{T}, & 0 \leq t \leq T/2, \\ 2F_0 \left(1 - \dfrac{t}{T}\right), & T/2 \leq t \leq T. \end{cases}$$

The solution reads

$$w_{\mathrm{G}} = \begin{cases} \dfrac{2F_0}{m_{\mathrm{eq}}\omega_{\mathrm{n}}^3 T} \left(\omega_{\mathrm{n}} t - \sin \omega_{\mathrm{n}} t\right), & t \leq T/2 \\ \dfrac{F_0}{m_{\mathrm{eq}}\omega_{\mathrm{n}}^3 T} \left(4 \sin \omega_{\mathrm{n}} t \cdot \cos \dfrac{\omega_{\mathrm{n}} T}{2} - 4 \cos \omega_{\mathrm{n}} t \cdot \sin \dfrac{\omega_{\mathrm{n}} T}{2} - 2 \sin \omega_{\mathrm{n}} t - 2(t - T)\right), & t \geq T/2 \end{cases}$$

(Q5) Replace the solid rectangular beam by a hollow beam, for example. Wings are usually thin walled structures.

Assignment 2

(Q1) Alternative 1: Lagrange's equation of motion. The kinetic kinetic energy is

$$T = \frac{1}{2}(M + m)\dot{x}^2 + mL \cos\theta \dot{x}\dot{\theta} + \frac{m}{2} L^2 \dot{\theta}^2.$$

and the potential energy is given by

$$V = \frac{1}{2} k(x - y)^2 + mgL(1 - \cos\theta).$$

Assume small angles such that $\sin\theta \approx \theta$, $\cos\theta \approx 1$, and $\dot{\theta}^2 \approx 0$. Use Lagrange's equation to derive the linearised equation system

$$\begin{bmatrix} M+m & mL \\ mL & mL^2 \end{bmatrix} \begin{pmatrix} \ddot{x} \\ \ddot{\theta} \end{pmatrix} + \begin{bmatrix} k & 0 \\ 0 & mgL \end{bmatrix} \begin{pmatrix} x \\ \theta \end{pmatrix} = \begin{pmatrix} kY_0 \sin\omega t \\ 0 \end{pmatrix}$$

Alternative 2: Newton's method. Draw separate free-body diagrams for the upper floor and for the mass. The equation of motion for the upper floor reads

$$\rightarrow: \quad T\sin\theta - k(x-y) = M\ddot{x}.$$

The equations of motion for the mass are

$$\nearrow: \quad -mg\sin\theta = m(\ddot{x}\cos\theta + L\ddot{\theta}),$$

and

$$\nwarrow: \quad T - mg\cos\theta = m(-\ddot{x}\sin\theta + L\dot{\theta}^2).$$

Finally, eliminate T between the $\rightarrow$ and $\nwarrow$ equations.

(Q2) Use the ansatz $\boldsymbol{x} = \boldsymbol{X}e^{i\omega t}$ and substitute in the equation of motion. Premultiply by $\boldsymbol{K}^{-1}$ gives after a term rearrangement

$$\boldsymbol{X} = \left(-\omega^2 \boldsymbol{K}^{-1}\boldsymbol{M} + \boldsymbol{I}\right)^{-1} \boldsymbol{K}^{-1}\boldsymbol{F}.$$

Using this particular solution and the given ratios, we can compute (best using Maple)

$$\boldsymbol{X} = \frac{-Y_0}{r^4 - (1 + p^2(1+\mu))\, r^2 + p^2} \begin{pmatrix} r^2 - p^2 \\ r^2/L \end{pmatrix}.$$

(Q3) Matrix iteration is not included in the course this year.

(Q4) Using the method in Window 4.5 in Inman, the modal equations become

$$\ddot{r}_1 + 2.4508 r_1 = 37.0557 Y_0 \sin\Omega t, \quad \ddot{r}_2 + 37.9147 r_1 = 5358.1149 Y_0 \sin\Omega t.$$

Assignment 3

Separation of variables and the fixed-fixed conditions give $\omega_{\mathrm{n}} = c\pi n/l$. Thus,

$$w(x,t) = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi x}{l}\right) \left(C_n \cos\left(\frac{c\pi n}{l} t\right) + D_n \sin\left(\frac{c\pi n}{l} t\right)\right)$$

From the initial condition, $D_n = 0$ and

$$\sum_{k=1}^{\infty} \sin\left(\frac{n\pi x}{l}\right) C_k = w_0 \sin\left(\frac{\pi x}{l}\right).$$

Multiply both sides by $\sin(\pi x/l)$ and integrate from 0 to l. Orthogonality of the sin function gives $C_n = w_0$ when $n = 1$ and $C_n = 0$ otherwise.