TSKS01 DIGITAL COMMUNICATION

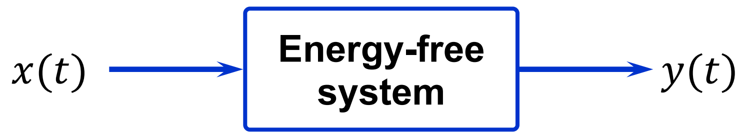
Repetition and Examples

SIGNALS AND SYSTEMS

Signals and Systems

Signals: **Voltages, currents, or other measurements**

Systems: **Manipulate/filter signals**



Energy-free system: No transients, constant input → constant output

---

Complex exponential: $e^{j2\pi ft} = \cos(2\pi ft) + j\sin(2\pi ft)$

Unit step: $u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$

Unit impulse: $\delta(t)$: $\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)$

Linear Systems

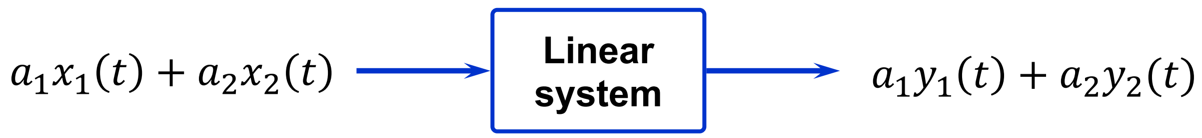
Definition: Let $x_1(t)$ and $x_2(t)$ be input signals to a one-input energy-free system, and let $y_1(t)$ and $y_2(t)$ be the corresponding output signals. If

$$y(t) = a_1 y_1(t) + a_2 y_2(t)$$

is the output corresponding to the input

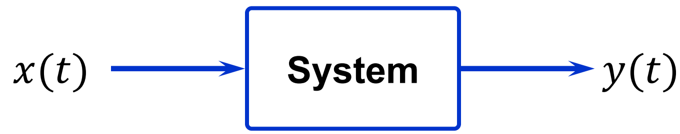
$$x(t) = a_1 x_1(t) + a_2 x_2(t),$$

for any $x_1(t)$, $x_2(t)$, a_1, and a_2, then the system is referred to as *linear*. A system that is not linear is referred to as *non-linear*.



Linear Systems – Example 1

Is the system $y(t) = x(t-1)$ linear?



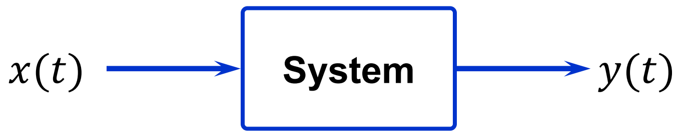
- Consider $x(t) = a_1 x_1(t) + a_2 x_2(t)$

- Then: $y(t) = a_1 x_1(t-1) + a_2 x_2(t-1) = a_1 y_1(t) + a_2 y_2(t)$

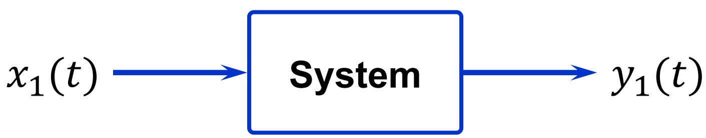
Linear combination of outputs of $x_1(t)$ and $x_2(t)$: ***Linear system***

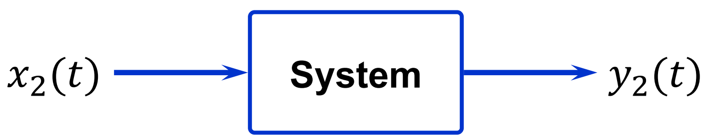
Linear Systems – Example 2

Is the system $y(t) = \left(x(t)\right)^2$ linear?



- Consider $x(t) = a_1 x_1(t) + a_2 x_2(t)$





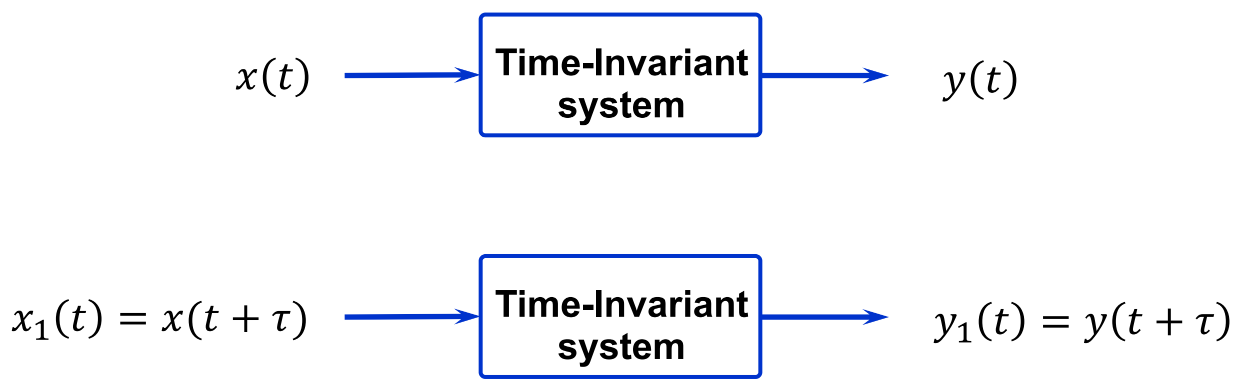
- Then: $y(t) = (a_1 x_1(t) + a_2 x_2(t))^2$
  $= a_1^2 x_1^2(t) + a_2^2 x_2^2(t) + 2a_1 a_2 x_1(t) x_2(t)$
- Different from $a_1 y_1(t) + a_2 y_2(t) = a_1 x_1^2(t) + a_2 x_2^2(t)$

Not a linear combination of outputs of $x_1(t)$ and $x_2(t)$: ***Non-linear***

Time-Invariant System

Time-invariant system

Definition: Let $x(t)$ be the input to an energy-free system and let $y(t)$ be the corresponding output. If $y(t+\tau)$ is the output for the input $x(t+\tau)$ for any $x(t)$, t, and τ, then the system is referred to as *time-invariant*. Otherwise the system is referred to as *time-varying*.



Time-Invariant System - Example

Is the system $y(t) = x(t-1)$ time-invariant?

- True output: $y_1(\mathrm{t}) = x_1(t-1) = x(t+\tau-1)$
- Output if time-invariant: $y(t+\tau) = x(t+\tau-1)$

- Since $y_1(t) = y(t+\tau)$: **Time-invariant**

Is the system $y(t) = \cos(2\pi f t)x(t)$ time-invariant?

- True output: $y_1(t) = \cos(2\pi f t)\, x_1(t) = \cos(2\pi f t)\, x(t+\tau)$
- Output if time-invariant: $y(t+\tau) = \cos(2\pi f(t+\tau))x(t+\tau)$

- Since $y_1(t) \neq y(t+\tau)$: **Time-varying**

Linear Time-Invariant (LTI) Systems

LTI system

Definition: A system that is both linear and time-invariant is referred to as a *linear time-invariant (LTI) system*.

Example: $y(t) = x(t-1)$ is an *LTI system*

Property: The input and output of LTI systems are related as