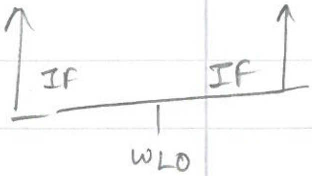
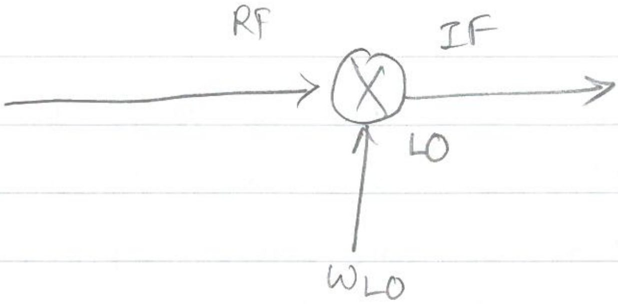
So what we have, is a received signal with both the sidebands.

3.1





This has to be down converted now to IF on the receiver side.

$$\Rightarrow V_{IF} = V_{rf}(t) \times V_{LO}(t)$$

$$V_{IF} = V_{RF}\left[\cos(\omega_{LO} - \omega_{IF})t + \cos(\omega_{LO} + \omega_{IF})t\right] \cdot V_{LO} \cdot \cos\omega_{LO}t$$

$$\left[\cos A * \cos B\right] = \tfrac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

So simplifying this; and cos(-x) = cos(x)

$$V_{IF} = \frac{V_{RF}\cdot V_{LO}}{xxx}\left[\cos(\omega_{LO})t \cdot \cos(\omega_{LO} - \omega_{IF})t + \cos(\omega_{LO})t \cdot \cos(\omega_{LO} + \omega_{IF})t\right]$$

$$= \frac{V_{RF}\cdot V_{LO}}{2}\left[\underline{\cos(\omega_{IF})t} + \cos(2\omega_{LO} - \omega_{IF})t + \underline{\cos(-\omega_{IF})t} + \cos(2\omega_{LO} + \omega_{IF})t\right]$$

$$= \frac{V_{RF}\cdot V_{LO}}{2}\left[\underbrace{2\cos(\omega_{IF})t}_{\text{frequency of interest}} + \underbrace{\cos(2\omega_{LO} - \omega_{IF})t}_{\text{pretty far away}} + \underbrace{\cos(2\omega_{LO} + \omega_{IF})t}_{\text{pretty far away}}\right]$$

$$= V_{RF}V_{LO}(\cos\omega_{IF})t$$

We see that both the sidebands mix to the same frequency i.e. IF.

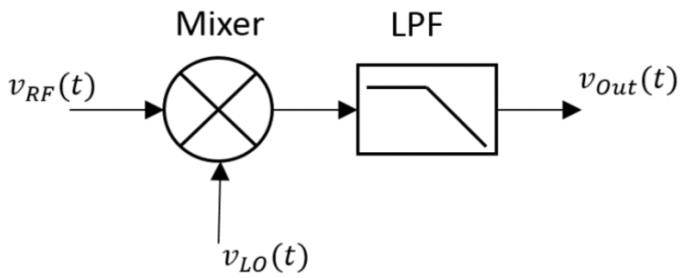
Note: This is a very important result and shows the problem of image frequency;

So, what this tells us is that on either side of LO we have two tones that are downconverted to the same IF. Hence, it is necessary to filter out the image frequency.

Data from another RF channel can come into IF and corrupt the data.

3.1 A double-sideband signal of the form $v_{RF}(t) = V_{RF}[\cos(\omega_{LO} - \omega_{IF})t + \cos(\omega_{LO} + \omega_{IF})t]$ is applied to a mixer with an LO (local oscillator) voltage given by $V_{LO}\cos\omega_{LO}t$. Derive the output of the mixer after low pass filtering.

Solution:



The resultant waveform after the mixing operation corresponds to:

$$v_{RF}(t)\cdot v_{LO}(t) = V_{RF}[\cos(\omega_{LO} - \omega_{IF})t + \cos(\omega_{LO} + \omega_{IF})t]\cdot V_{LO}\cos\omega_{LO}t$$

$$= V_{RF}V_{LO}\cos(\omega_{LO} - \omega_{IF})t\cos\omega_{LO}t + V_{RF}V_{LO}\cos(\omega_{LO} + \omega_{IF})t\cos\omega_{LO}t$$

$$= \frac{V_{RF}V_{LO}}{2}\left[\cos(-\omega_{IF})t + \underbrace{\cos(2\omega_{LO} - \omega_{IF})t}_{high\ freq\ comp} + \cos(\omega_{IF})t + \underbrace{\cos(2\omega_{LO} + \omega_{IF})t}_{high\ freq\ comp}\right]$$

After the LPF, $v_{Out}(t)$ equals to:

$$v_{Out}(t) = \frac{V_{RF}V_{LO}}{2}[\cos(-\omega_{IF})t + \cos(\omega_{IF})t]$$

$$= \frac{V_{RF}V_{LO}}{2}\left[\underbrace{\cos(\omega_{IF})t}_{image} + \cos(\omega_{IF})t\right]$$

Notice that after the LPF, the high-frequency components are filtered out, remaining only two tones located at ω_{IF}. One of the tones corresponds to the image and interferes with the other tone.

3.2 Recommend design parameters for an AM standard broadcast receiver that is to operate with an IF frequency f_{IF} of 455 kHz:

a. Calculate the required LO frequency f_{LO} when the receiver is tuned to a carrier frequency $f_C = 540$ kHz, assuming that the frequency of the LO is <u>above</u> the frequency of the received signal.

b. Calculate the required LO frequency f_{LO} when the receiver is tuned to carrier frequency $f_C = 540$ kHz, assuming that the frequency of the LO is <u>below</u> the frequency of the received signal.

NB. In "Lab 2 measurements", you will measure this on such an AM reciever.

Answer: (a) $f_{LO} = 995$ kHz, (b) $f_{LO} = 85$kHz.

Solution:

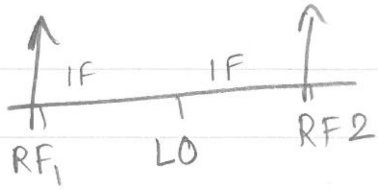
The IF is the difference between the carrier frequency and the local oscillator frequencies:

a. $f_{LO} > f_C$: $f_{IF} = f_{LO} - f_C \Rightarrow f_{LO} = 995$ kHz

b. $f_{LO} < f_C$: $f_{IF} = f_{LO} - f_C \Rightarrow f_{LO} = 85$ kHz

Both combinations of LO will work. Which one to implement is up to the designer and practical limitations of the designed circuit.

3.3



RF = 600 MHz
IF = 80 MHz

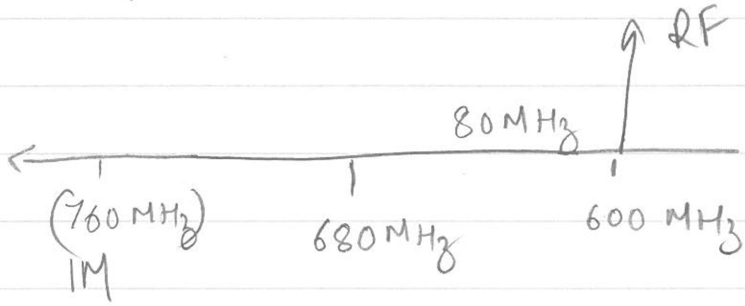
We know from the previous problem that in order to achieve an IF of IF, we have two options.

<u>Option 1</u>

$$f_{LO} - f_{RF_2} = f_{IF}$$

or $$f_{LO} = f_{RF_2} + f_{IF}$$

With $f_{RF} = 600$ MHz, $f_{IF} = 80$ MHz
$f_{LO} = 680$ MHz;

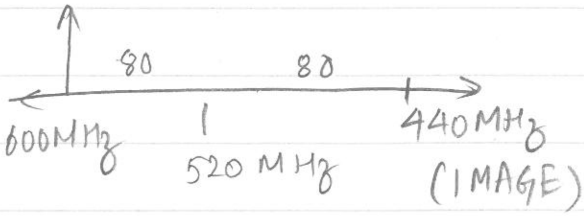


$\Rightarrow$ Image frequency is 760 MHz.

<u>Similarly,</u>

$$f_{RF_1} - f_{LO} = f_{IF}$$

$$f_{LO} = f_{RF_1} - f_{IF} \Rightarrow f_{LO} = 600 - 80 = \underline{\underline{520 \text{ MHz}}}$$

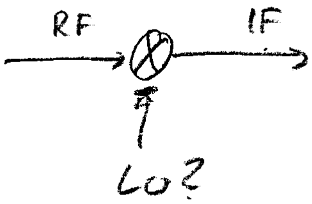


$\Rightarrow$ Image frequency is $\underline{\underline{440 \text{ MHz}}}$

Note: In summary, prior to the mixer, making sure that the image frequency is clean

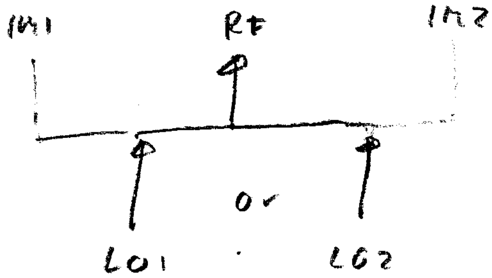
3.3

Alternative



RF = 600 MHz

IF = 80 MHz



Option 1) $LO_1 = f_{RF} - f_{IF} = 600 - 80 = 520$ MHz

$IM1 = f_R - 2 \cdot f_{IF} = 600 - 160 = 440$ MHz

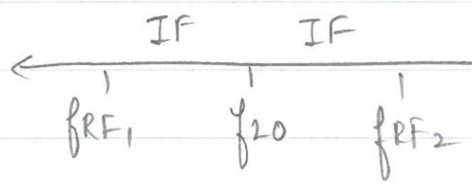
2) $LO_2 = f_{RF} + f_{IF} = 600 + 80 = 680$ MHz

$IM2 = f_R + 2 \cdot f_{IF} = 600 + 160 = 760$ MHz

3.4

This problem is similar to (xxxxxx) 2.8

f_{RF}

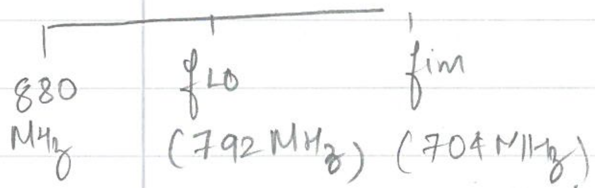


So, in case 1; we have

$f_{LO} = f_{RF} - f_{IF} = 880 - 88 = 792$ MHz.

This implies $f_{image} = 792$ MHz $- 88$

$= \underline{\underline{704 \text{ MHz}}}$



Similarly, solving the other f_{LO}.

$f_{LO} = f_{RF_2} + f_{IF}$

$= 880 + 88 = 968$ MHz.

$\Rightarrow f_{image} = 968 + 88 = \underline{\underline{1056 \text{ MHz}}}$

Three ways to mitigate the reception of an image signal.

a) Channel filter to filter out the image frequency.

b) Increase IF such that the image is far away. Easier to filter and will be easily rejected.

c) Use a zero IF. architecture; this removes the image altogether.

So what happens.

when $f_{IF} = 0$ and $f_{RF} = f_{LO}$

The image is the received signal band itself. So the image rejection is solved.
Of course, symmetric modulation or phase seperated baseband signals are needed.
However, it has its own issues.

d) Lastly, use an image reject receiver if a LOW-IF is needed.

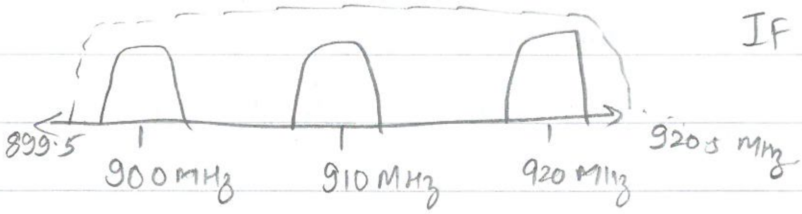
e) Dual conversion i.e. using two IFs

High IF good image suppression but low channel selectivity

Low IF good channel selectivity but difficult image rejection.

3.5

Similar to the previous two problems we have the following frequency plan



IF = 10 MHz.

To find out if the receiver will pick up any image frequencies, check the LOs and the possible image frequencies.

Case.	RF.	LO	Image
1	900 MHz	910 MHz	920 MHz
2	900 MHz	890 MHz	880 MHz
3	910 MHz	900 MHz	890 MHz
4	910 MHz	920 MHz	930 MHz
5	920 MHz	910 MHz	900 MHz
6	920 MHz	930 MHz	940 MHz

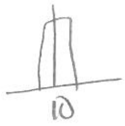
In case 1, with lower sideband mixing image is INBAND

In case 6, with upper sideband mixing, image is also inband

Now, the IF frequencies can be generalized by

$$f_{IF} = \pm m f_{RF} \pm n f_{LO}$$

When we have no harmonics, $f_{IF} = \pm f_{RF} \pm f_{LO}$.



Since channel BW is 1 MHz, the channel is from 9.5 - 10.5 MHz.
The IF from the spurs should not fall into this region.

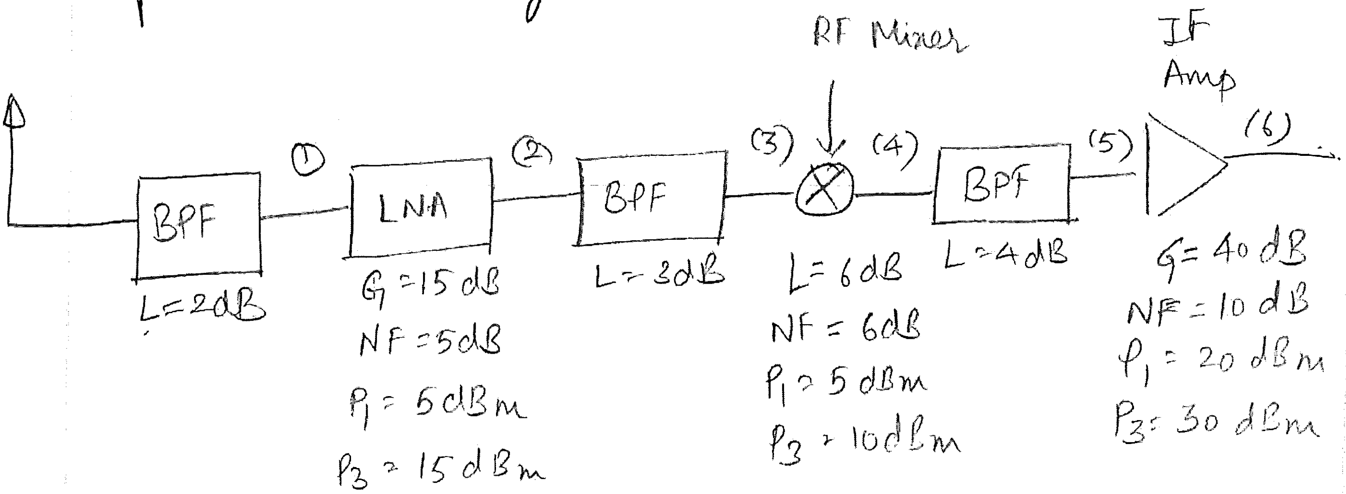
$\pm m\,RF \pm n\,LO$.
Substitute values of m and n to verify this.

In this case spurs do not fall into the IF band.

3.6

Tutorial - 3

This problem is again a cascaded system. We need to calculate its NF and use to calculate the input and sensitivity.



a)

Writing this as follows.

BPF 1	$G_1 = -2dB = .63$	$NF_1 = 2dB = 1.58$
LNA	$G_2 = 15dB = 31.62$	$NF_2 = 5dB = 3.16$
BPF 2	$G_3 = -3dB = .5$	$NF_3 = 3dB = 2$
Mixer	$G_4 = -6dB = .25$	$NF_4 = 6dB = 4$
BPF 3	$G_5 = -4dB = .4$	$NF_5 = 4dB = 2.5$
IF amp	$G_6 = 40dB = 10^4$	$NF_6 = 10\ dB = 10$

We know from the previous tutorial that

$$NF = NF_1 + \frac{NF_2 - 1}{G_1} + \frac{NF_3 - 1}{G_1 G_2} \cdots \frac{NF_6 - 1}{G_1 G_3 G_4 G_5}$$

$$= 1.58 + \frac{2.16}{(0.63)} + \frac{1}{(31.62)(0.63)} + \frac{3}{(0.63)(31.62)(0.5)} + \frac{1.5}{(0.63)(31.62)(0.5)(0.25)}$$

$$+ \frac{9}{(0.63)(31.62)(0.5)(0.25)(0.4)}$$

$$NF = 1.58 + 3.43 + \frac{3}{+1/20\ 10} + \frac{1.5}{2.5} + \frac{9}{1} \quad \text{approx}$$

$$= 14.91 \simeq 15 = \underline{\underline{11.8\ dB}}$$

14.9618 11.7498

Now, output SNR required = 12 dB
Assuming T = 300K, as before from tutorial 2

b) $P_{sen} = -174\ dBm/H_z + NF + 10\log(B) + SNR$

$$= \quad + 11.8 + 10\log(50e^3) + 12$$

$$= -103.2\ dBm = \boxed{47.86 \times 10^{-15}}\ W$$

c) $P = \frac{V_{rms}^2}{R} \Rightarrow V = \sqrt{PR}$

$$= \sqrt{(47.86 \times 10^{-15})(50)}$$

$$V_{rms} = \boxed{1.55\mu}\ V$$

If V_{peak}, then $V_{rms}\sqrt{2} = \underline{\underline{2.19\ \mu V}}$

(d) Now consider an input power P_1.
The power at output of the amps and the input of the mixers is needed i.e. at (2), (3), (6).

$\Rightarrow$

The power at point 2 is $P - 2 + 15 = P + 13$ dB
(3) $P - 2 + 15 - 3 \Rightarrow P + 10$
(6) $P + 40$ dB.

When $P_1 = -90$ dBm

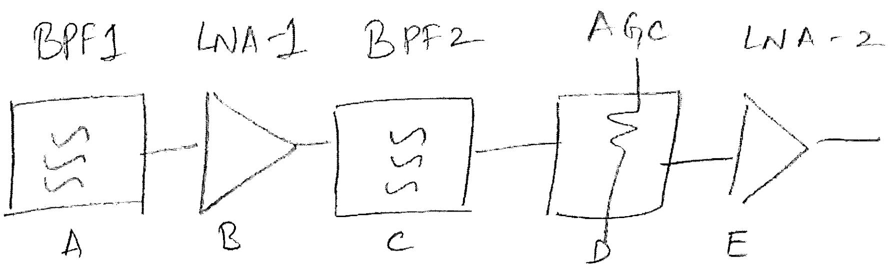
$P_{(2)} = -77$ dBm
$P_{(3)} = -80$ dBm
$P_{(6)} = -50$ dBm

When $P_1 = -30$ dBm

$P_{(2)} = -17$ dBm
$P_{(3)} = -20$ dBm
$P_{(6)} = 10$ dBm.

In both cases, P_{1dB} and P_3 are not violated for these components. So we are safe.

3.7



	G	N	OIP3 [W]
BPF (A)	.4	2.5	10^7
LNA-1 (B)	12.6	1.6	5×10^{-3}
BPF (C)	.63	1.6	10^7
AGC (D)	.8	1.25	3.162
LNA-2 (E)	12.6	1.6	5×10^{-3}

$$\frac{1}{OIP_{3\,TOT}} = \frac{1}{G_B G_C G_D G_E} \times \frac{1}{OIP_{3A}} + \frac{1}{G_C G_D G_E\, OIP_{3B}} + \frac{1}{G_D G_E\, OIP_{3C}} + \frac{1}{G_E\, OIP_{3D}} + \frac{1}{OIP_{3E}}$$

$\Rightarrow$ $OIP_{3\,tot} = 6.36$ dBm

NF = 6.4 dB

G = 15 dB.

Moving E to after D,

NF = 6.142 dB

OIP_3 = 3.67 dBm

Although the linearity of the last two components is high, their low gain still reduces the overall linearity.

Quite a simple substitution problem.

3.8

A repeat type cascaded linearity and noise figure calculation. Nothing special.
Students should solve this on their own.

3.9 Derive all the necessary equations before this problem.

The Receivers characteristics have been given.

$NF = 6\,dB$, $G = 30\ dB$.

$P_{1dB(output)} = 21\ dBm \Rightarrow P_{1dB(input)} = -9\ dBm$

$P_{OIP_3(output)} = 33\ dBm \Rightarrow P_{IIP_3(input)} = 3\ dBm$

$SNR_{output} = 8\ dB$, $BW = 20\ MHz$.

a) Dynamic Range Linear. (not called explicitly linear in the course book)

We know from (2.4.9), the dynamic range linear. takes into account the minimum limitation from Psen. and the upper limitation from P_{1dB}.

∴ We know, $P_{sen} = -174\ dBm/Hz + 10\log B + NF + SNR_{min}$

$$\Rightarrow P_{sen} = -174 + 10\log(20\times10^6) + 6 + 8$$
$$= -86.99 \simeq -87\ dBm \quad -(1)$$

∴ P_{1dB} (referred to input) $= -9\ dBm$

$$\Rightarrow DR_{linear} = -9 - P_{sen}$$
$$= -9 + 87 = \underline{\underline{78\ dB}}$$

b) SFDR, this is limited by the third order intercept products. (2.4.2)

$$SFDR = \frac{2\,(P_{IIP_3} + 174\ dBm - NF - 10\log B)}{3} - SNR_{min}$$

$$= \underline{\underline{57.32\ dB}}$$

An important <u>conclusion</u>

SFDR can be more restrictive that the <u>dynamic</u> <u>range linear</u>

3.10 A cell phone receiver operates at room temperature (T = 20 °C) and has the following specifications: noise figure NF = 20 dB, bandwidth B = 1 MHz, signal-to-noise-ratio SNR = 0 dB. Also, the non-linearity of the receiver circuit can be described by the following output signal $y(x)$ function: $y(x) = 2x - 0.267x^3$. The input signal is x = 0.1 sin ωt V

Calculate:

a. the 1 dB compression point.

b. the input intercept point IIP3.

c. receiver sensitivity.

d. dynamic range.

Answers: a. 10 dBm (input-referred), b. 20 dBm, c. -94 dBm (using T=27 °C), d. 104.4 dBm.

Solution:

a. A quantitative measure of an amplifier's non-linearity is its 1 dB compression point, which can be calculated directly from the non-linear transfer function:

$y(x) = 2x - 0.267x^3 = \alpha_1 x + \alpha_3 x^3 \Rightarrow \alpha_1 = 2, \alpha_3 = -0.267.$

From Lecture 5, slide 26:

$$20\log\left|\alpha_1 + \frac{3}{4}\alpha_3 A_{in,1dB}^2\right| = 20\log|\alpha_1| - 1 \text{ dB}.$$

$$A_{in,1dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}.$$

$A_{in,1dB}$ is the V_p of the input voltage in volts (Razavi, p.18)

A_{in}= sqrt (0.145 * 2 / 0.267) = 1.042 V_p

If the source of the signal is assumed to be 50 Ω, then the power is $P_{in,db} = V_p^2/(2*50)$ = 10.9 mW = 10.4 dBm.

b. Similarly, IIP3 can be calculated (lecture 5, slide 43):

$$A_{IIP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}.$$

=> A = 3.16 V => IIP3 = 20 dBm.

"Sanity check": I P_{1dB} + 9.6 dB = IIP3. Yes!

c. Sensitivity can be calculated from the Eq. 14 in the Appendix. Note that the formula is given for T=300 K (27 °C), but let us simplify things a little bit and assume operation at this temperature instead of T = 18 °C. The difference is the "174 dBm/Hz" term, but it does not change so much.

$P_{in(min)}$ = Sensitivity = -174 dBm/Hz + 10log(B) + NF + SNR (in dB).

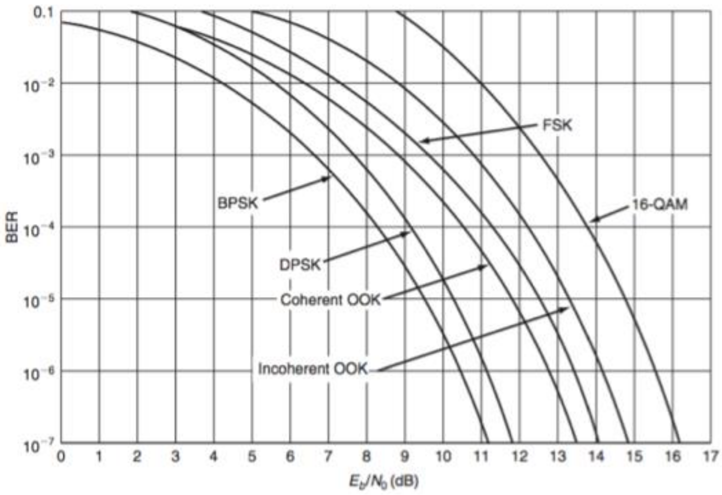
B = 1E6, NF = 20 dB, SNR = 0 => Sensitivity = -94 dBm.

c. DR, eq. 15: DR = P_{1dB} (input) – Psens = 10.4 dBm – (-94 dBm) = 104.4 dB.

3.11 A transceiver is designed for the following system characteristics: Access method is half duplex (TDD). Modulation is QPSK with raised cosine shaping, roll-off factor = 0.3. Channel BW=1.6 MHz. Data rate = 1200 kb/s. BER <1E-5@sensitity of -96 dBm.

What is the maximum acceptable noise figure for the reciever (called “reference noise figure”)?

The relation between BER and SBR is given below.



Answer: 7.2 dB

Solution:

Start by calculating the minimum SNR for the receiver.

From Lecture 8, slide 50 (2018):

$$SNR = \frac{Signal\ Power}{Noise\ Power} = \frac{E_b}{N_0}\frac{R_b}{B}$$

From the plot, we estimate for QPSK modulation and BER = 1E-5, a value for E_b/N_0 of 10.5 dB.

R_b = 1200E3.

B: channel BW = 1.6 MHz. This is the maximum BW we can use, so the BW after the pulse shaping must not be larger than this.

Let us check! Equation 2 (in the appendix) gives the BW. T_b = 1/1200E3, roll-off factor = 0.3 = α. => BW = 1.56 MHz ≈ 1.6 MHz.

If we write the SNR formula in dB, we get SNR [dB] = E_b/N_0 [dB] + 10*log (1.2E6 / 1.6E6) ≈ 10.5 dB – 1.25 dB = 8.75 dB.

Continue by calculating the total noise figure for the receiver.

We use equation 7 (appendix) and rearrange it:

$NF = P_{in} + 174 - 10*\log(BW) - SNR_{min}$

P_{in} is the minimum input recevied signal, i.e. the sensitivity S_{min}, which is given as -96 dBm.

NF = -96 + 174 – 10*log (1.6E6) – 8.75 = 7.2 dB.

To this number, the losses of a switch (1 dB for TDD) should be accounted for as well as some additional margin (assume 1 dB), making the NF of the receiver in the order of 5 dB.