TSKS02 Telecommunication

Lecture 4

Analog Modulation Methods

– AM, PM, FM, Pre-emphasized FM

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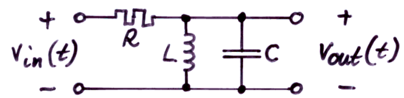
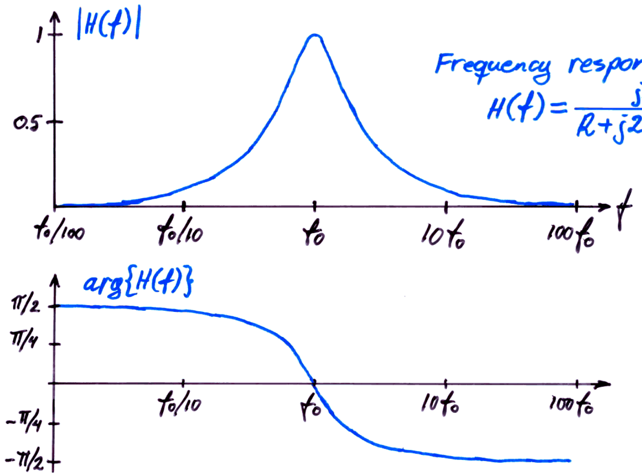
A Simple BP Filter



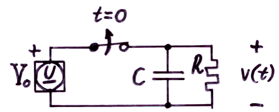


Frequency response:

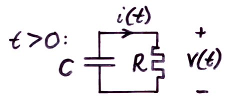
$$H(f) = \frac{j2\pi fL}{R + j2\pi fL - 4\pi^2 f^2 RLC}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Discharging a Capacitor



Initial condition $v(0) = V_0$



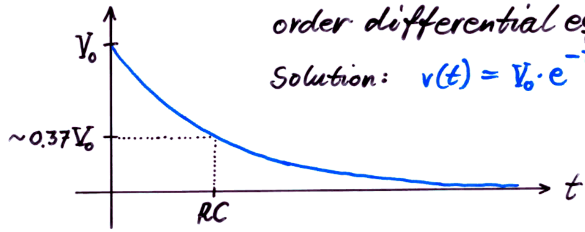
$$v(t) = R \cdot i(t) = -RC\frac{d}{dt}v(t)$$

$$i(t) = -C\frac{d}{dt}v(t)$$

$$\Rightarrow RC\frac{d}{dt}v(t) + v(t) = 0$$

This is the standard example of a first order differential equation.

Solution: $v(t) = V_0 \cdot e^{-t/RC}$ for $t \geq 0$



Amplitude Modulation

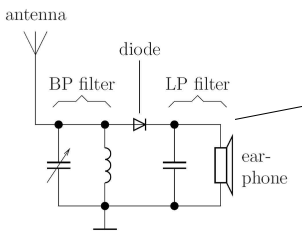
- The first technique used for radio broadcasts.
- A linear modulation technique.
- Simple to analyze.
- Simple demodulation.
- Noise sensitive.

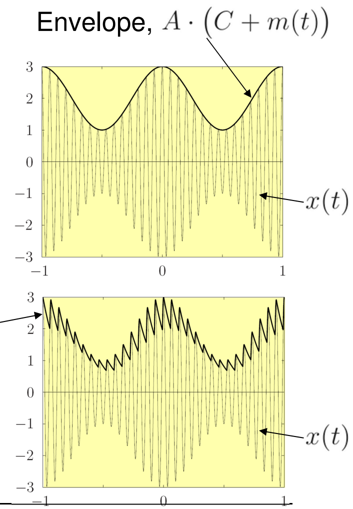
Amplitude Modulation

Standard AM:

$$x(t) = A \cdot \big(C + m(t)\big) \cos(2\pi f_c t)$$

Crystal receiver, an envelope detector, first demodulator of standard AM:



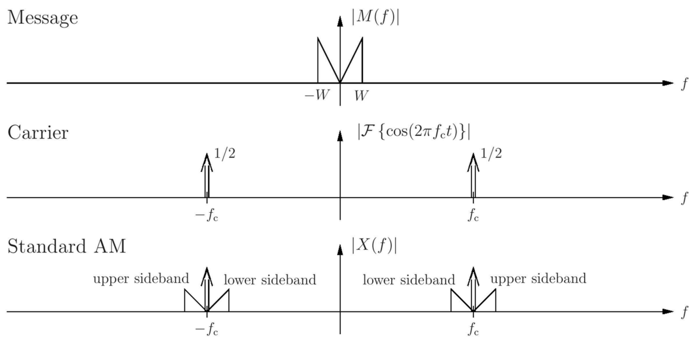


Spectrum of Standard AM

$$x(t) = A \cdot \big(C + m(t)\big) \cos(2\pi f_c t)$$

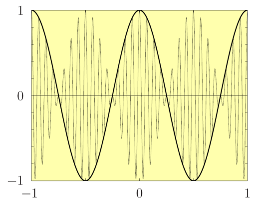
$$X(f) = \mathcal{F}\{AC\cos(2\pi f_c t)\} + \mathcal{F}\{A\,m(t)\cos(2\pi f_c t)\}$$

$$= \frac{AC}{2}\big(\delta(f - f_c) + \delta(f + f_c)\big) + \frac{A}{2}\big(M(f - f_c) + M(f + f_c)\big),$$



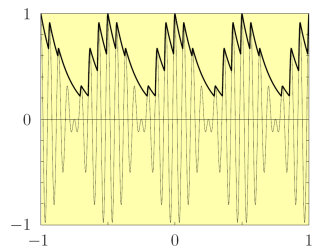
AM-SC – Suppressed Carrier

AM-SC: $x(t) = A\,m(t)\cos(2\pi f_c t)$

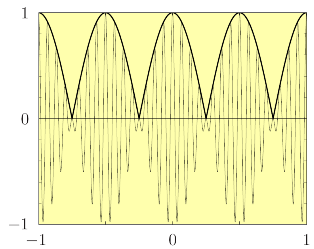


Demodulating AM-SC with an envelope detector:

Crystal receiver output



Envelope



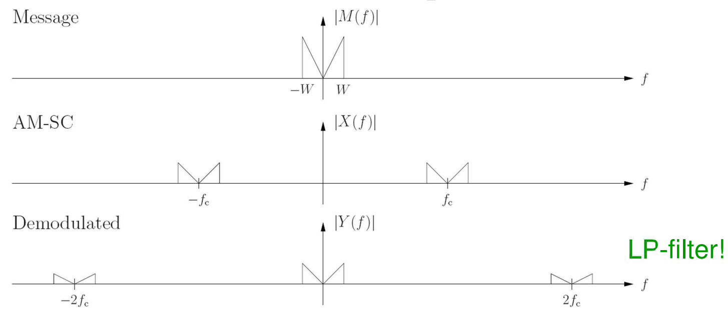
Spectrum of AM-SC – and Demodulation

$$x(t) = A\,m(t)\cos(2\pi f_c t) \quad \Rightarrow \quad X(f) = \frac{A}{2}\big(M(f - f_c) + M(f + f_c)\big)$$

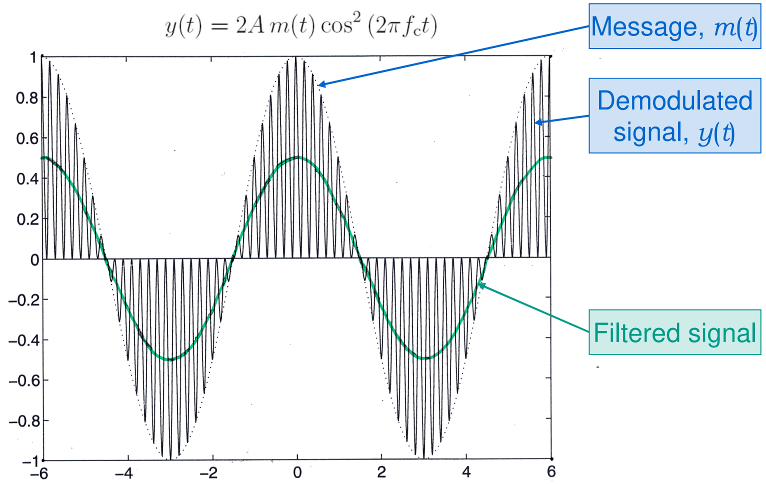
Coherent demodulation:

$$y(t) = 2\,x(t)\cos(2\pi f_c t) = 2A\,m(t)\cos^2(2\pi f_c t) = A\,m(t)\big(1 + \cos(4\pi f_c t)\big),$$

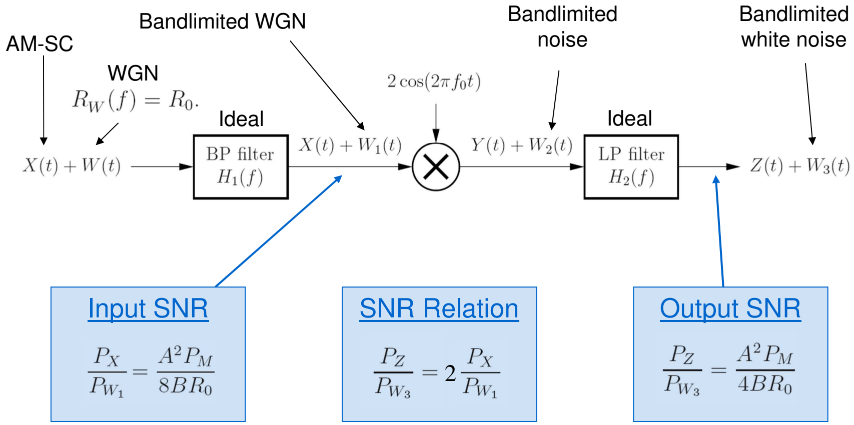
$$Y(f) = A\big(X(f - f_c) + X(f + f_c)\big) = A \cdot M(f) + \frac{A}{2}\big(M(f - 2f_c) + M(f + 2f_c)\big).$$



Coherent Demodulation of AM-SC in the Time Domain



Noise Impact on AM-SC



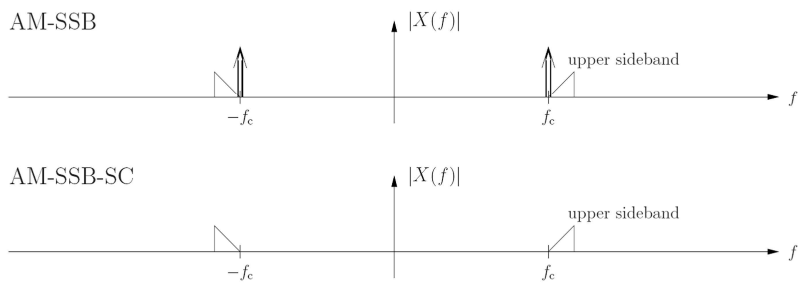
Twice the bandwidth – Twice the SNR.

AM-SSB – Single Side-Band

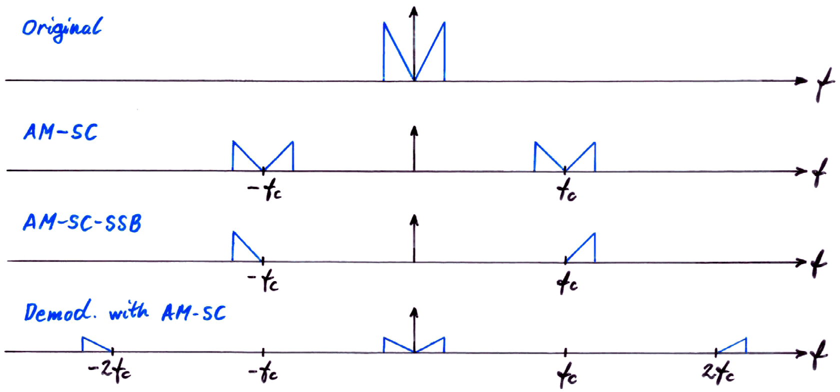
AM (SC) uses twice as much bandwidth as needed.

Each sideband contains all the information.

Filter out one of the sidebands.



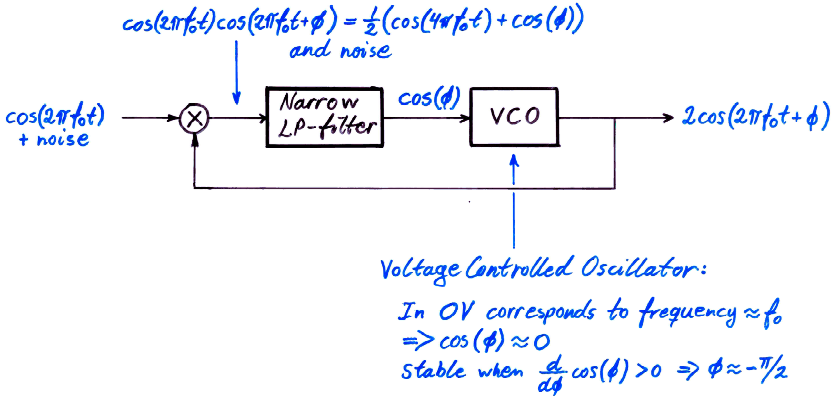
Demodulate SSB



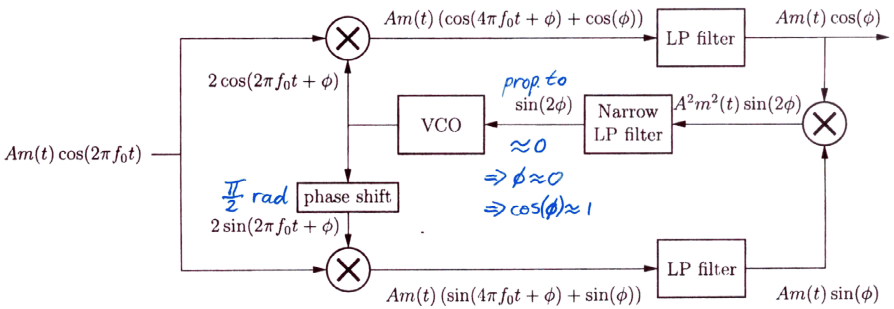
LP-filter again!

Phased-Locked Loop

An automatic control system



Costas Loop



Angle Modulation

- The major modulation techniques used in radio broadcasts today are examples of angle modulation.
  - FM – Frequency Modulation
  - PM – Phase Modulation
- Nonlinear modulation techniques.
- Complicated to analyze.
- Still fairly simple demodulation.
- Less sensitive to noise than AM.

Angle Modulation – Modulation Indices

Angle modulation: $x(t) = A \cdot \cos\left(2\pi f_c t + \underbrace{\phi\{m(t)\}}_{\text{Momentary phase}}\right)$

Phase deviation: $\phi_d(t) = \phi\{m(t)\}$, for mean 0.

Phase modulation index: $\mu_p = \phi_{d,\max} = \max|\phi_d(t)|$

Momentary frequency: $f_{\text{mom}}(t) = \frac{1}{2\pi} \cdot \frac{d}{dt}\left(2\pi f_c t + \phi\{m(t)\}\right) = f_c + \frac{1}{2\pi} \cdot \frac{d}{dt}\phi\{m(t)\}$

Frequency deviation: $f_d(t) = f_{\text{mom}}(t) - f_c = \frac{1}{2\pi} \cdot \frac{d}{dt}\phi\{m(t)\}$

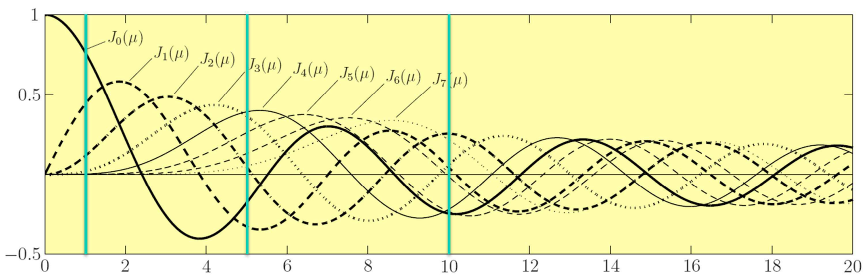
Frequency modulation idx: $\mu_f = \frac{f_{d,\max}}{B}$ $\qquad f_{d,\max} = \max|f_d(t)|$

Spectrum of Angle Modulation 1(2)

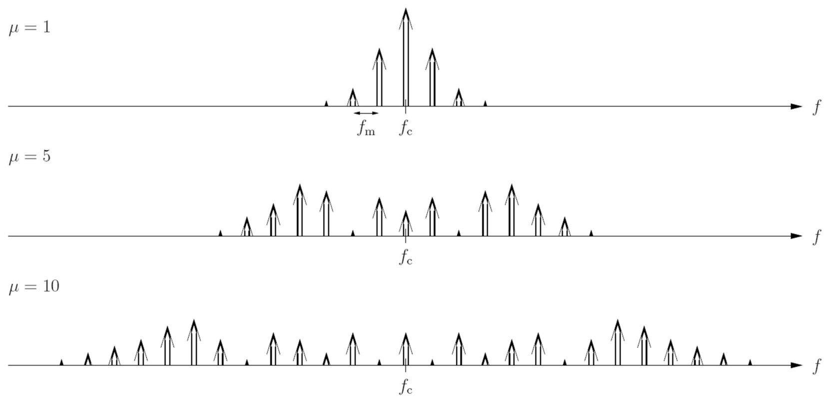
Example: $x(t) = A \cdot \cos\left(2\pi f_c t + \mu \sin(2\pi f_m t)\right) = \sum_{n=-\infty}^{\infty} A \cdot J_n(\mu) \cos\left(2\pi (f_c + n f_m) t\right)$

Spectrum: $X(f) = \sum_{n=-\infty}^{\infty} \frac{A \cdot J_n(\mu)}{2} \Big(\delta(f + f_c + n f_m) + \delta(f - f_c - n f_m) \Big).$

Bessel functions of the first kind of order n: $J_n(\mu) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(n+k)!} \left(\frac{\mu}{2}\right)^{n+2k}$



Spectrum of Angle Modulation 2(2)



Carsons rule: Bandwidth ≈ $2(\mu+1) f_m = 2\left(1 + \frac{1}{\mu}\right) f_{d,max}.$

PM – Phase Modulation

Momentary phase: $\phi\{m(t)\} = a \cdot m(t),$

Signal: $x(t) = A \cdot \cos\left(2\pi f_c t + a \cdot m(t)\right).$

Momentary frequency: $f_{mom}(t) = \frac{1}{2\pi} \cdot \frac{d}{dt}\left(2\pi f_c t + a \cdot m(t)\right) = f_c + \frac{a}{2\pi} \cdot \frac{d}{dt} m(t),$

Frequency deviation: $f_d(t) = \frac{a}{2\pi} \cdot \frac{d}{dt} m(t).$

Peak frequency dev.: $f_{d,max} = \frac{a}{2\pi} \cdot \max \left| \frac{d}{dt} m(t) \right|$

Frequency mod. idx: $\mu_f = \frac{a}{2\pi B} \cdot \max \left| \frac{d}{dt} m(t) \right|$

FM – Frequency Modulation

Momentary phase: $\phi\{m(t)\} = a \int m(t)\, dt,$ $\phi\{m(t)\} = a \int_{t_0}^{t} m(\tau)\, d\tau,$

Signal: $x(t) = \cos\left(2\pi f_c t + a \int m(t)\, dt\right)$

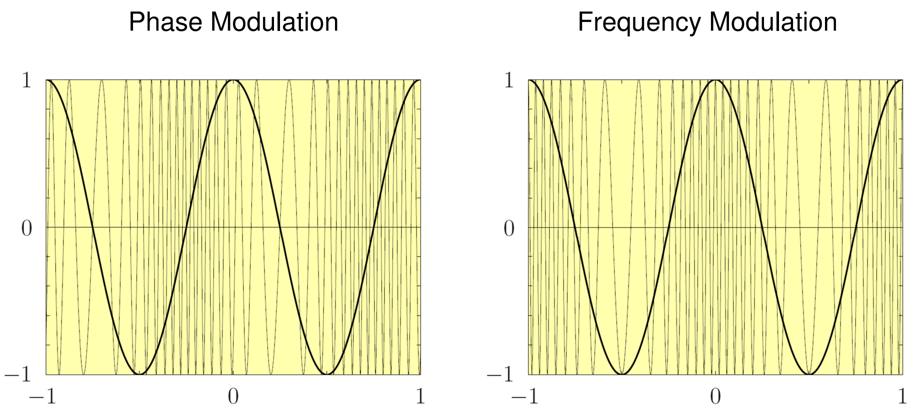
Momentary frequency: $f_{mom}(t) = \frac{1}{2\pi} \cdot \frac{d}{dt}\left(2\pi f_c t + a \int m(t)\, dt\right) = f_c + \frac{a}{2\pi} \cdot m(t).$

Frequency deviation: $f_d(t) = \frac{a}{2\pi} \cdot m(t).$

Peak frequency dev.: $f_{d,max} = \frac{a}{2\pi} \cdot \max |m(t)|$

Frequency mod. idx: $\mu_f = \frac{a}{2\pi B} \cdot \max |m(t)|$

Angle Modulation in the Time Domain



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