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Modern Physics, TNE046

Exam, 20 March 2023, Answers and short solutions

1. (a) False (should be squared). True. The corresponding wave equations consistent with special relativity are the Dirac equation and the Klein-Gordon eguation. General relativity is not yet united with QM at all. False (increases to 4). False (1000 times too much, should be a few eV).
   (b) The photoelectric equation can be written $hc/\lambda - \phi = K$, where $\phi = 4,26$eV is the work function for silver (PH 8.1) and K is the kinetic energy the potential difference should match. The particles are electrons, so the energy becomes eV, which gives us $V = 0,70$V.

2. The conservation laws critical here are the conservation of energy and the conservation of momentum. For energy: $p_{\text{photon}} c = \gamma_u m_e c^2 + \gamma_u m_p c^2$. For momentum (in the direction of the incomming photon): $p_{\text{photon}} = \gamma_u m_e u \cos\theta + \gamma_u m_p u \cos(-\theta)$. With $m_p = m_e$ and dividing the two equations one obtains $\cos\theta = c/u$ which is impossible since $\cos\theta \leq 1$ and $c/u > 1$.

3. (a) Only when $E < 0$ is the particle truly confined to the well in this assignment, else there is always due to the tunnel effect at least a slight probability it will escape. (b) Regardless of energy, the particle is more confined within $-a \leq x \leq a$ than a particle in a zero potential. Even with $E > V_0$ the particle might be refleced at the walls at $\pm a$.

4. (a) Both terms correspond to the energy-value $E_3 = me^4/(8\epsilon_0^2 h^2) \cdot 1/3^2$. The time-dependence is given by the same exponential factor for both states, which becomes $\Psi(r,\theta,\phi,t) = \psi(r,\theta,\phi)e^{-i\omega t}$ with $\omega = E_3/\hbar$. (b) In spherical coordinates, the volume is given by $0 \leq \theta \leq \pi/2$. The probability becomes then

$$\int_0^\infty \int_0^{\pi/2} \int_0^{2\pi} |R(r)|^2 \frac{1}{\sqrt{5}} \left(Y_{22}^*(\theta,\phi) + 4Y_{20}^*(\theta,\phi)\right) \frac{1}{\sqrt{5}} \left(Y_{22}(\theta,\phi) + 4Y_{20}(\theta,\phi)\right) r^2 \sin\theta \, d\phi \, d\theta \, dr.$$

   The functions can be taken from PH 6.4. The radial part integrates to unity by construction, while the ϕ-dependence parts of the mixed spherical harmonics integrate to zero. The whole expression turns into

$$2\pi \frac{1}{5} \frac{15}{32\pi} \int_0^{\pi/2} \sin^5\theta \, d\theta + 2\pi \frac{4}{5} \frac{5}{16\pi} \int_0^{\pi/2} (3\cos^2\theta - 1)^2 \sin\theta \, d\theta = 1/10 + 4/10 = 1/2.$$

5. The usual 1D infinite well has $E = n^2 \left(\pi^2\hbar^2/(2m_e L^2)\right)$. Solving for n yields $n = \sqrt{2m_e}L/(\pi\hbar)\sqrt{E}$ and from here directly since we are in one-dimension $D(E) = dn/dE = \sqrt{2m_e}L/(\pi\hbar) \cdot 1/(2\sqrt{E})$. But we have electrons, spin must be taken into account, so each energy-level is two-fold degenerate, i.e., $D(E) = dn/dE = \sqrt{2m_e}L/(\pi\hbar) \cdot 1/\sqrt{E}$.

6. (a) The energy of such a photon must exceed hc/λ, which implies $\lambda_{\max} \leq hc/E_g = 886$nm. (b) The effective mass is the ratio of the *external* force to the acceleration. In the one-dimensional case, which is what we have treated in the course, it is defined as $m_{\text{eff}} = \hbar^2/(d^2E/dk^2)$. For an electron in the free-electron-model we have $E = \hbar^2 k^2/(2m_e)$, which gives $m_{\text{eff}} = m_e$.