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Modern Physics, TNE046

Exam, 18 March 2024, Answers and short solutions

1. (a) False, False, False, True.

   (b) The accelerating potential V gives the the kinetic energy eV. The energies necessary to accelerate from rest to $0,9c$ and from $0,9c$ to $0,99c$, respectively, are the differences in energy, $\Delta K = \gamma_{0,9c} m_e c^2 - \gamma_{0c} m_e c^2 = 1,294 m_e c^2$ and $\Delta K = \gamma_{0,99c} m_e c^2 - \gamma_{0,9c} m_e c^2 = 4,79 m_e c^2$. The voltages become $1,294 m_e c^2/e = 660\text{kV}$ and $4,79 m_e c^2/e = 2,4\text{MV}$ (using $m_e c^2 = 511\text{keV}$).

2. (a) Make the normal ansatz with separation of variables, i.e., $\psi(x,y,z) = F(x)G(y)H(z)$. See textbook for continuation (Harris pp 234 – 235).
   (b) 1D has $E_n = (n + \frac{1}{2})\hbar\omega$, with $\omega = \sqrt{k/m}$. In x, y, z-dimensions the energy becomes $E_{n_x,n_y,n_z} = (n_x + n_y + n_z + \frac{3}{2})\hbar\omega$.
   (c) First excited state is obtained when one of the quantum numbers is increased to one, that is the energy $E_{1,0,0} = \frac{5}{2}\hbar\omega$. This can be obtained also by quantum numbers $(0,1,0)$ and $(0,0,1)$, i.e., there is a threefold degeneration.

3. (a) Normalization, $\int_{-\infty}^{\infty} |\psi(x)|^2\, dx = 1$ which gives $|C|^2 \int_0^{\infty} x e^{-2ax^2}\, dx = |C|^2/(4a) = 1$. Best choice (simplicity) is $C = 2\sqrt{a}$.
   (b) $\int_0^{1/\sqrt{a}} |\psi(x)|^2\, dx = 1 - e^{-2}$.

4. (a) $E_2 = -13,6\text{ eV}/2^2 = -3,4\text{ eV}$.
   (b) Other quantum numbers are: (ℓ, m_ℓ, m_s). For $n = 2$ they can have the values: $(0,0,\pm 1/2)$, $(1,-1,\pm 1/2)$, $(1,0,\pm 1/2)$, $(1,1,\pm 1/2)$.
   (c) The spin quantum number, m_s does not come from the solution of the Schrödinger equation. It is new entity, totally unexpected when it was discovered in 1922, due to the the magnetic dipole moment it gives rise to. The origin is the quantum mechanical property "spin", often said to be a relativistic effect in quantum mechanics.

5. MB-distribution $N(E_i) = (g_i/Z)e^{-E_i/kT}$. Hydrygen-like atom gives degeneracy as $g_1 = 2$, $g_2 = 8$ (cf assignment 4 above). Fraction asked for $N(E_2)/N(E_1) = (g_2/Z)e^{-E_2/kT}/(g_1/Z)e^{-E_1/kT}$ With $E_i = -E_H Z^2/i^2$ and $kT = 1/40$ eV, the fraction becomes $4 \cdot 10^{-1260}$. A quite small number.

6. (a) The number of Fermions, $N = \int_0^{\infty} N_{FD}(E) D(E)\, dE$. With the normal "cold-temperature approximation" and the given expression for $D(E)$ it becomes $N = \int_0^{E_{FD}} 1 m_e A/(\pi\hbar^2)\, dE = m_e A E_F/(\pi\hbar^2)$. That is, $E_F = (N/A)\pi\hbar^2/m_e$.
   (b) $E_F = 3,8 \cdot 10^{-20}\text{J} = 0,24\text{eV}$.
   (c) $T_F = 2800\text{K}$, an order of magnitude colder than for the 3D-case (see PH T7.1), but still enough to consider room-temperature as cold.