TSTE05 Elektronik & mätteknik

Föreläsning 4

Växelströmsteori: Introduktion

Labförberedelser

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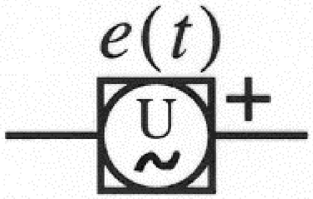
Växelströmsteori

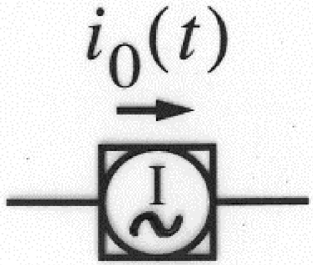
Tidsberoende storheter:

Spänning	$u(t)$
Ström	$i(t)$
Effekt	$p(t)$

Ideal spänningskälla

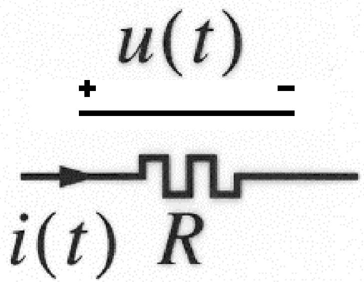


Ideal strömkälla



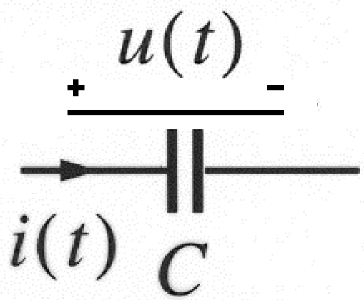
Växelströmsteori – Passiva komponenter

Resistans



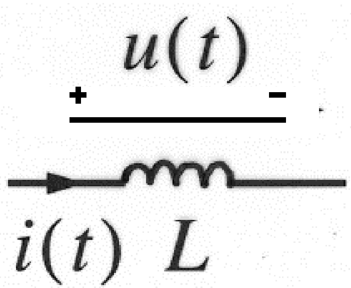
$$u(t) = Ri(t)$$

Kapacitans



$$i(t) = C\frac{d}{dt}u(t)$$

Induktans



$$u(t) = L\frac{d}{dt}i(t)$$

Kapacitans – Kondensatorer

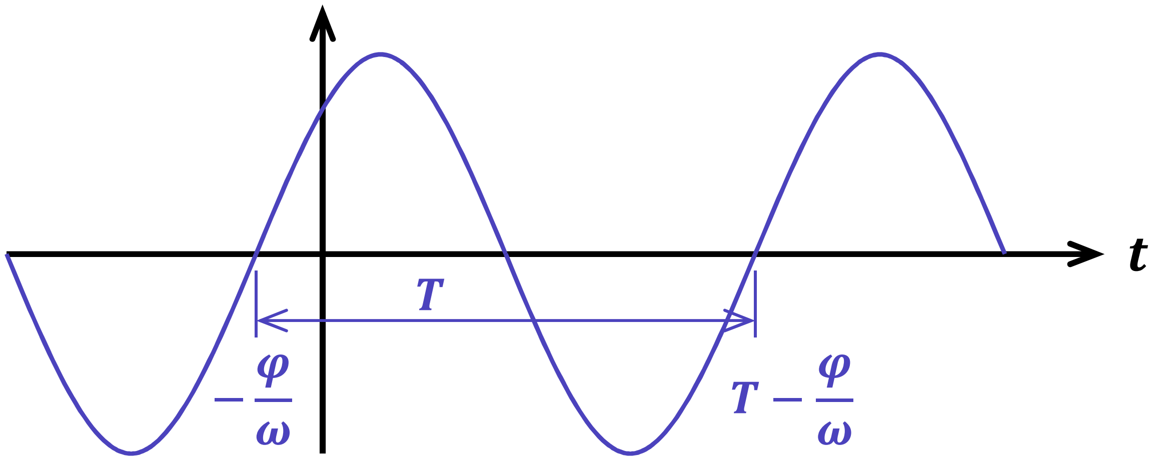
Källa: Wikipedia

Induktans – Spolar

Källa: Wikipedia

Stationär sinussignal

$$x(t) = \hat{X}\sin(\omega t + \varphi)$$



Symbol	Förklaring
$x(t)$	Momentanvärde
$\hat{X}$	Amplitud (toppvärde)
ω	Vinkelfrekvens [rad/s]
φ	Fasvinkel [rad]
T	Periodtid [s]
f	Frekvens [Hz]

$$f = \frac{1}{T} \qquad \omega = 2\pi f$$

Momentan effekt

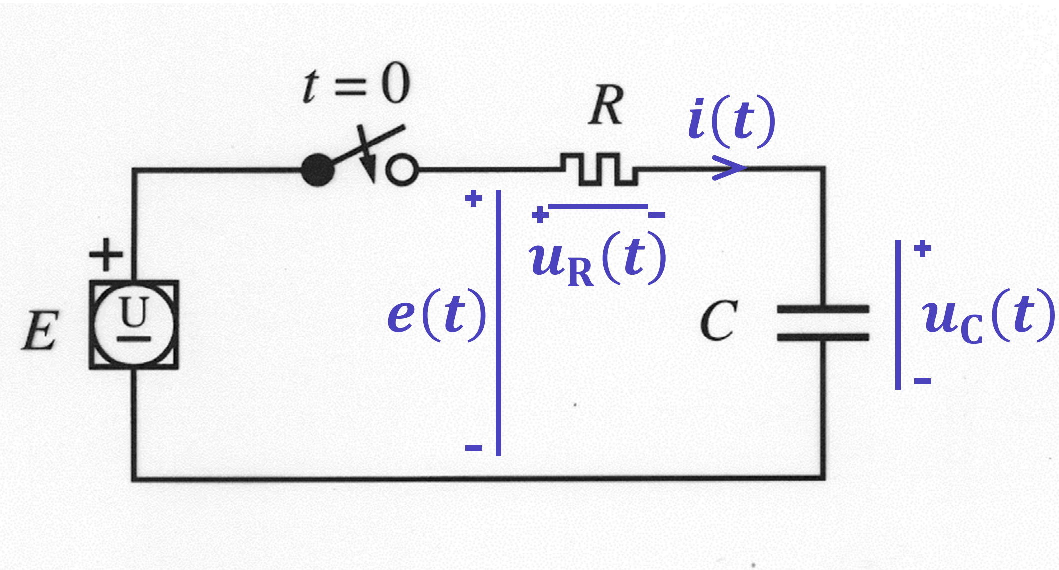
$$p(t) = u(t)i(t)$$

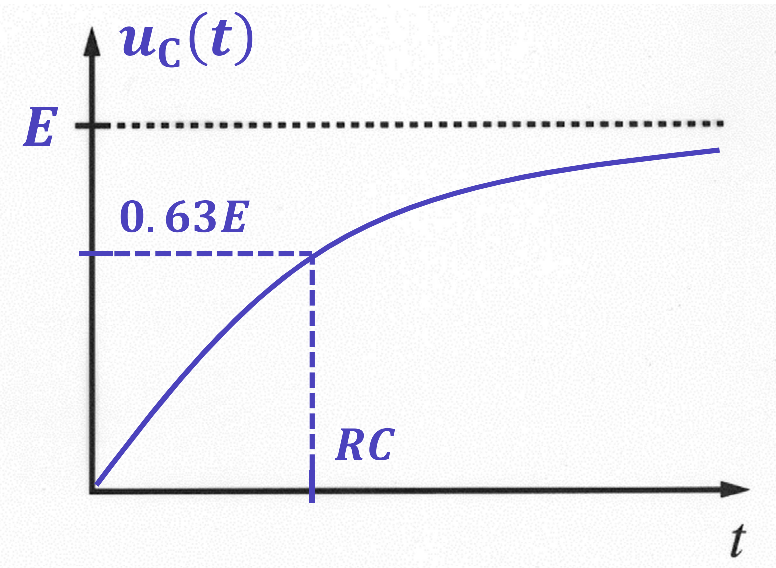
Aktiv effekt: $$P = \frac{1}{T}\int_0^T p(t)\,dt$$

Effektivvärde: $$X_e = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt} = \frac{\hat{X}}{\sqrt{2}}$$

Sinus

Uppladdning av en kapacitans





Initialtillstånd: $u_C(0-) = 0$ $\quad$ $e(t) = \begin{cases} 0, & t < 0, \\ E, & t \geq 0. \end{cases}$ $\quad$ $u_C(t) + u_R(t) = e(t)$

$$u_R(t) = Ri(t) = RC\frac{d}{dt}u_C(t)$$

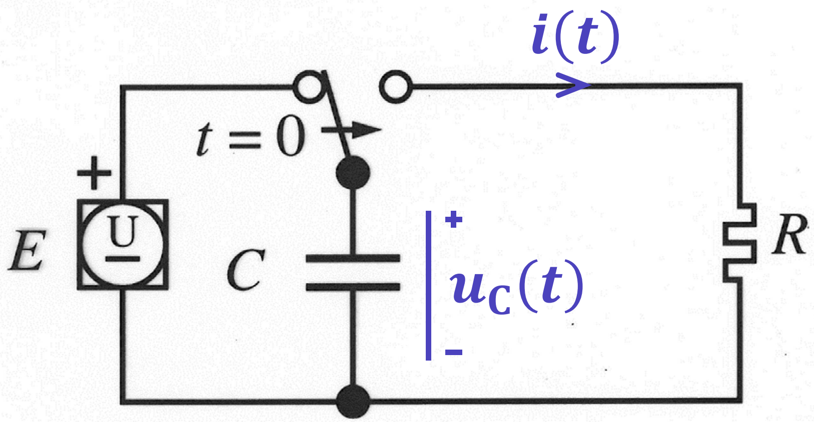
$$i(t) = C\frac{d}{dt}u_C(t)$$

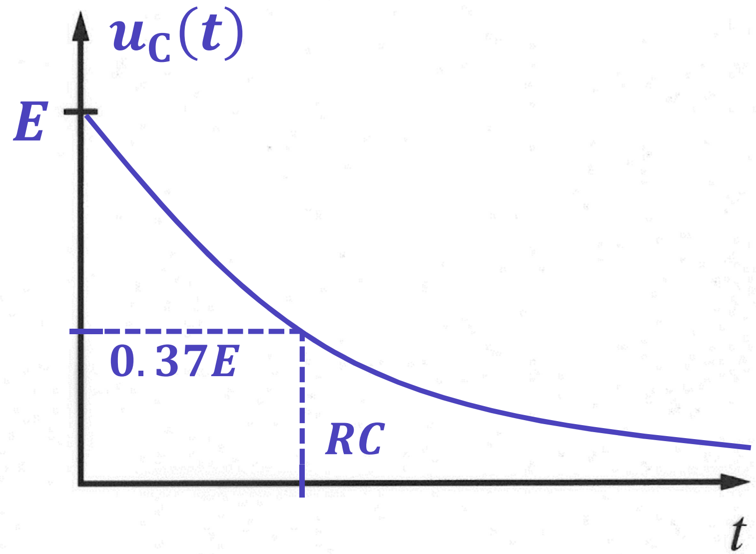
$t \geq 0$: $\quad u_C(t) + RC\dfrac{d}{dt}u_C(t) = E$

Homogen och partikulär lösning ⇒

$$u_C(t) = \left(1 - e^{-t/RC}\right)E$$

Urladdning av kapacitans





Initialtillstånd: $u_C(0-) = E$

$\boldsymbol{t \geq 0}$:

$$u_C(t) = Ri(t) = -RC\frac{d}{dt}u_C(t)$$

$$i(t) = -C\frac{d}{dt}u_C(t)$$

$$u_C(t) + RC\frac{d}{dt}u_C(t) = 0$$

Homogen (och partikulär) lösning ⇒

$$u_C(t) = Ee^{-t/RC}$$

jω-metoden

1. Ersätt strömmar, spänningar och källor med deras komplexa motsvarigheter:

$$a(t) = \hat{A}\sin(\omega t + \varphi) \Rightarrow$$

$$A = \hat{A}e^{j\varphi} = b + jc$$

$$b = \hat{A}\cos\varphi \qquad c = \hat{A}\sin\varphi$$

2. Ersätt R, L, C med deras impedanser:

$$Z_L = j\omega L \qquad Z_C = \frac{1}{j\omega C} \qquad Z_R = R$$

3. Lös problemet med likströmsteori.

4. Gör omvändningen till punkt 1:

$$A = \hat{A}e^{j\varphi} = b + jc \Rightarrow$$

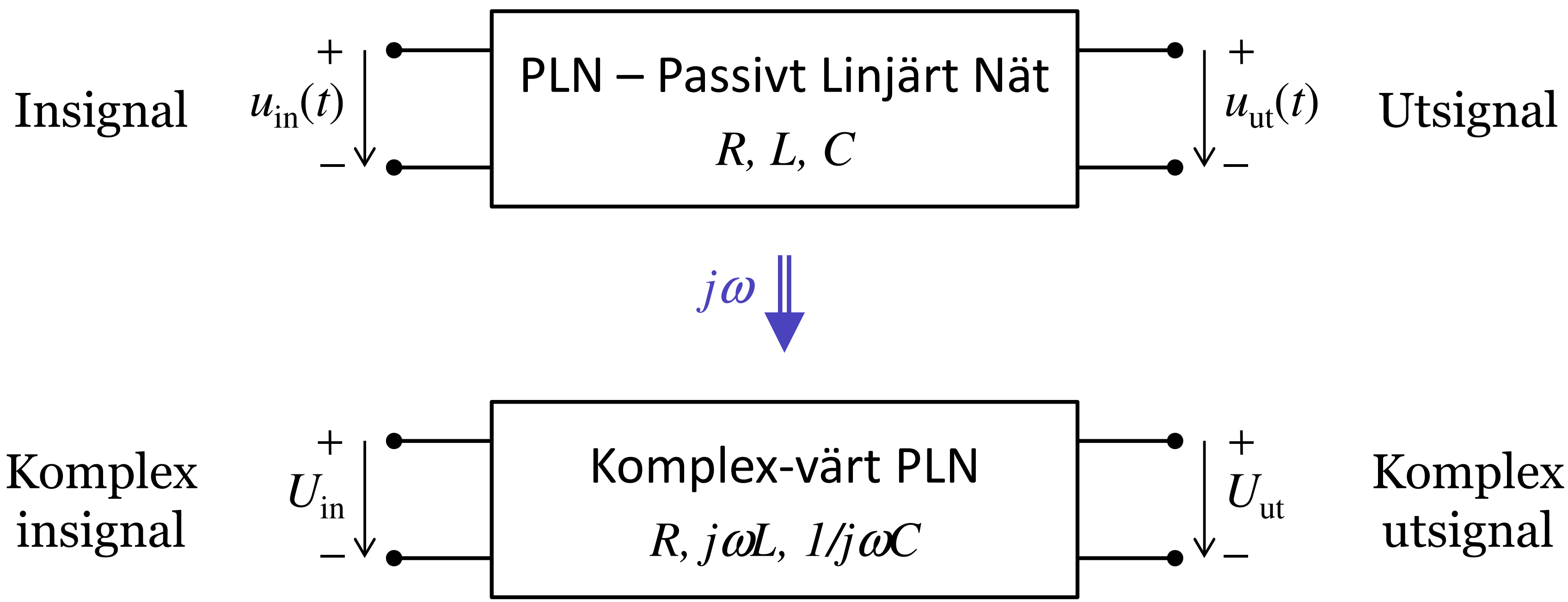
$$a(t) = \hat{A}\sin(\omega t + \varphi)$$

$$\hat{A} = \sqrt{b^2 + c^2}$$

$$\varphi = arg(b + jc) = \operatorname{atan}\frac{c}{b} \qquad (\pm\pi)$$

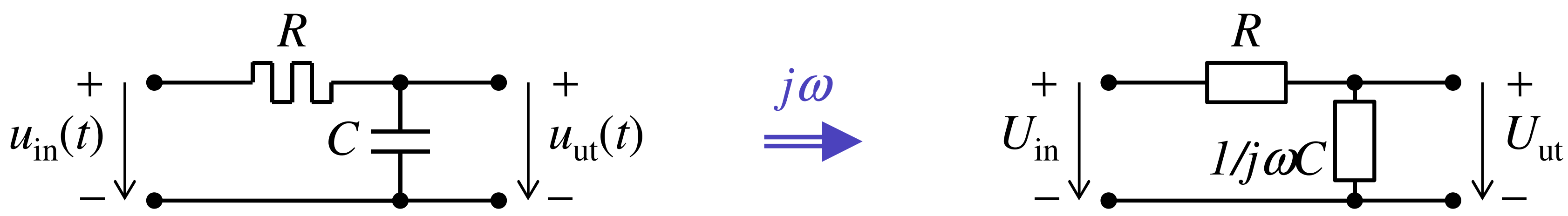
Om $b < 0$

Passiva filter – Introduktion



Samband: $U_{ut} = \underbrace{H(\omega)}_{\text{Frekvensfunktion}} U_{in}$ $\qquad H(\omega) = \underbrace{|H(\omega)|}_{\text{Amplitudkarakteristik}} \cdot e^{j\underbrace{\arg\{H(\omega)\}}_{\text{Faskarakteristik}}} = U_{ut}/U_{in}$

Passiva filter – Exempel 1(2)



Spänningsdelning ger

$$U_{\text{ut}} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} U_{\text{in}} = \frac{1}{1 + j\omega RC} U_{\text{in}}$$

Frekvensfunktion

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

Amplitudkarakteristik

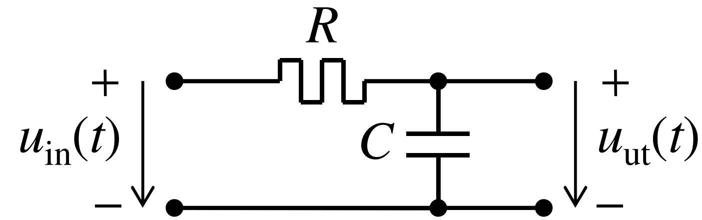
$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

Faskarakteristik

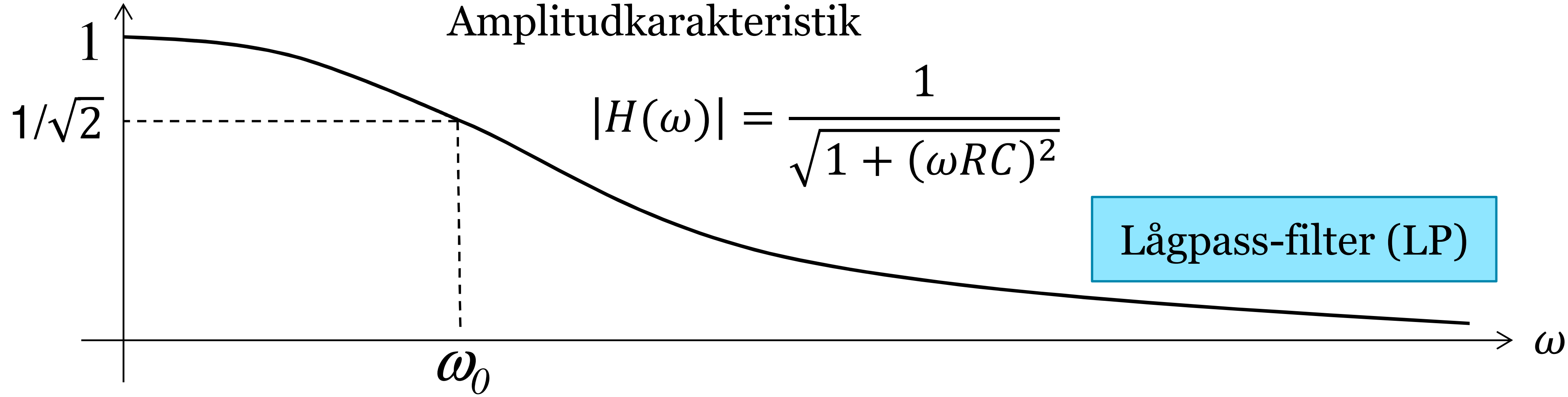
$$\arg\{H(\omega)\} = -\arctan(\omega RC)$$

$$u_{\text{in}}(t) = \hat{U}_{\text{in}} \sin(\omega t + \varphi) \quad \Rightarrow \quad u_{\text{ut}}(t) = \hat{U}_{\text{in}} |H(\omega)| \sin(\omega t + \varphi + \arg\{H(\omega)\})$$

Passiva filter – Exempel 2(2)



Amplitudkarakteristik

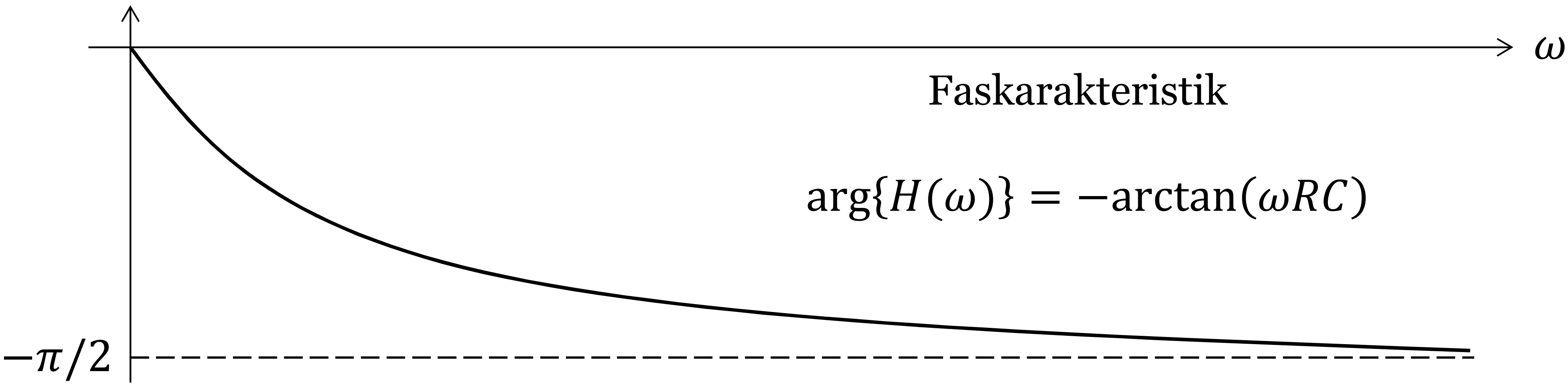


$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

Lågpass-filter (LP)

Gränsvinkelfrekvens

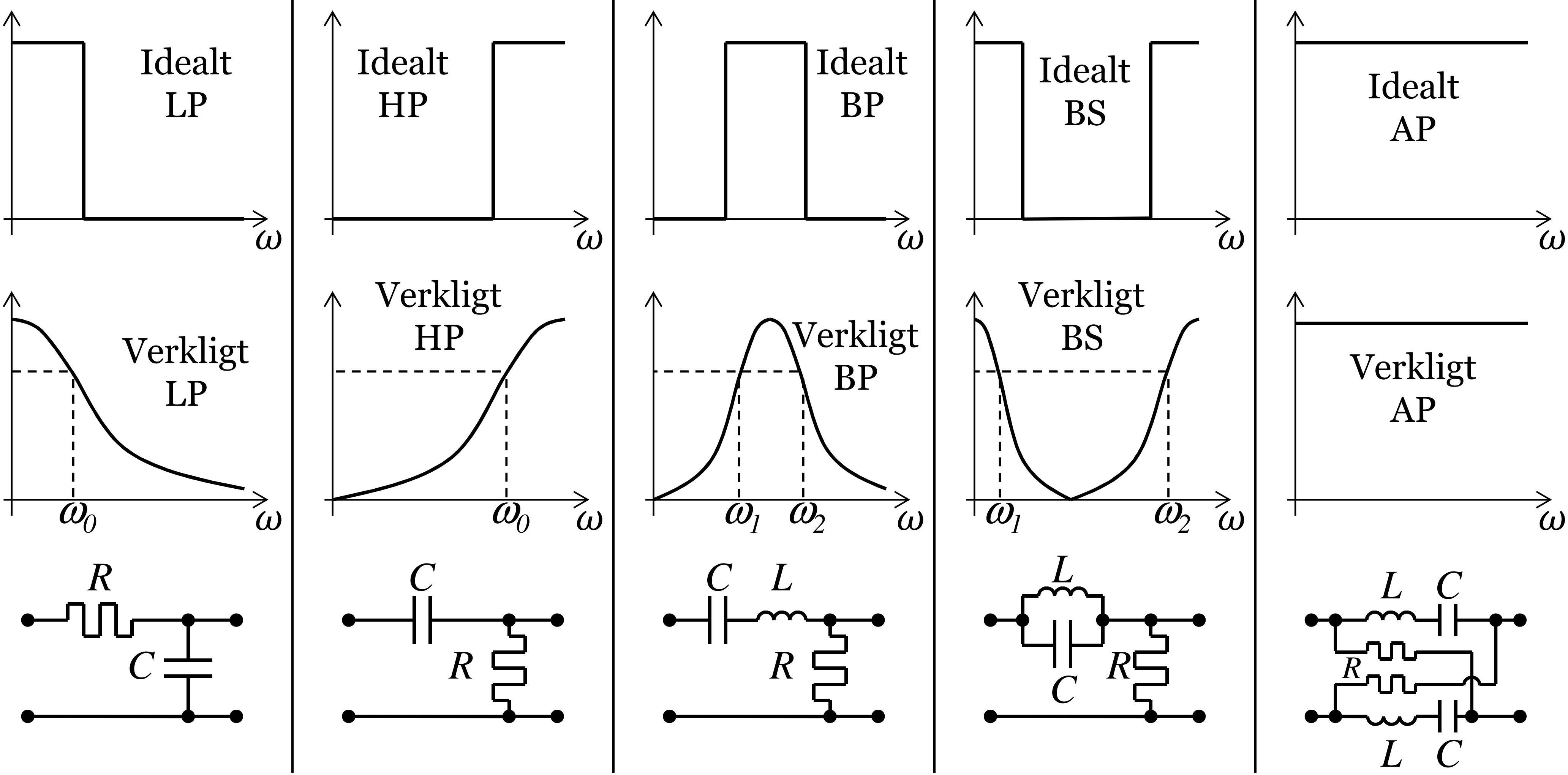
Faskarakteristik



$$\arg\{H(\omega)\} = -\arctan(\omega RC)$$

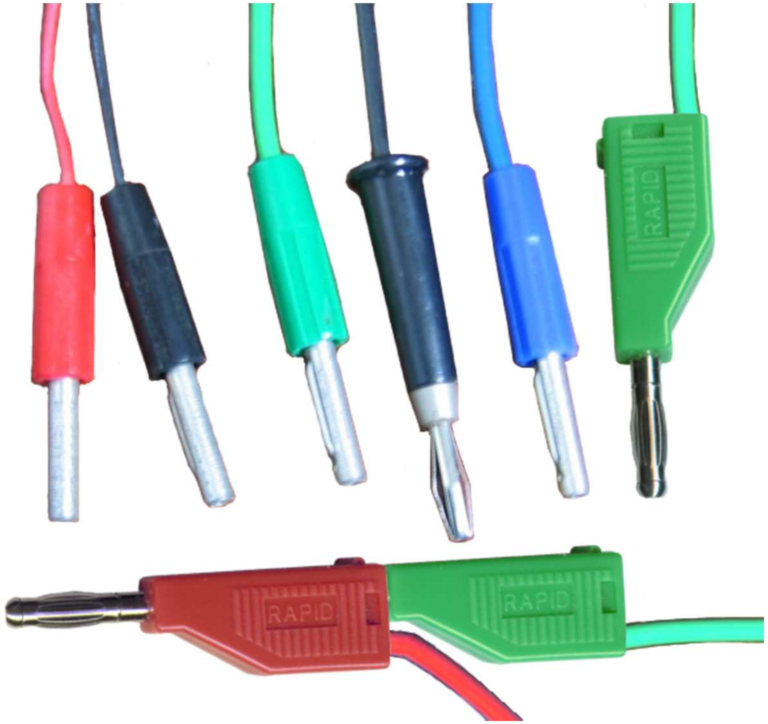
Olika frekvensselektiva filter

Lågpass (LP) | **Högpass (HP)** | **Bandpass (BP)** | **Bandspärr (BS)** | **Allpass (AP)**



Labutrustningen – Kontakter

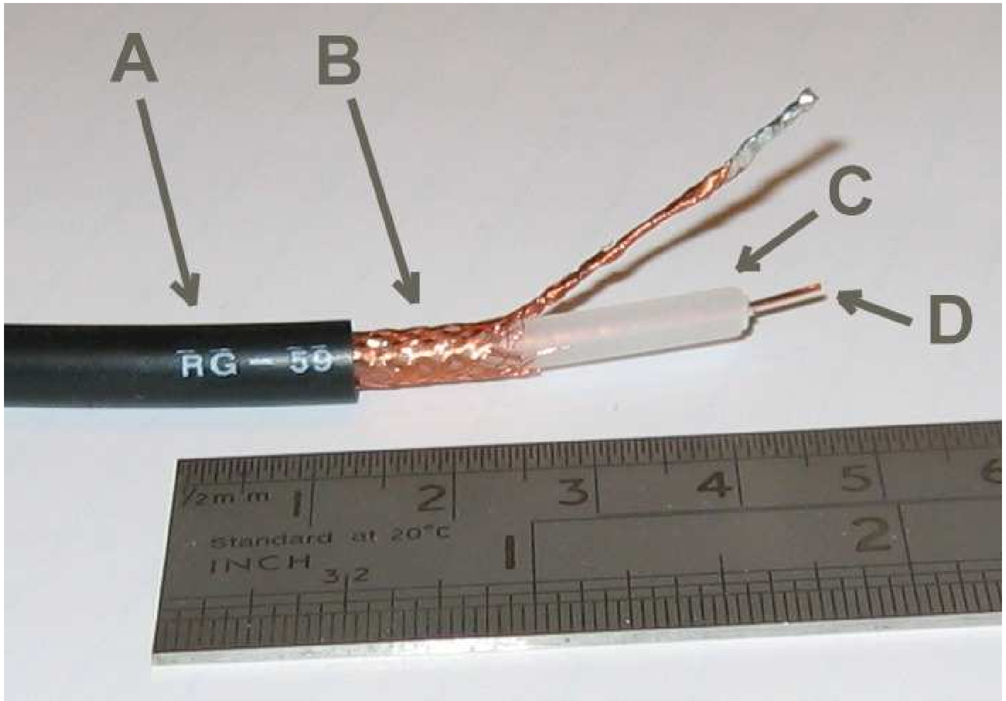
Banankontakter



BNC-kontakter

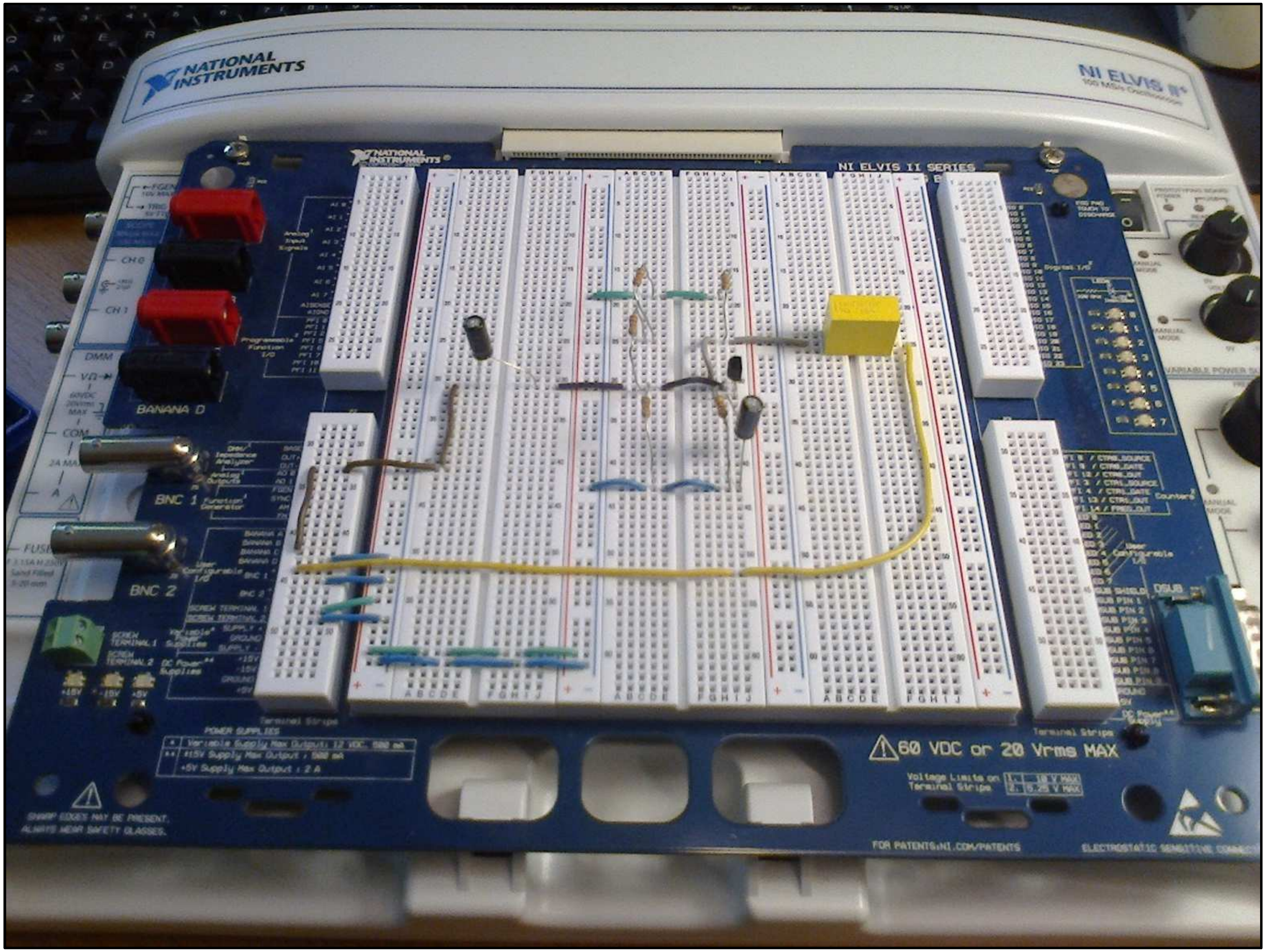
Källa: Wikipedia

Labutrustningen – Koaxialkablar

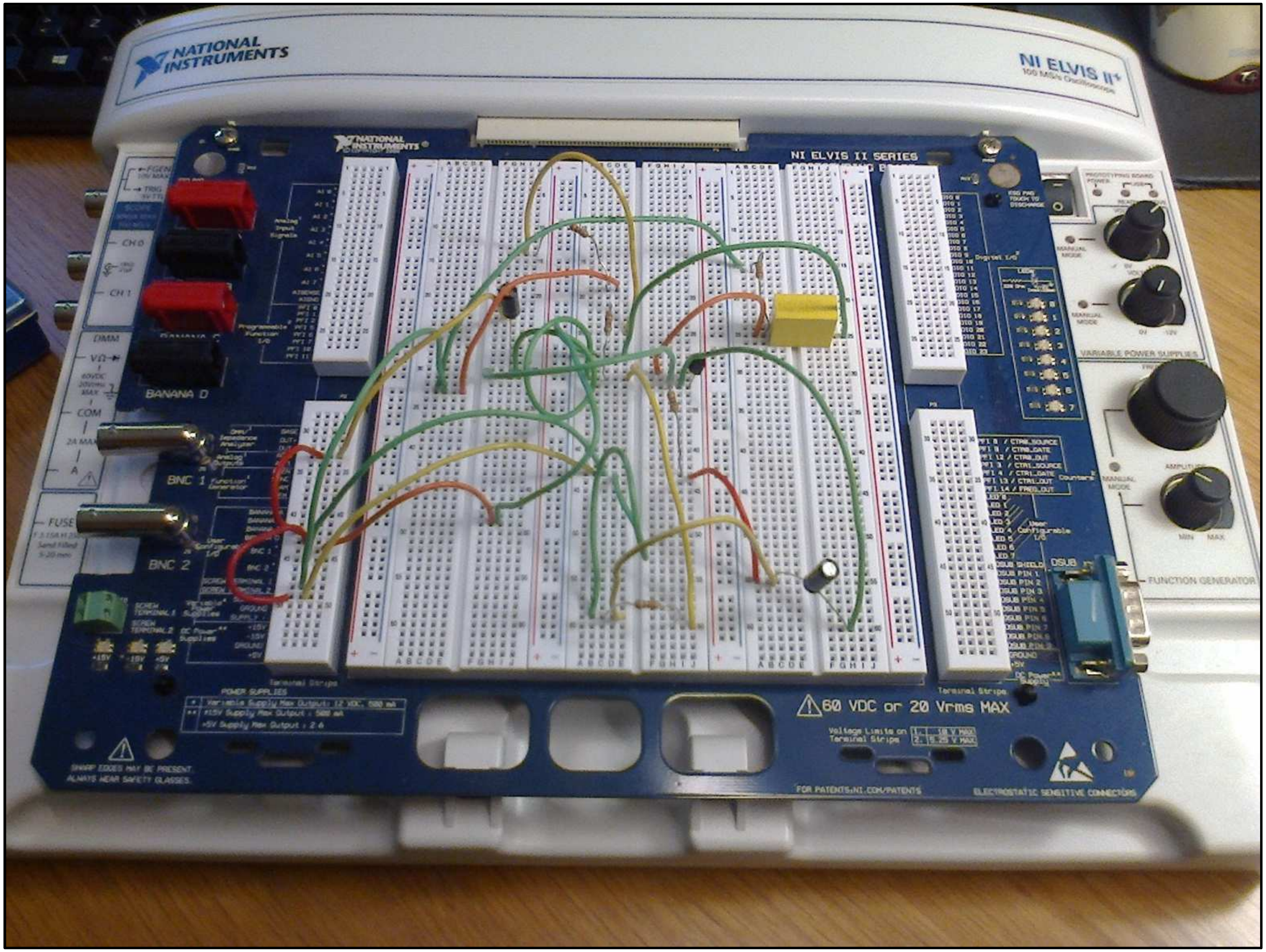


Källa: Wikipedia

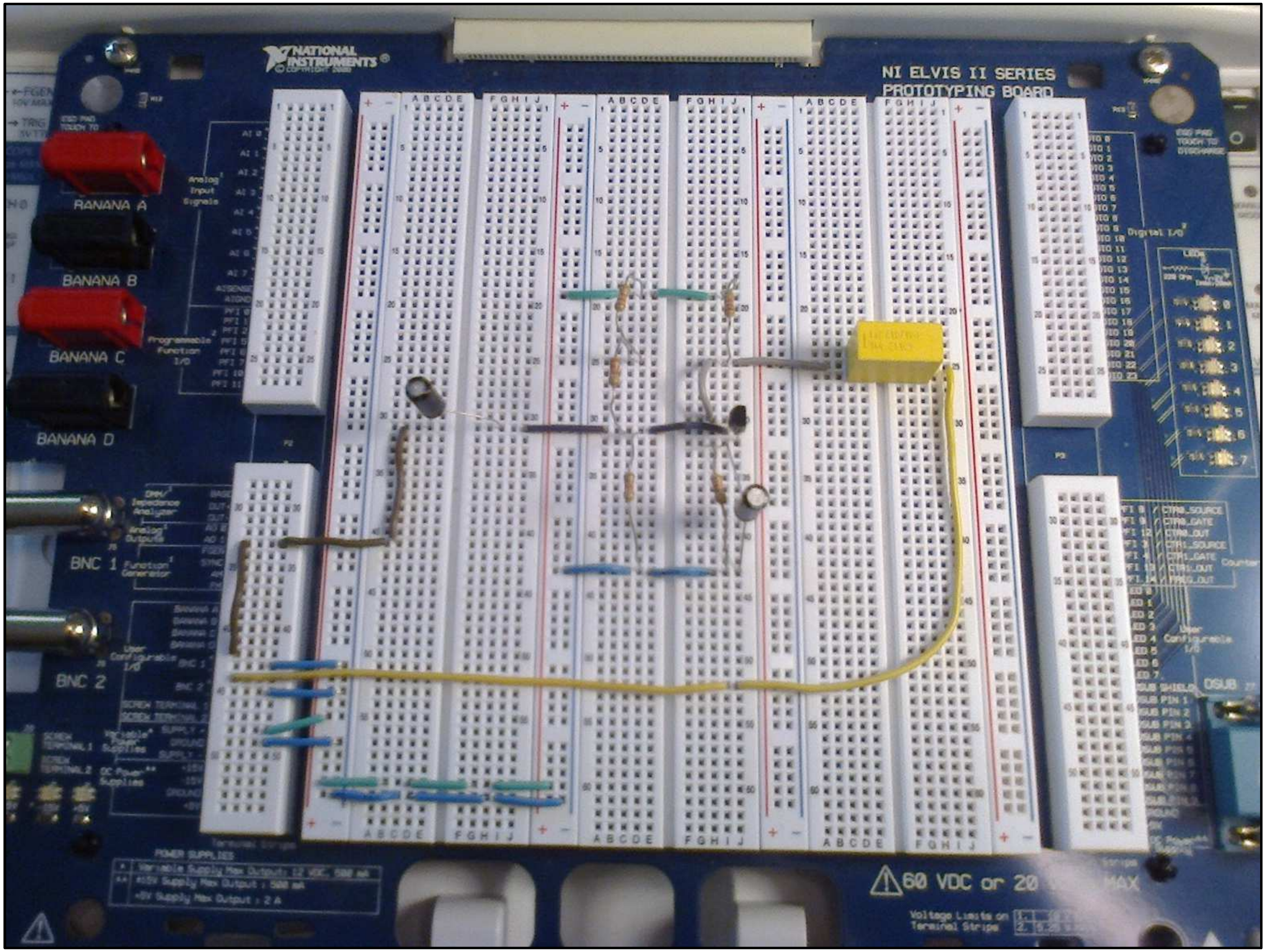
Labutrustningen – Elvis II – Prydlig uppkoppling



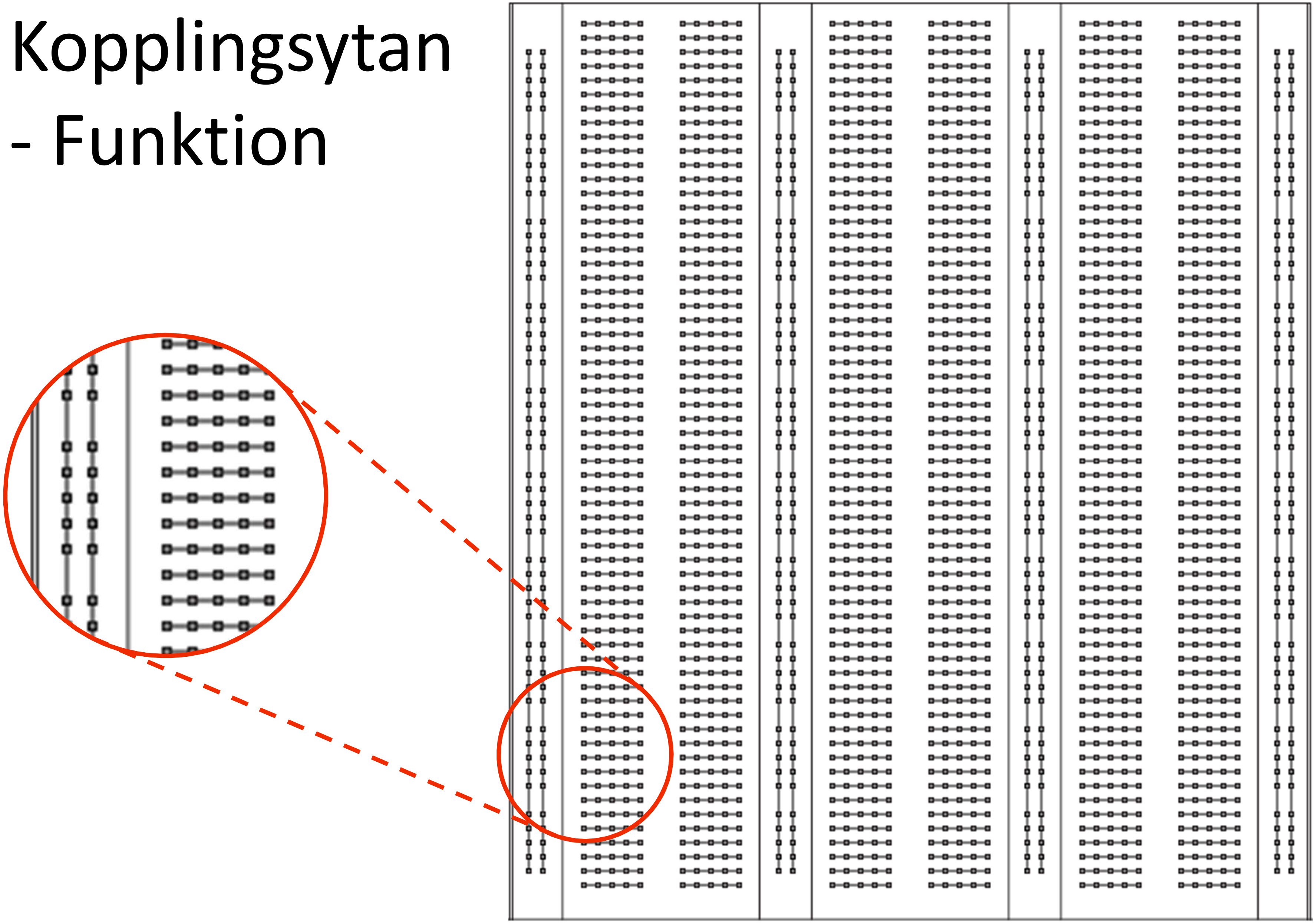
Elvis II – Mindre prydlig uppkoppling



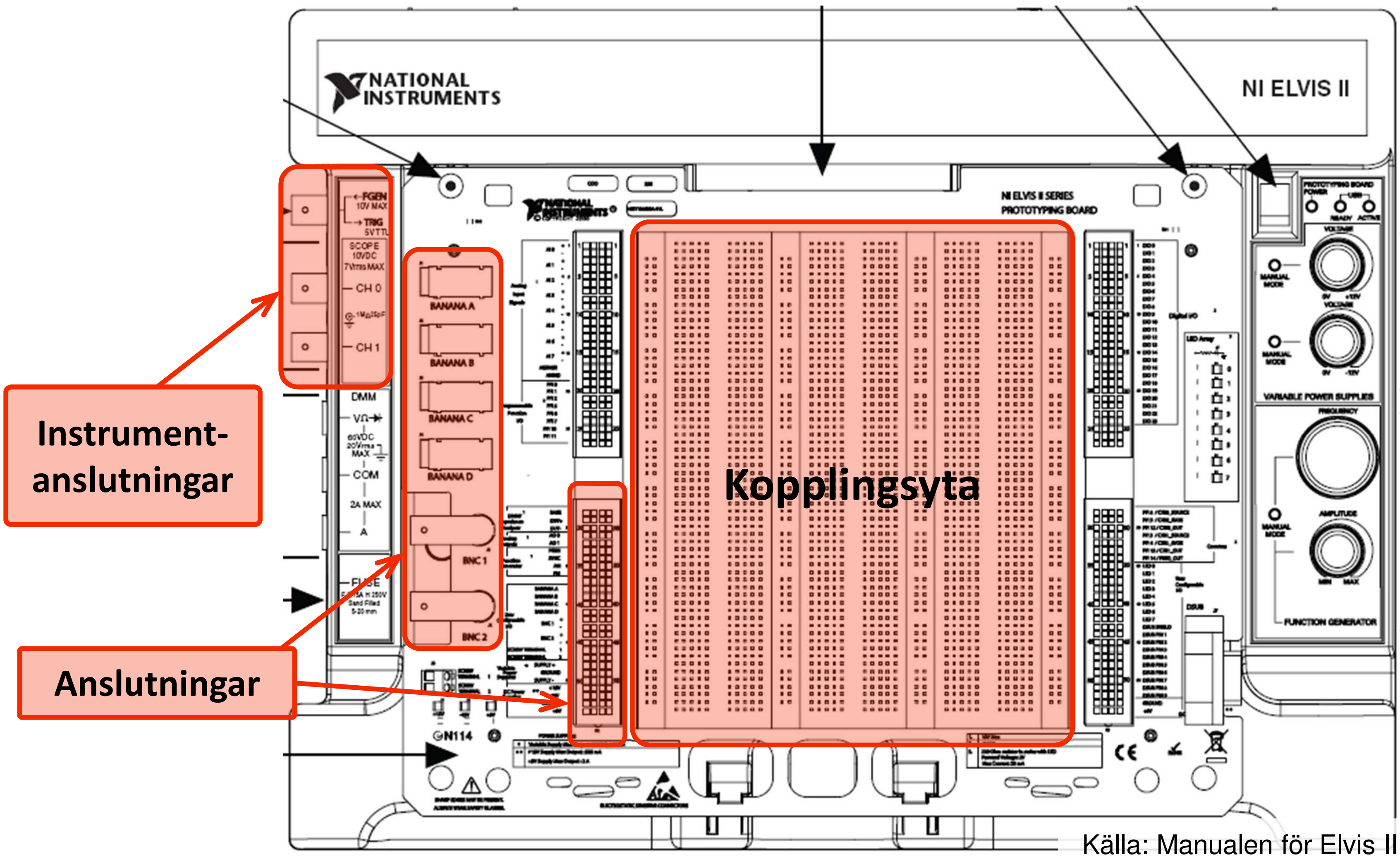
Elvis II – Kopplingsytan, breadboard



Kopplingsytan
- Funktion

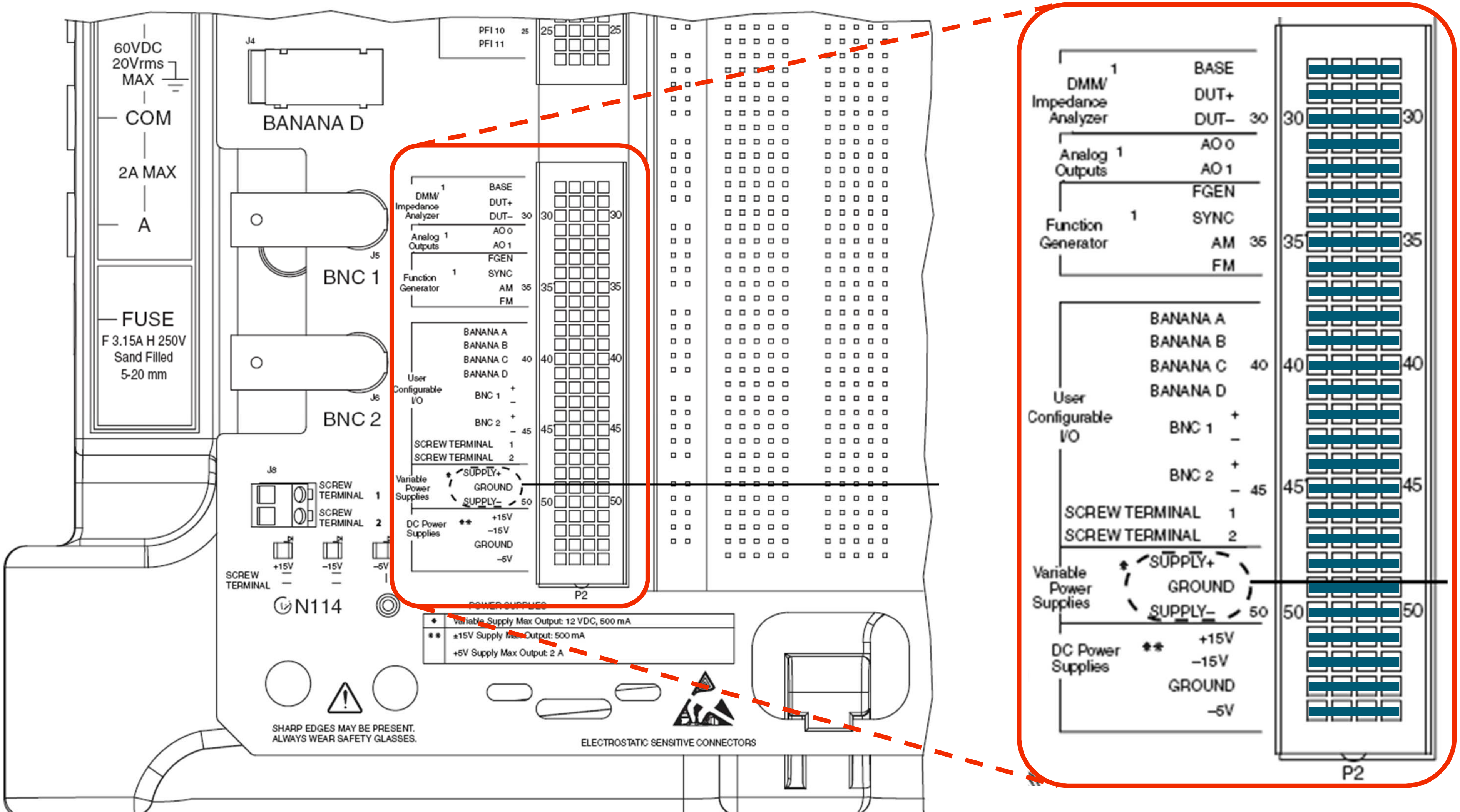


Elvis II – Översikt



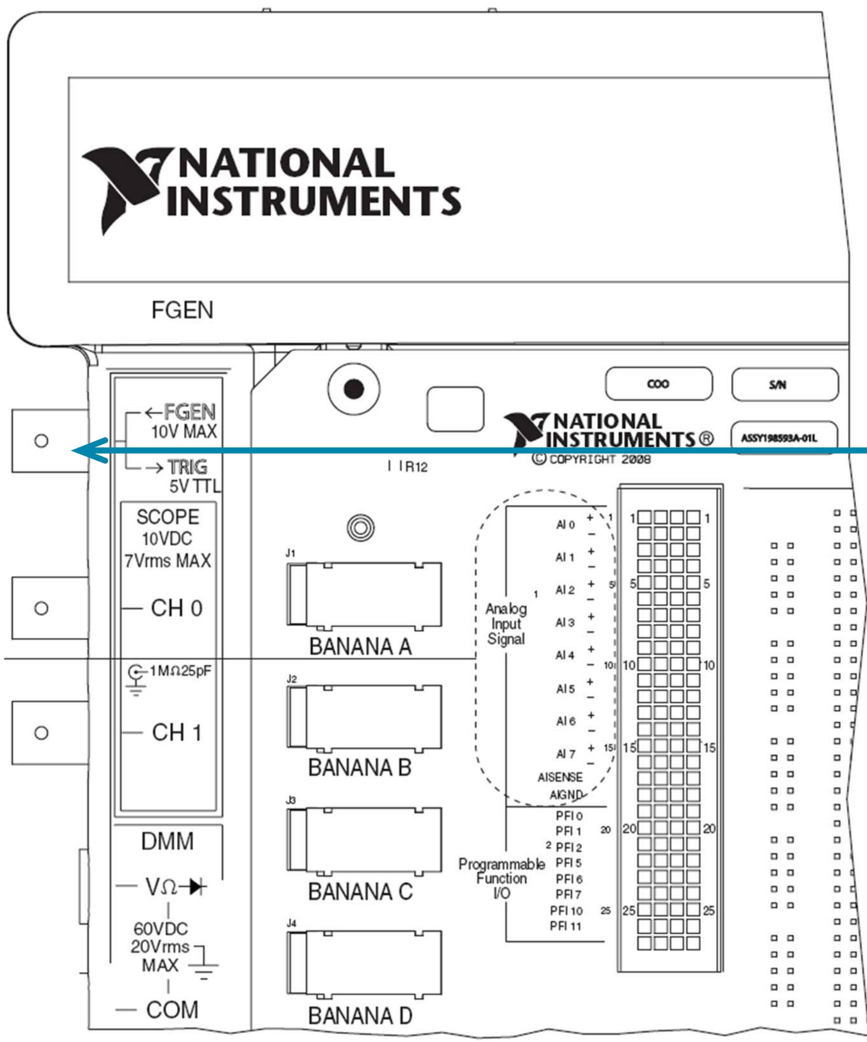
Källa: Manualen för Elvis II

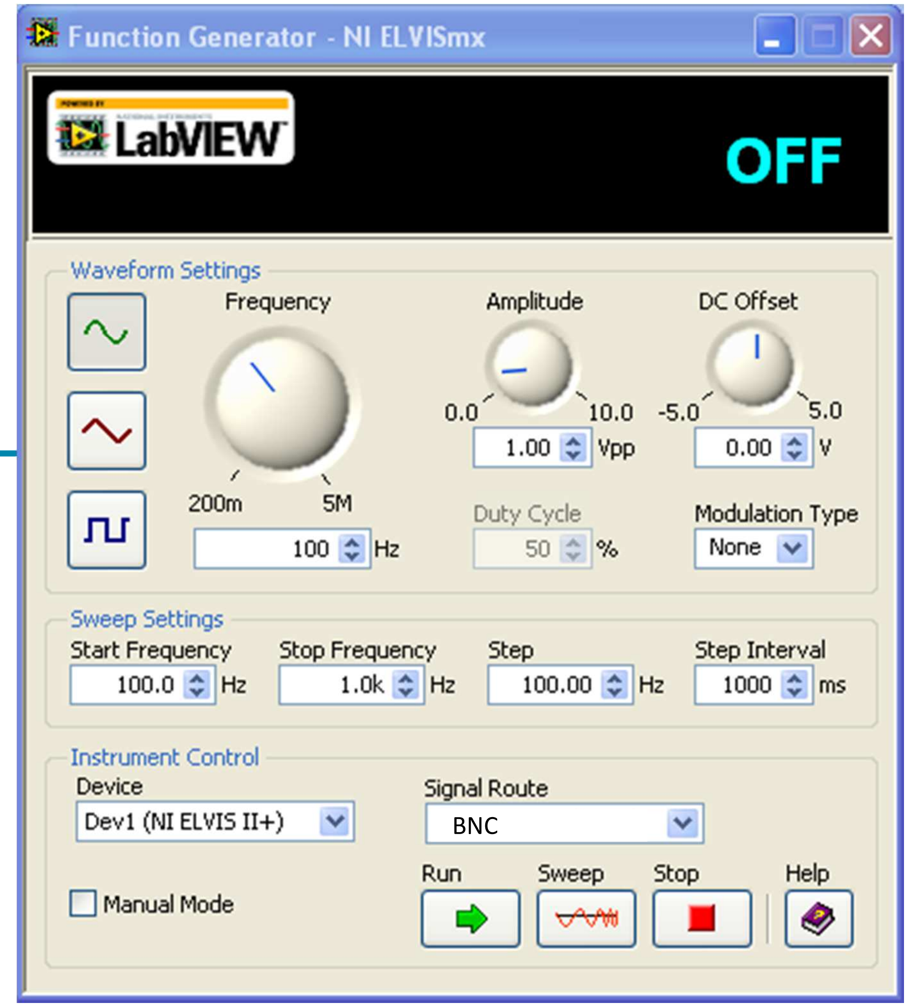
Elvis II – Nedre vänstra hörnet



Källa: Manualen för Elvis II

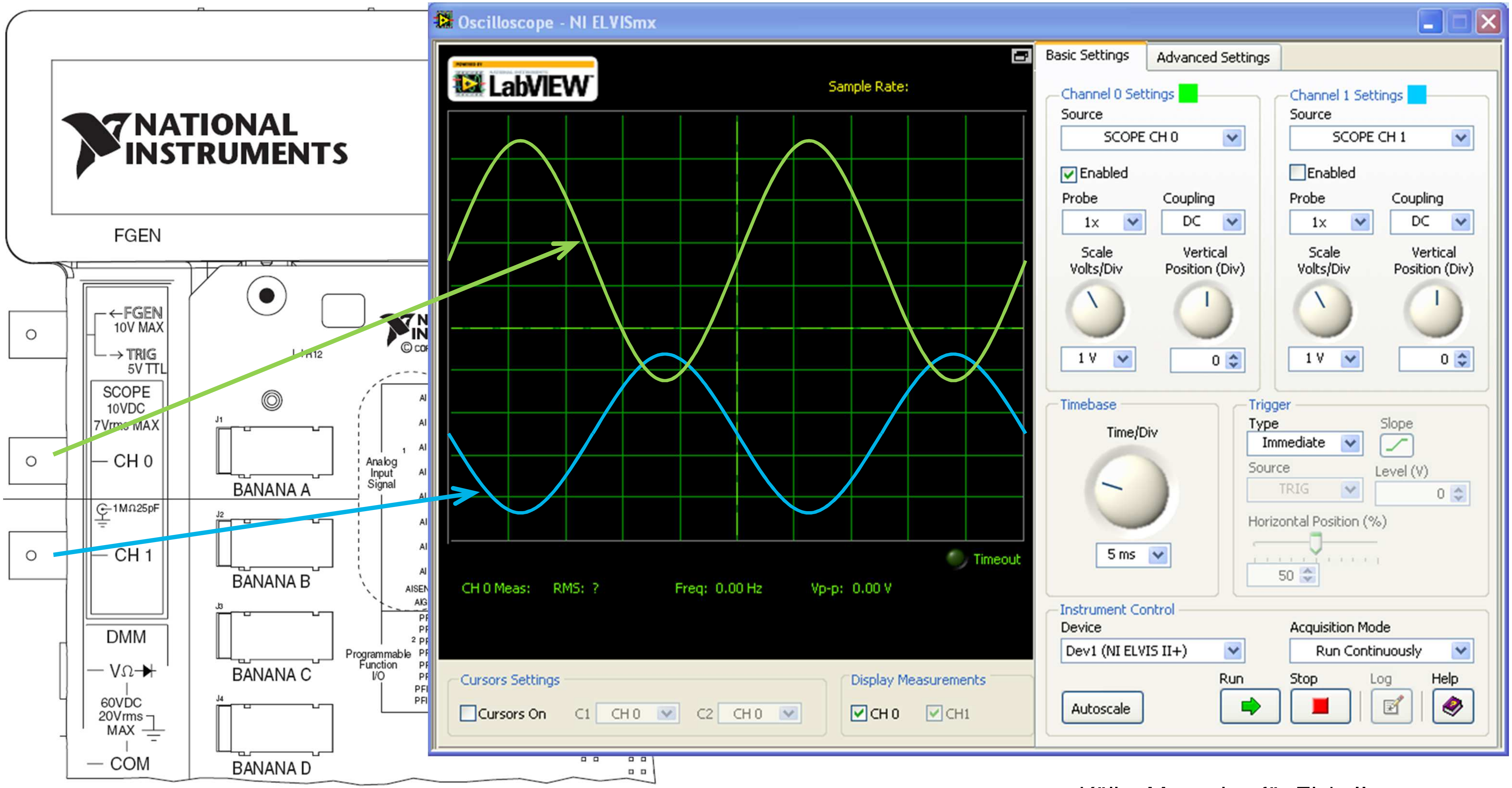
Elvis II – Funktionsgeneratorn





Källa: Manualen för Elvis II

Elvis II – Oscilloskopet



Källa: Manualen för Elvis II

jω-metoden

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$$\varphi = arg(b + jc) = \operatorname{atan}\frac{c}{b} \qquad (\pm\pi)$$

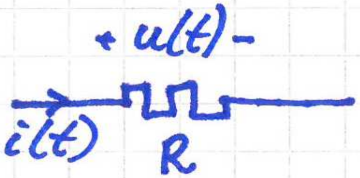
Om $b < 0$

Härledning jω-metoden 1(2)

$$u(t) = \hat{U}\cdot\sin(\omega t + \phi_u) = \hat{U}\,\mathrm{Im}\{e^{j(\omega t+\phi_u)}\} = \mathrm{Im}\{\underbrace{\hat{U}e^{j\phi_u}}_{U}\cdot e^{j\omega t}\}$$

$$i(t) = \hat{I}\cdot\sin(\omega t + \phi_i) = \hat{I}\cdot\mathrm{Im}\{e^{j(\omega t+\phi_i)}\} = \mathrm{Im}\{\underbrace{\hat{I}e^{j\phi_i}}_{I}\cdot e^{j\omega t}\}$$

Resistans:



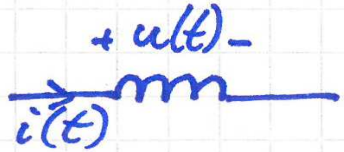
$$u(t) = R\,i(t)$$

$$\mathrm{Im}\{U\cdot e^{j\omega t}\} = R\cdot\mathrm{Im}\{I\cdot e^{j\omega t}\} = \mathrm{Im}\{RI\cdot e^{j\omega t}\}$$

En lösning: $U = R\cdot I$

Härledning jω-metoden 2(2)

Induktans



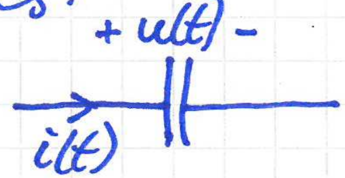
$$u(t) = L\frac{d}{dt}i(t)$$

$$Im\{U\cdot e^{j\omega t}\} = L\frac{d}{dt}Im\{I\cdot e^{j\omega t}\} = Im\{LI\cdot\frac{d}{dt}e^{j\omega t}\}$$

$$= Im\{L\cdot I\cdot j\omega\cdot e^{j\omega t}\}$$

Motsv. lösning: $U = j\omega L\cdot I$

Kapacitans:



$$i(t) = C\frac{d}{dt}u(t)$$

$$Im\{I\cdot e^{j\omega t}\} = C\frac{d}{dt}Im\{U\cdot e^{j\omega t}\} = Im\{CU\frac{d}{dt}e^{j\omega t}\}$$

$$= Im\{CUj\omega e^{j\omega t}\}$$

Motsv. lösn: $I = j\omega C\cdot U \Rightarrow U = \frac{1}{j\omega C}\cdot I$

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