TSEK02: Radio Electronics Lecture 5: TX Nonlinearity Considerations

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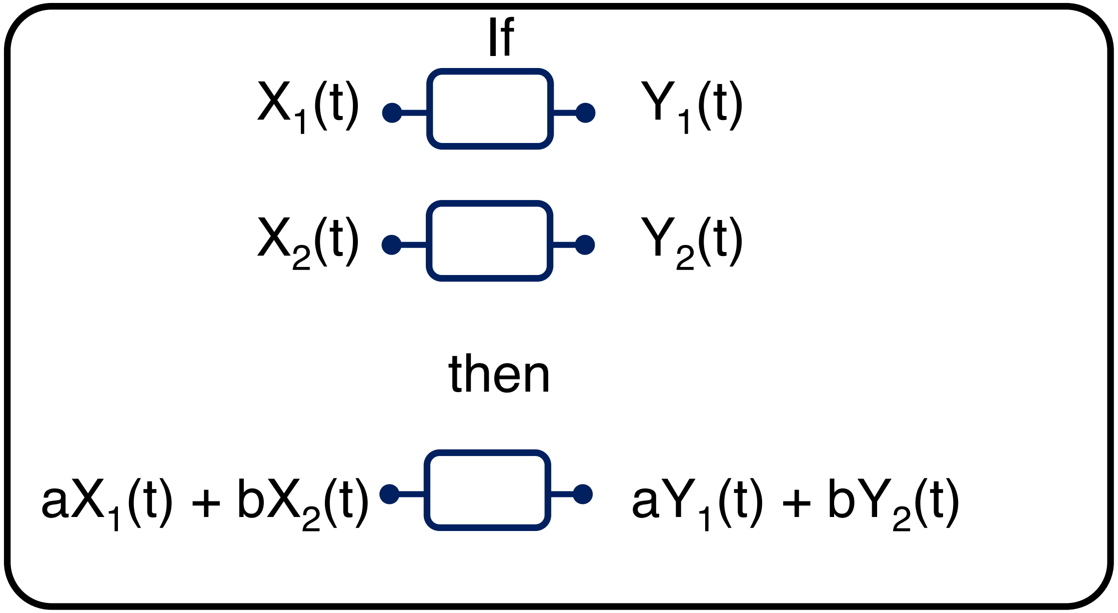
Nonlinearity Issues

- **Definition (2.1.2)**
- Modeling Nonlinearities
- Effects of Nonlinearity
  - Harmonic Distortion
  - Gain Compression
  - Intermodulation
- Characterization of Nonlinearities
- Cascaded Nonlinear Stages

Linear and Nonlinear Systems

- A system is said to be linear if it follows the superposition rule:



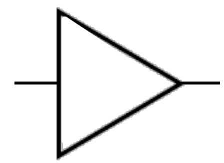
- A system which is not linear (i.e. does not follow the superposition rule), is nonlinear

Example - I

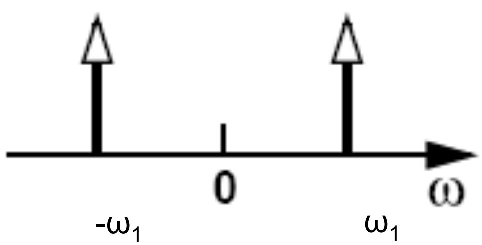
Vout = G * Vin

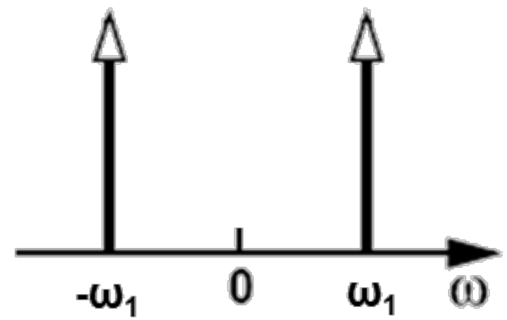
linear

$A \cos \omega_1 t$



$G * A \cos \omega_1 t$





Output contains the same frequency components as input, they are just stronger

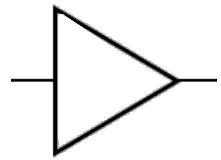
Example - II

Vout = G * Vin

linear

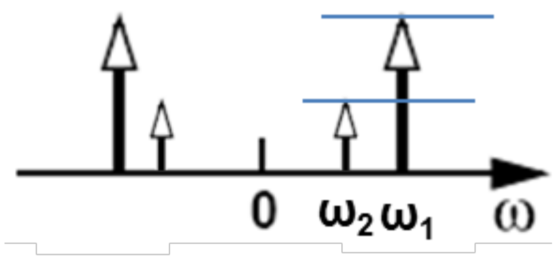
$A \cos \omega_1 t$

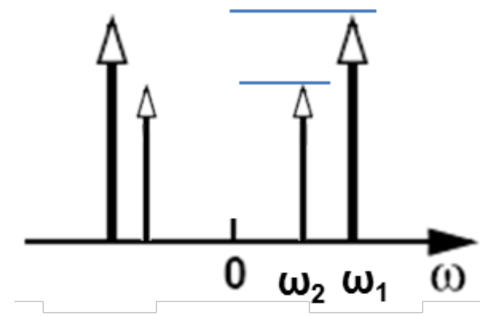
$B \cos \omega_2 t$



$G * A \cos \omega_1 t$

$G * B \cos \omega_2 t$





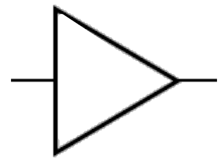
Both inputs are amplified by the same amount

Example - III

$V_{out} = G(V_{in}) * V_{in}$

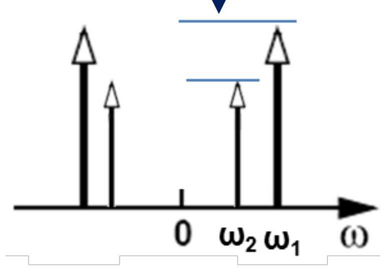
nonlinear!

$A' \cos \omega_1 t$
$B' \cos \omega_2 t$

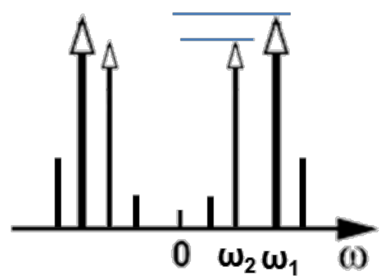


$K_1 \cos (2\omega_1 - \omega_2)\, t$
$A'' \cos \omega_1 t$
$B'' \cos \omega_2 t$
$K_2 \cos (2\omega_2 - \omega_1)\, t$

Input signals are stronger now



Different tones are amplified differently



New frequencies appear at the output

Always check....

- In reality, systems behave linearly only under specified conditions
  - Input level
  - Bias
  - Load impedance
  - …

- Always check if these conditions are met before assuming linear operation!

Time Variance

- A system is time-invariant if a time shift in its input results in the same time shift in its output.

If $y(t) = f\,[x(t)]$

then $y(t-\tau) = f\,[x(t-\tau)]$

- Time-variant = response depends on the time of origin.

Nonlinearity Issues

- Definition
- **Modeling Nonlinearities (2.1.3)**
- Effects of Nonlinearity
  - Harmonic Distortion
  - Gain Compression
  - Intermodulation
- Characterization of Nonlinearities
- Cascaded Nonlinear Stages

Nonlinearity: Memoryless and Static System

- The input/output characteristic of a *memoryless* or *static* nonlinear system can be approximated with a polynomial.

$$y(t) = \alpha x(t),$$

linear

$$y(t) = \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \cdots$$

nonlinear

Why model? Why not the ”right” thing?

- Nonlinearities often have physical origins.
- For analysis and computer simulation purposes, the physical phenomenon should be described by a mathematical model.
- Here we use Taylor Series for modeling nonlinearities:

$$V_{out}(t) = \alpha_1 V_{in} + \alpha_2 V^2_{in} + \alpha_3 V^3_{in} + \dots$$

$$\alpha_1 = [\partial V_{out}/\partial V_{in}]_{Vin=0}$$

$$\alpha_2 = [\partial^2 V_{out}/\partial V_{in}^2]_{Vin=0}$$

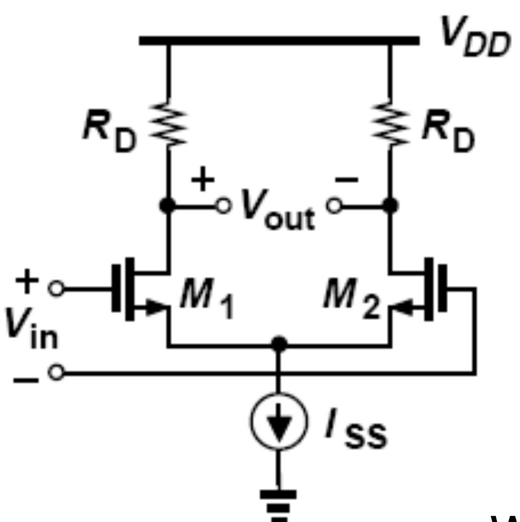
$$\alpha_3 = [\partial^3 V_{out}/\partial V_{in}^3]_{Vin=0}$$

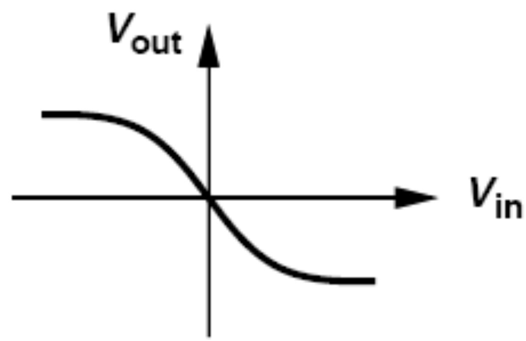
...

- Accuracy of a model is usually limited to the range of input level

Example

- RF amplifiers usually exhibit nonlinear behavior for large input signals, typically as shown below





We will approximate this behavior with a Taylor serier expansion

$V_{out} = \alpha_1 V_{in} + \alpha_2 V^2_{in} + \alpha_3 V^3_{in} + ...$

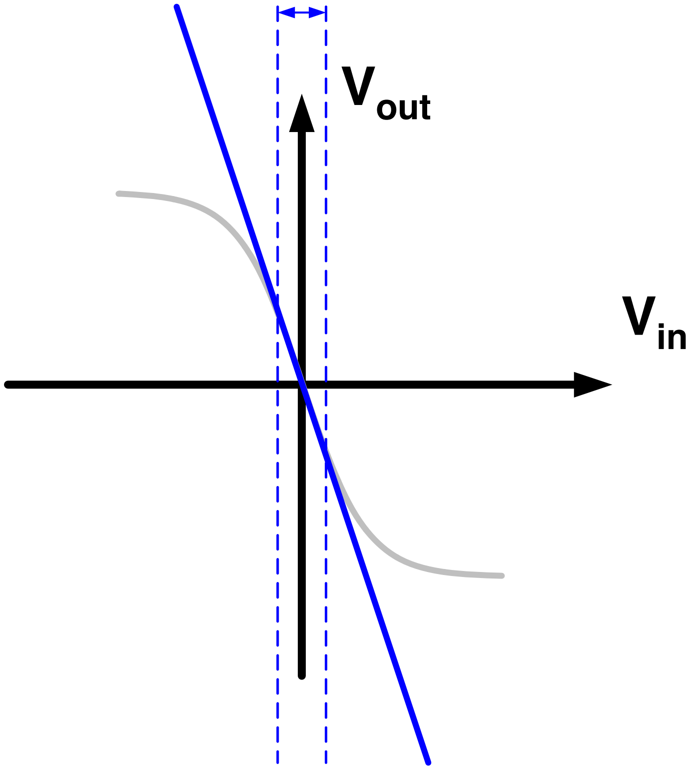
Example

Linear Approximation

– Valid for limited input levels

– α_1 is often called linear gain

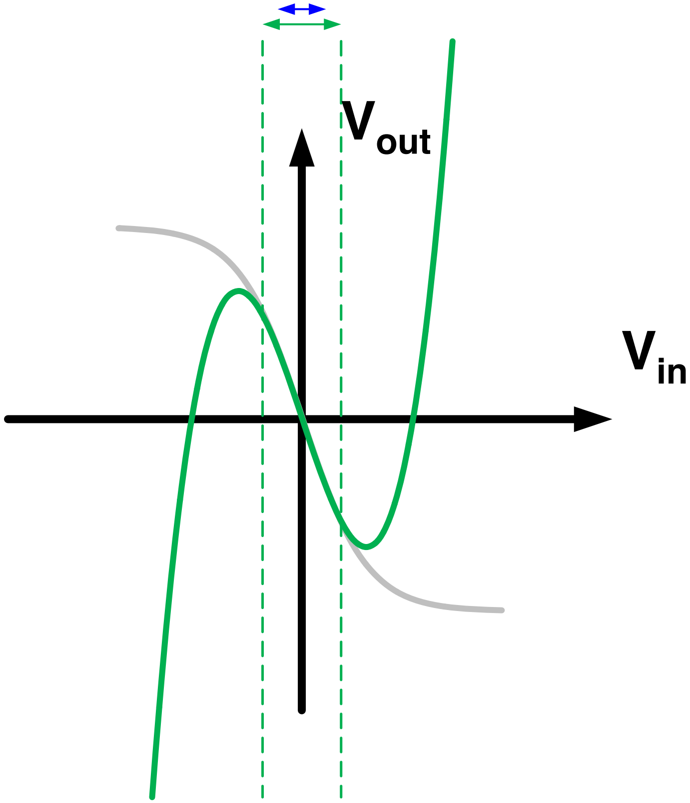
$$V_{out} = \alpha_1 V_{in}$$



Example

More terms should be added as we wish to extent the validity of the model

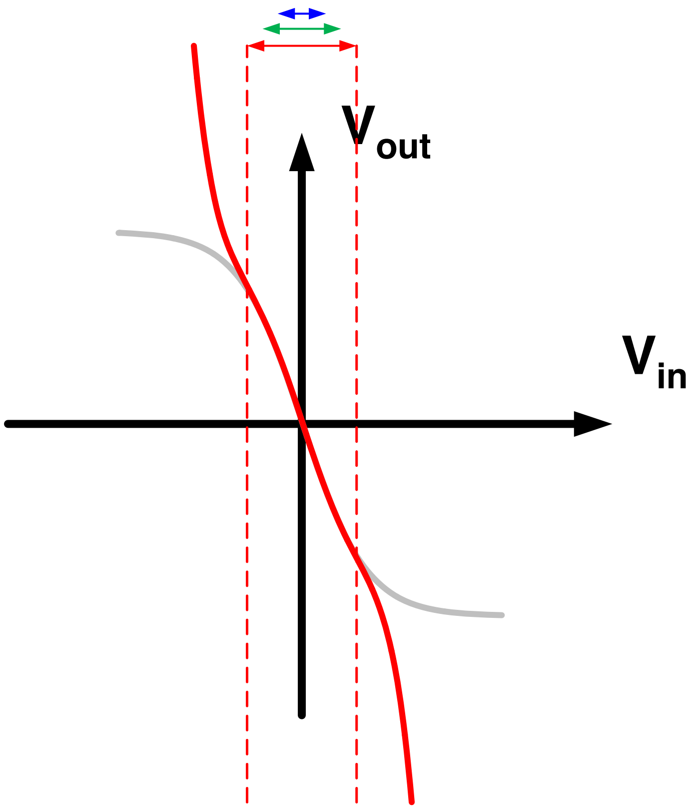
$$V_{out} = a_1 V_{in} + a_3 V^3_{in}$$



Example

More terms should be added as we wish to extent the validity of the model

$$V_{out} = a_1 V_{in} + a_3 V^3_{in} + a_5 V^5_{in}$$



Nonlinearity Issues

- Definition
- Modeling Nonlinearities
- **Effects of Nonlinearity**
  - **Harmonic Distortion (2.2.1)**
  - Gain Compression
  - Intermodulation
- Characterization of Nonlinearities
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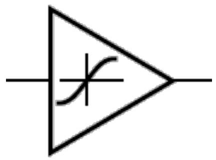
Effects of Nonlinearity

- Consider a nonlinear system

x(t)  $y(t) = \alpha_1 V_{in} + \alpha_2 V^2_{in} + \alpha_3 V^3_{in} + \ldots$

Let us apply a single-tone (*A cos ωt*) to the input and calculate the output:

$$
\begin{aligned}
y(t) &= \alpha_1 A \cos \omega t + \alpha_2 A^2 \cos^2 \omega t + \alpha_3 A^3 \cos^3 \omega t \\
&= \alpha_1 A \cos \omega t + \frac{\alpha_2 A^2}{2}(1 + \cos 2\omega t) + \frac{\alpha_3 A^3}{4}(3 \cos \omega t + \cos 3\omega t) \\
&= \frac{\alpha_2 A^2}{2} + \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right) \cos \omega t + \frac{\alpha_2 A^2}{2} \cos 2\omega t + \frac{\alpha_3 A^3}{4} \cos 3\omega t.
\end{aligned}
$$

DC | **Fundamental** | **Second Harmonic** | **Third Harmonic**

Effects of Nonlinearity

$$
\begin{aligned}
y(t) &= \alpha_1 A \cos \omega t + \alpha_2 A^2 \cos^2 \omega t + \alpha_3 A^3 \cos^3 \omega t \\
&= \alpha_1 A \cos \omega t + \frac{\alpha_2 A^2}{2}(1 + \cos 2\omega t) + \frac{\alpha_3 A^3}{4}(3 \cos \omega t + \cos 3\omega t) \\
&= \frac{\alpha_2 A^2}{2} + \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right) \cos \omega t + \frac{\alpha_2 A^2}{2} \cos 2\omega t + \frac{\alpha_3 A^3}{4} \cos 3\omega t.
\end{aligned}
$$

DC | **Fundamental** | **Second Harmonic** | **Third Harmonic**

- Observations:
  - Even-order harmonics result from α_j with even j, and same for odd
  - n^{th} harmonic grows in proportion to A^n

Example 2.6

The transmitter in a 900-MHz GSM cellphone delivers 1 W of power to the antenna. Explain the effect of the harmonics of this signal.

Solution:

The second harmonic falls within another GSM cell phone band around 1800 MHz and must be sufficiently small to negligibly impact the other users in that band. The third, fourth, and fifth harmonics do not coincide with any popular bands but must still remain below a certain level imposed by regulatory organizations in each country. The sixth harmonic falls in the 5-GHz band used in wireless local area networks (WLANs), e.g., in laptops. Figure 2.8 summarizes these results.

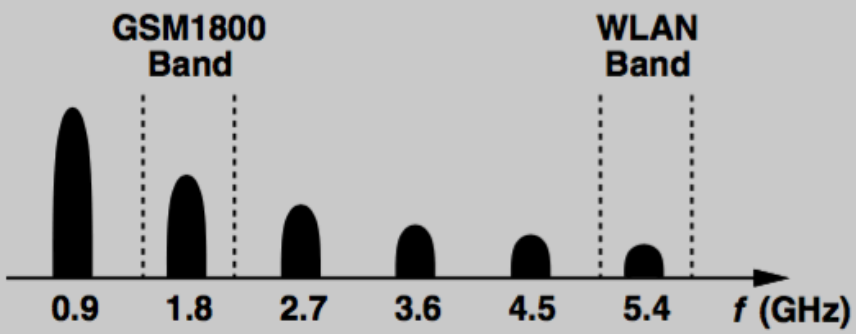
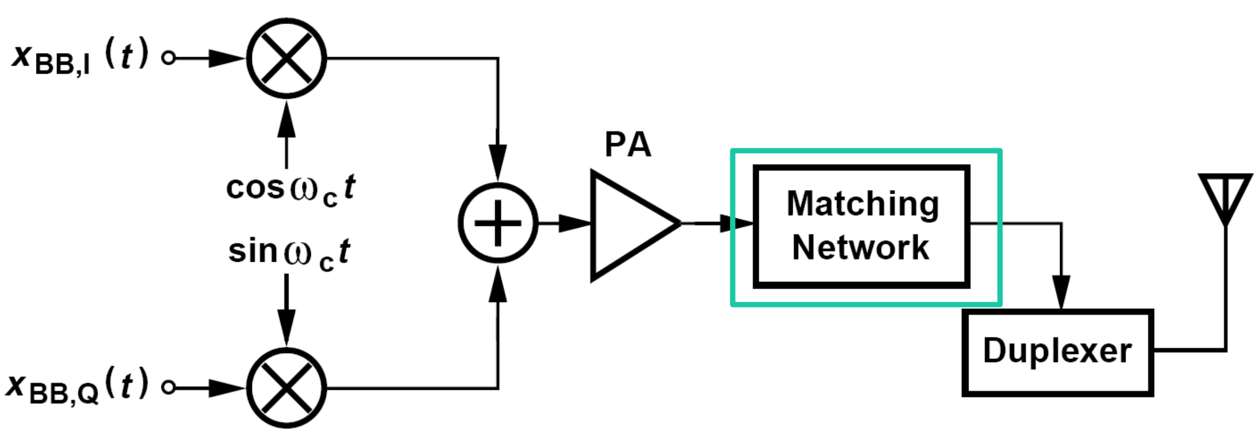


Figure 2.8 *Summary of harmonic components.*

Harmonics: less problem in real circuits, matching network works as a filter.



Nonlinearity Issues

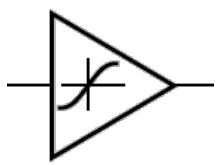
- Definition
- Modeling Nonlinearities
- **Effects of Nonlinearity**
  - Harmonic Distortion
  - **Gain Compression (2.2.2)**
  - Intermodulation
- Characterization of Nonlinearities
- Cascaded Nonlinear Stages

Gain

- We define gain as ratio of signal levels at the same frequency

$$\text{Gain} = \frac{\left(\alpha_1 + \tfrac{3}{4}\alpha_3 A_{in}^2\right) A_{in}}{A_{in}} = \alpha_1 + \tfrac{3}{4}\alpha_3 A_{in}^2$$

$$X(t) = A_{in} \cos \omega t$$

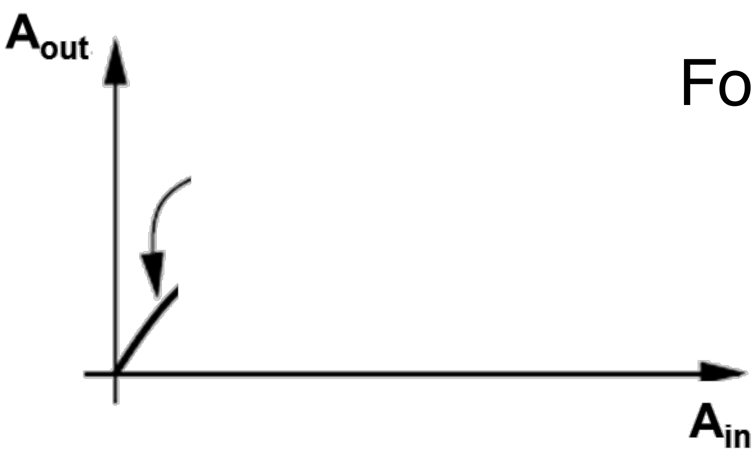


$$\begin{aligned} Y(t) &= \tfrac{1}{2}\alpha_2 A_{in}^2 \\ &+ \left(\alpha_1 + \tfrac{3}{4}\alpha_3 A_{in}^2\right) A_{in} \cos \omega t \\ &+ \tfrac{1}{2}\alpha_2 A_{in}^2 \cos 2\omega t \\ &+ \tfrac{1}{4}\alpha_3 A_{in}^3 \cos 3\omega t + \dots \end{aligned}$$

Gain

- We define gain as ratio of signal levels at the same frequency

$$\text{Gain} = \frac{(\alpha_1 + {}^3/_4\alpha_3 A^2_{in})\, A_{in}}{A_{in}} = \alpha_1 + {}^3/_4\alpha_3 A^2_{in}$$

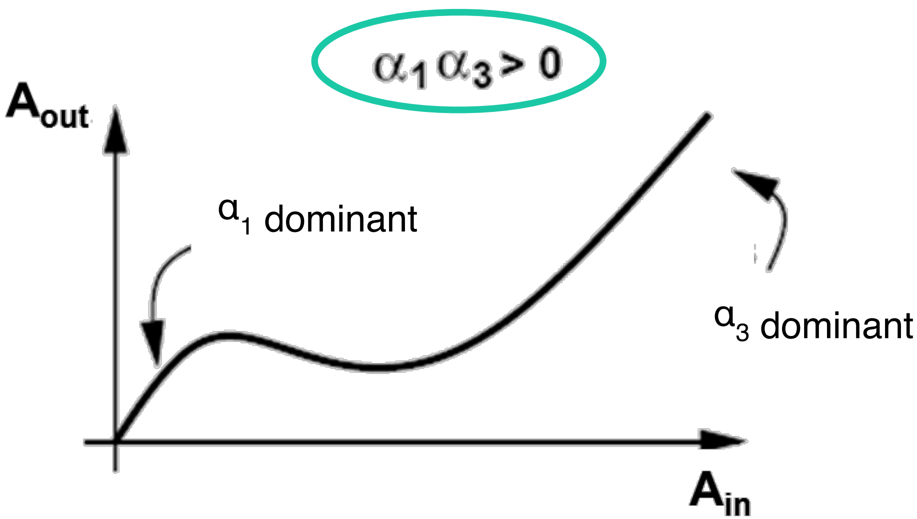


For small input levels, gain is α_1 and it is linear

Gain (expansive)

- We define gain as ratio of signal levels at the same frequency

$$\text{Gain} = \frac{(\alpha_1 + {}^3/_4\alpha_3 A^2_{in})\, A_{in}}{A_{in}} = \alpha_1 + {}^3/_4\alpha_3 A^2_{in}$$

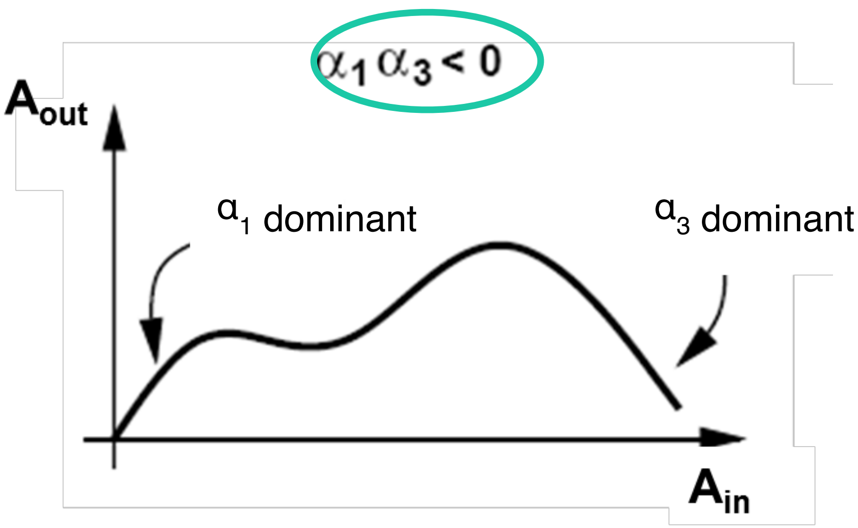


For larger input levels, depending on sign of $\alpha_1\alpha_3$ gain may expand

Gain (compressive)

- We define gain as ratio of signal levels at the same frequency

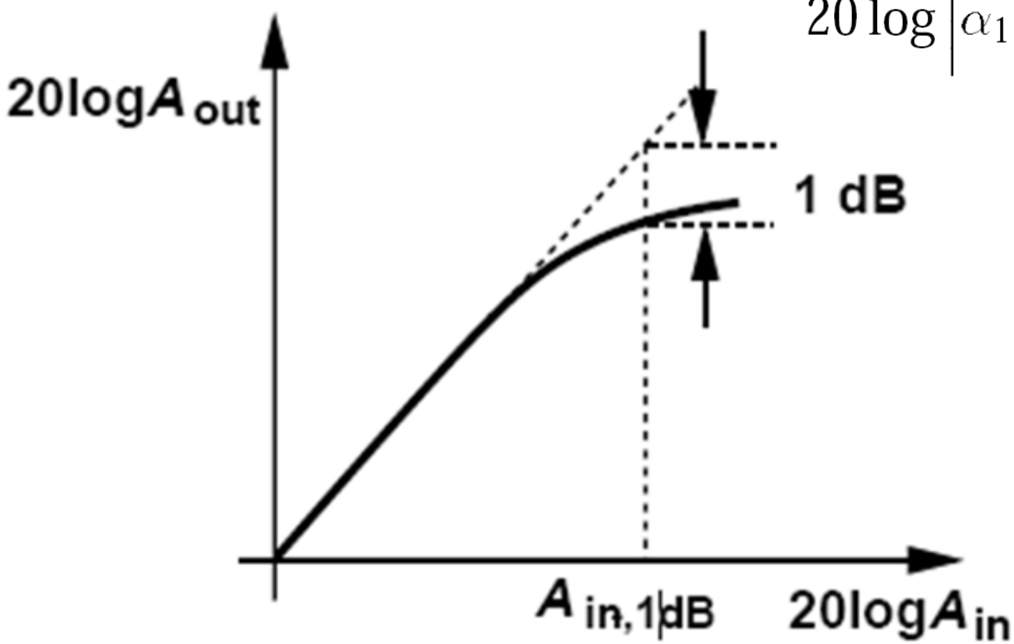
$$\text{Gain}=\frac{\left(\alpha_1+{}^3/_4\alpha_3A^2_{in}\right)A_{in}}{A_{in}}=\alpha_1+{}^3/_4\alpha_3A^2_{in}$$



For larger input levels, depending on sign of $\alpha_1\alpha_3$, gain may expand or compress. Many electronic systems have compressive gain behavior.

Gain Compression (1dB, P_{1dB},P_{-1dB})

- Eventually at large enough signal levels, output power does not follow the input power



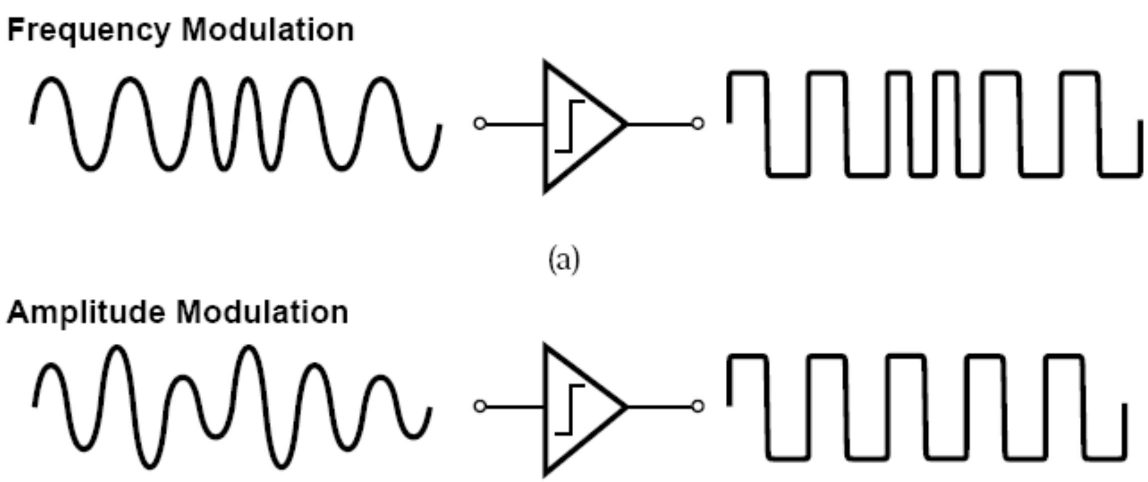
$$20 \log \left| \alpha_1 + \frac{3}{4} \alpha_3 A_{in,1dB}^2 \right| = 20 \log |\alpha_1| - 1 \text{ dB}.$$

$$A_{in,1dB} = \sqrt{0.145 \left| \frac{\alpha_1}{\alpha_3} \right|}.$$

The P_{1dB} point correlates well to loss of linear behavior, getting out-of-spec in standards (EVM, ACPR, etc.) so for linear applications, operation beyond this point is useless.

Effect of compression for FM/PM and AM-modulated signals

- FM signal carries no information in its amplitude and hence tolerates compression.
- AM contains information in its amplitude, hence distorted by compression



Example 2.7

A 900-MHz GSM transmitter delivers a power of 1 W to the antenna. By how much must the second harmonic of the signal be suppressed (filtered) so that it does not desensitize a 1.8-GHz receiver having $P_{1dB} = -25$ dBm? Assume the receiver is 1 m away (Fig. 2.13) and the 1.8-GHz signal is attenuated by 10 dB as it propagates across this distance.

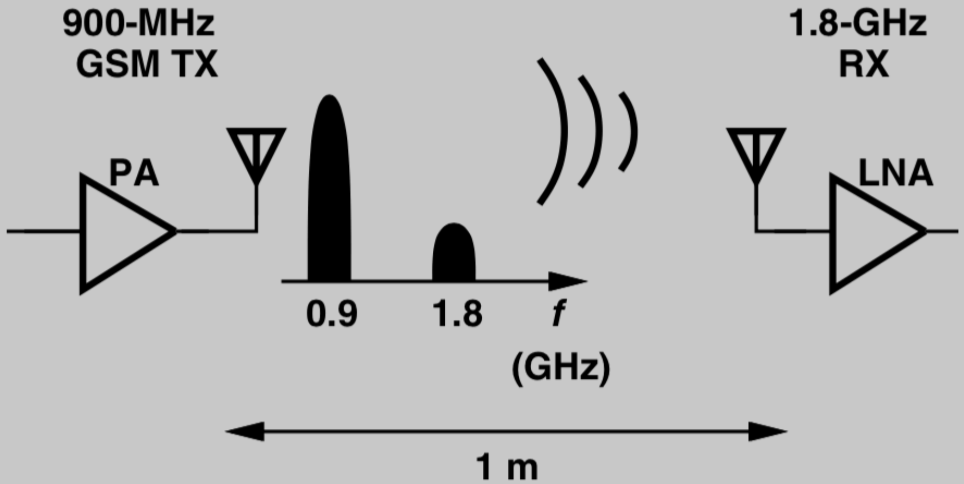


Figure 2.13 *TX and RX in a cellular system.*

Solution:

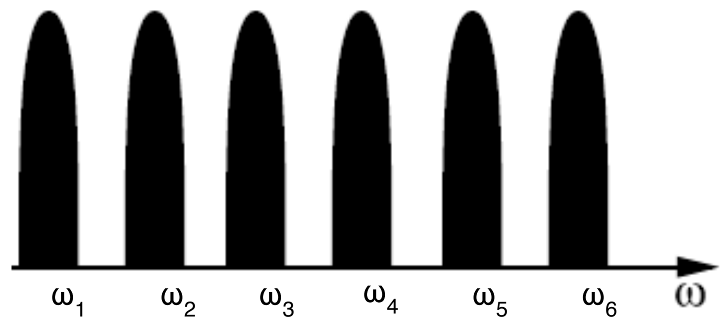
The output power at 900 MHz is equal to +30 dBm. With an attenuation of 10 dB, the second harmonic must not exceed −15 dBm at the transmitter antenna so that it is below P_{1dB} of the receiver. Thus, the second harmonic must remain at least 45 dB below the fundamental at the TX output. In practice, this interference must be another several dB lower to ensure the RX does not compress.

Nonlinearity Issues

- Definition
- Modeling Nonlinearities
- **Effects of Nonlinearity**
  - Harmonic Distortion
  - Gain Compression
  - **Intermodulation (2.2.4)**
- Characterization of Nonlinearities
- Cascaded Nonlinear Stages

Why do we care about intermodulation?

- Communication systems often use the available frequency band to transmit multiple channels

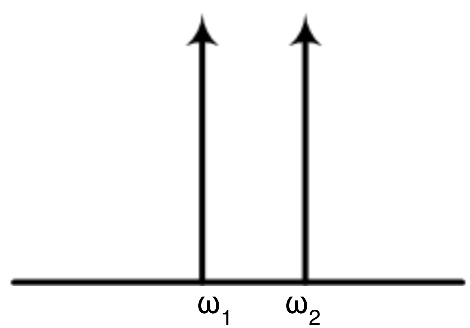


- What happens to these signals when the transmitter exhibits nonlinear behavior?

Intermodulation (IM)

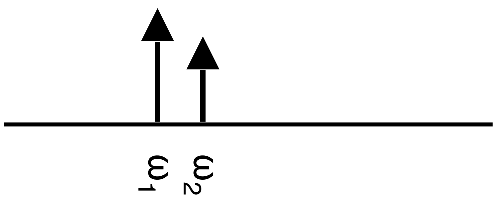
For ease of analysis we only consider two channels

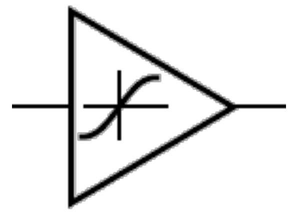
and approximate them as single-tone sinusoidal



Intermodulation

$A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$

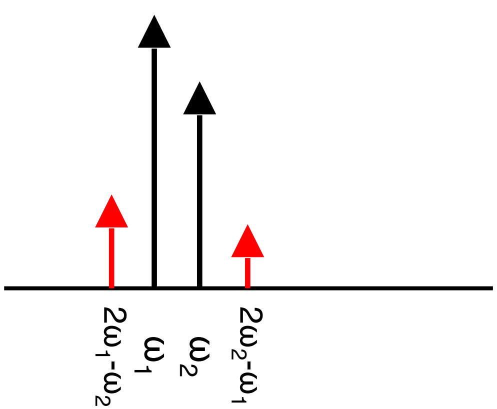




$$
\begin{aligned}
& \alpha_1(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t) \\
+ & \alpha_2(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^2 \\
+ & \alpha_3(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^3
\end{aligned}
$$

$$
\begin{aligned}
= & \tfrac{1}{2}\alpha_2 (A^2_1 + A^2_2) \\
+ & [\alpha_1 + \tfrac{3}{4}\alpha_3 A^2_1 + \tfrac{3}{2}\alpha_3 A^2_2]\, A_1 \cos\omega_1 t \\
+ & [\alpha_1 + \tfrac{3}{4}\alpha_3 A^2_2 + \tfrac{3}{2}\alpha_3 A^2_1]\, A_2 \cos\omega_2 t \\
+ & [\alpha_2 A_1 A_2] \cos (\omega_1 \pm \omega_2)t \\
+ & [\tfrac{1}{2}\alpha_2 A^2_1] \cos 2\omega_1 t \\
+ & [\tfrac{1}{2}\alpha_2 A^2_2] \cos 2\omega_2 t \\
+ & [\tfrac{3}{4}\alpha_3 A^2_1 A_2] \cos (2\omega_1 + \omega_2)t \\
+ & [\tfrac{3}{4}\alpha_3 A^2_2 A_1] \cos (2\omega_2 + \omega_1)t \\
+ & [\tfrac{3}{4}\alpha_3 A^2_1 A_2] \cos (2\omega_1 - \omega_2)t \\
+ & [\tfrac{3}{4}\alpha_3 A^2_2 A_1] \cos (2\omega_2 - \omega_1)t \\
+ & [\tfrac{1}{4}\alpha_3 A^3_1] \cos 3\omega_1 t \\
+ & [\tfrac{1}{4}\alpha_3 A^3_2] \cos 3\omega_2 t
\end{aligned}
$$

Third Order Intermodulation (IM3) products



Intermodulation

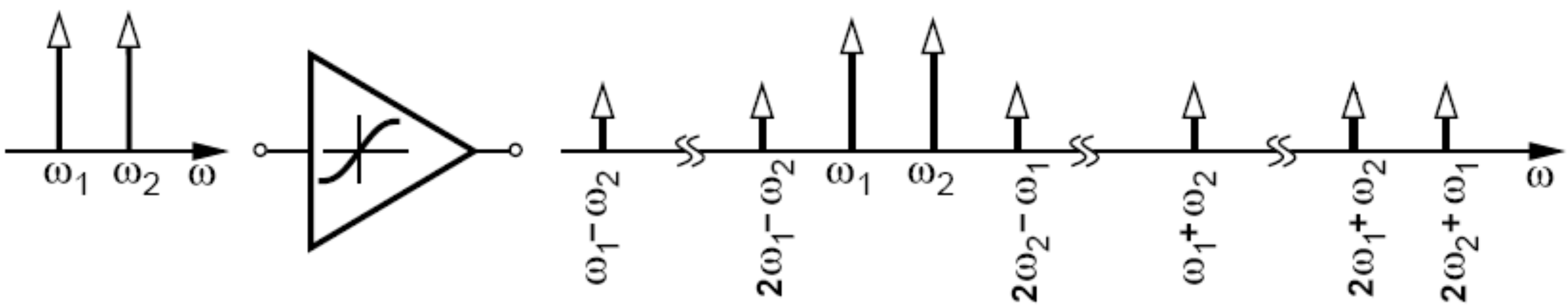
Fundamental components:

$$\omega = \omega_1,\ \omega_2 :\ \left(\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2\right)\cos\omega_1 t + \left(\alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2^3 + \frac{3}{2}\alpha_3 A_2 A_1^2\right)\cos\omega_2 t$$

Intermodulation products:

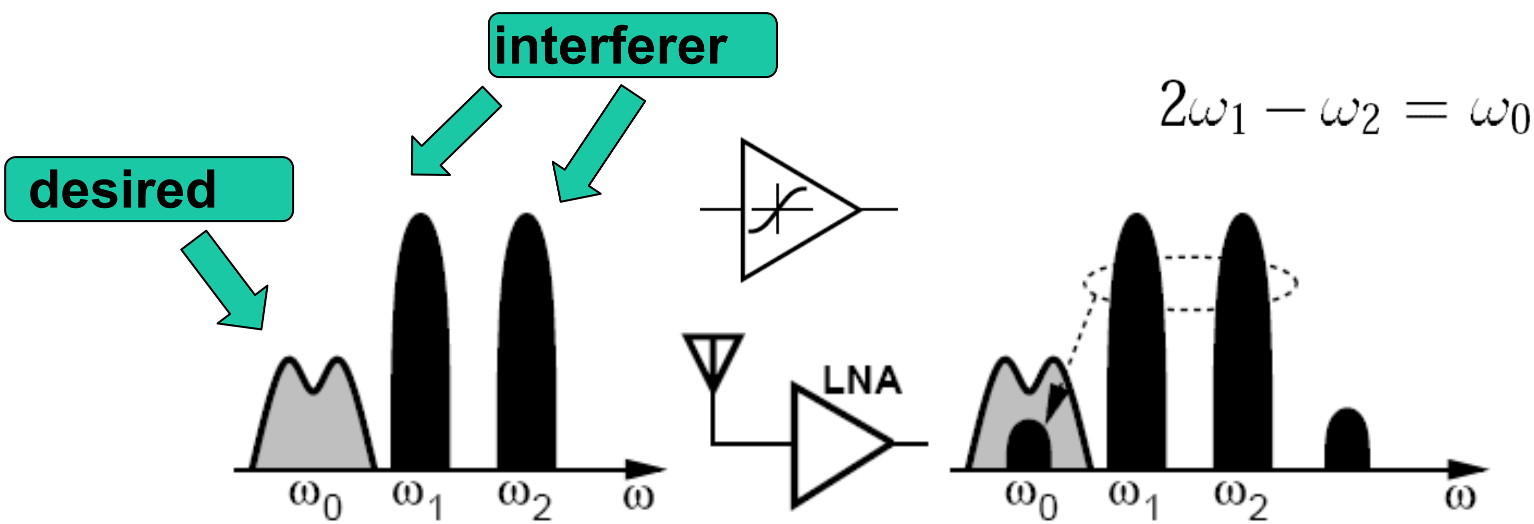
$$\begin{aligned}
\omega &= 2\omega_1 \pm \omega_2 :\ \frac{3\alpha_3 A_1^2 A_2}{4}\cos(2\omega_1 + \omega_2)t + \frac{3\alpha_3 A_1^2 A_2}{4}\cos(2\omega_1 - \omega_2)t \\
\omega &= 2\omega_2 \pm \omega_1 :\ \frac{3\alpha_3 A_1 A_2^2}{4}\cos(2\omega_2 + \omega_1)t + \frac{3\alpha_3 A_1 A_2^2}{4}\cos(2\omega_2 - \omega_1)t
\end{aligned}$$

Intermodulation



Intermodulation

- IM3 products do not interfere with main tones, so why should we be worried?
- They interfere with adjacent channels!
- Intermodulation products are troublesome both in transmitter and in receiver.



Intermodulation

Example 2.9

Suppose four Bluetooth users operate in a room as shown in Fig. 2.17. User 4 is in the receive mode and attempts to sense a weak signal transmitted by User 1 at 2.410 GHz.

Example 2.9 *(Continued)*

At the same time, Users 2 and 3 transmit at 2.420 GHz and 2.430 GHz, respectively. Explain what happens.

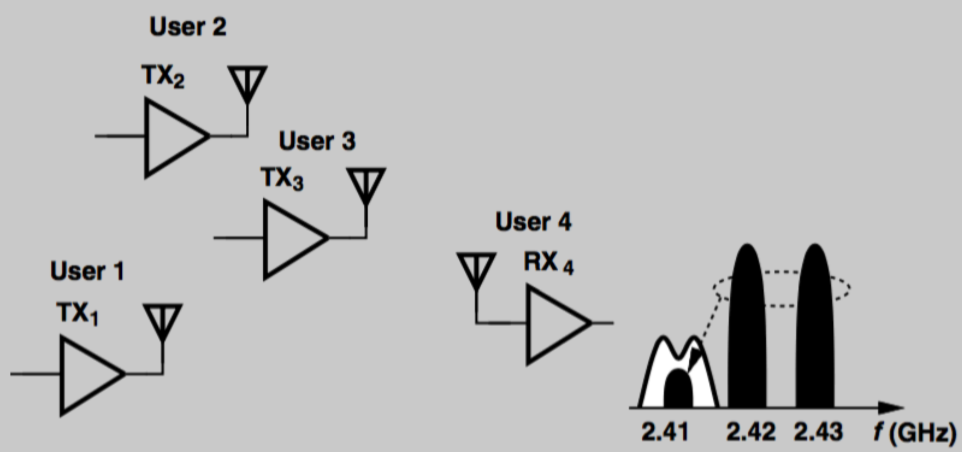


Figure 2.17 *Bluetooth RX in the presence of several transmitters.*

Solution:

Since the frequencies transmitted by Users 1, 2, and 3 happen to be equally spaced, the intermodulation in the LNA of RX_4 corrupts the desired signal at 2.410 GHz.

Nonlinearity Issues

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- Cascaded Nonlinear Stages

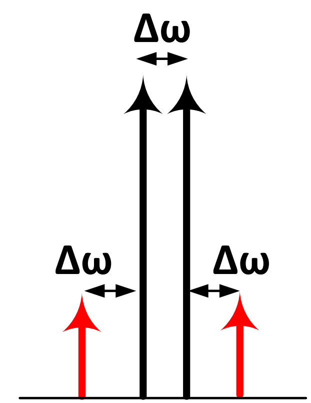
Intermodulation - Characterization

- We generate a test signal for Intermodulation characterization
  - Test Signal : $A_1 \cos \omega_1 t + A_2 \cos \omega_2 t = A \cos \omega_1 t + A \cos (\omega_1 + \Delta\omega)t$
  - Assumptions:
    - $A_1 = A_2 = A$
    - $\Delta\omega = \omega_2 - \omega_1$

this is called
a two-tone test

- We write the output signal again (after filtering high frequency components)

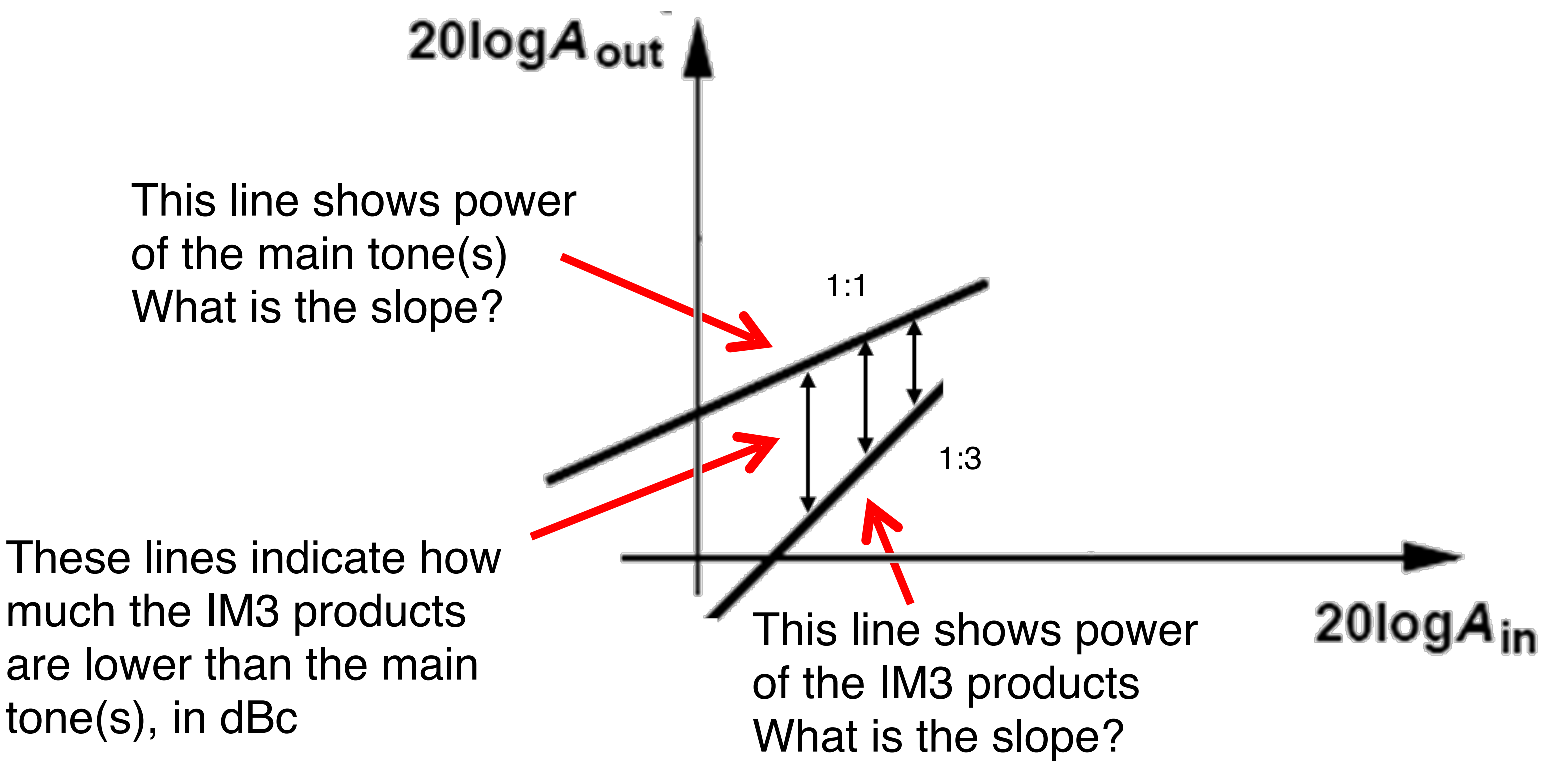
$$\text{Out} = [{}^3/_4 \alpha_3 A^3] \cos (\omega_1 - \Delta\omega)t$$
$$+[\alpha_1 A + {}^9/_4 \alpha_3 A^3] (\cos\omega_1 t + \cos (\omega_1 + \Delta\omega)t)$$
$$+[{}^3/_4 \alpha_3 A^3] \cos (\omega_1 + 2\Delta\omega)t$$



1) Frequency separations are the same
2) Before compression, main tones grow with A and IM3 products grow with A^3

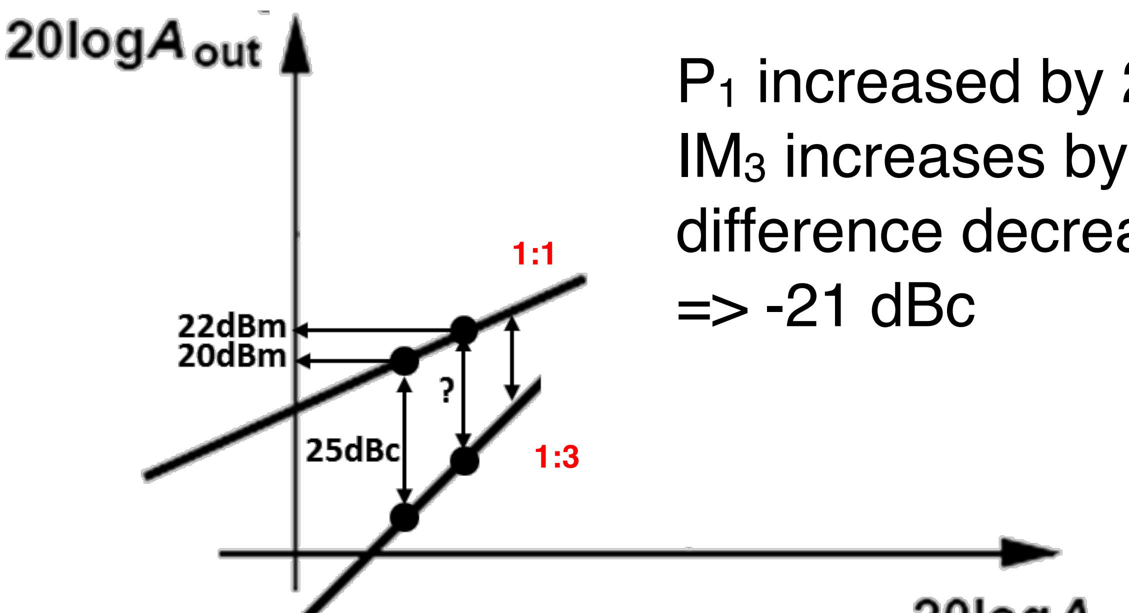
Intermodulation - Characterization

- Recall the Pin-Pout plot, and let us draw the main tones and the IM3 products



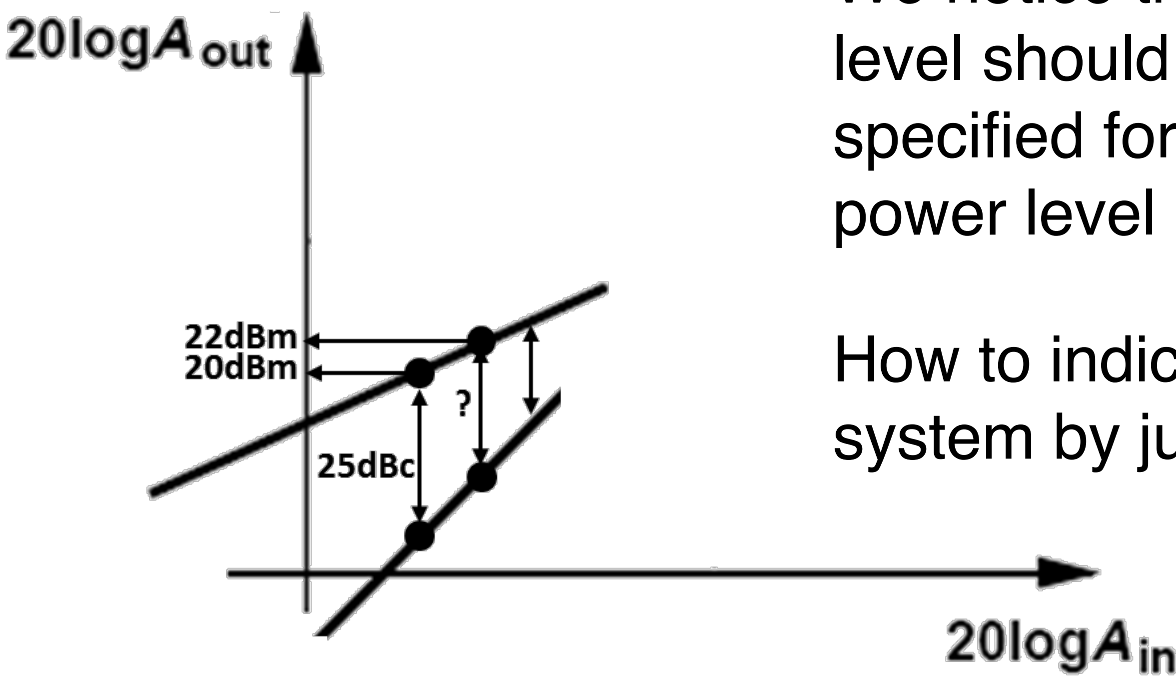
Example

If we measure relative IM3 to be -25 dBc at output power of 20 dBm, what would it be at 22 dBm?



P_1 increased by 2 dB (1:1)
IM_3 increases by 6 dB (1:3)
difference decreases by 4 dB,
=> -21 dBc

Intermodulation – Intercept Point



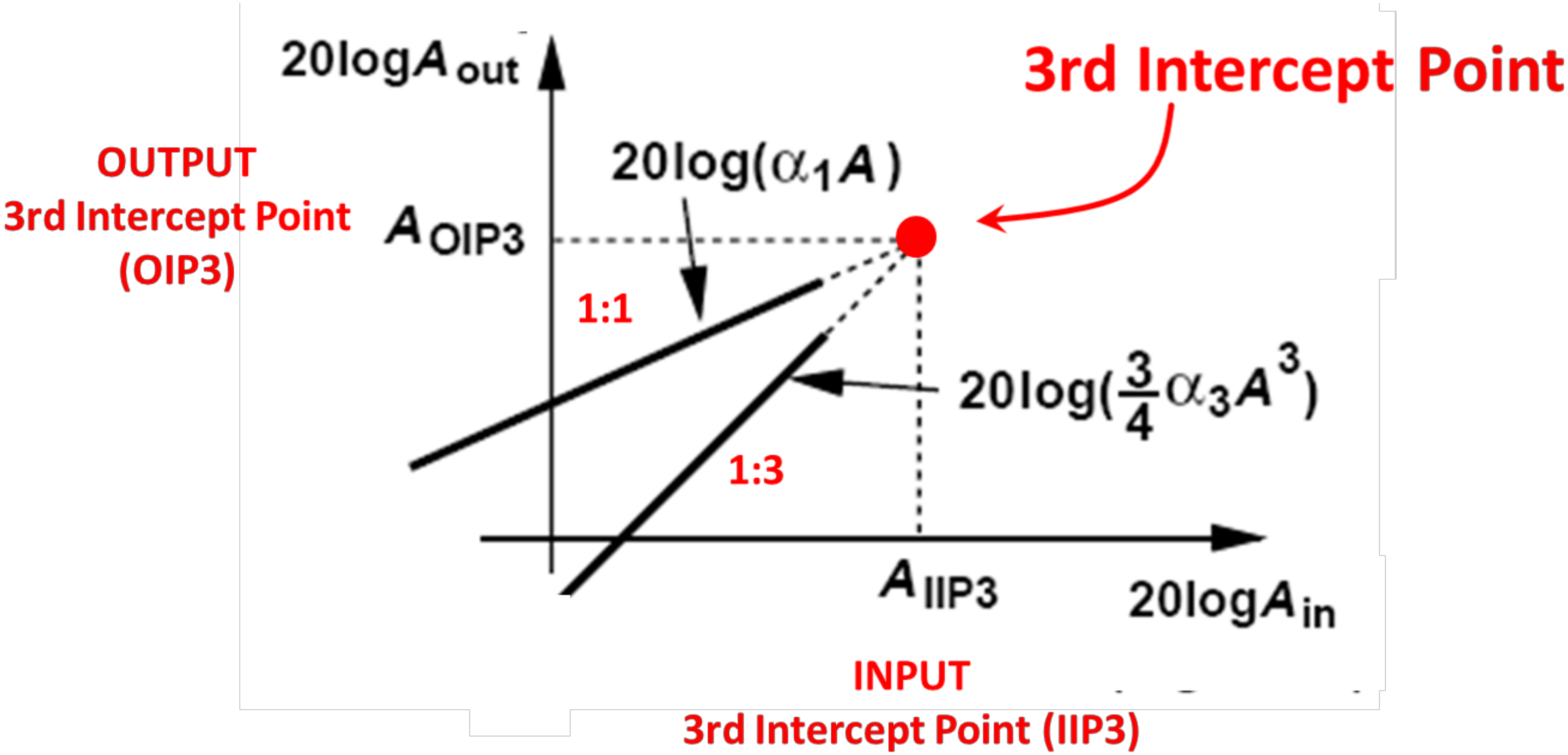
We notice that relative IM_3 level should always be specified for an output power level

How to indicate linearity of a system by just one number?

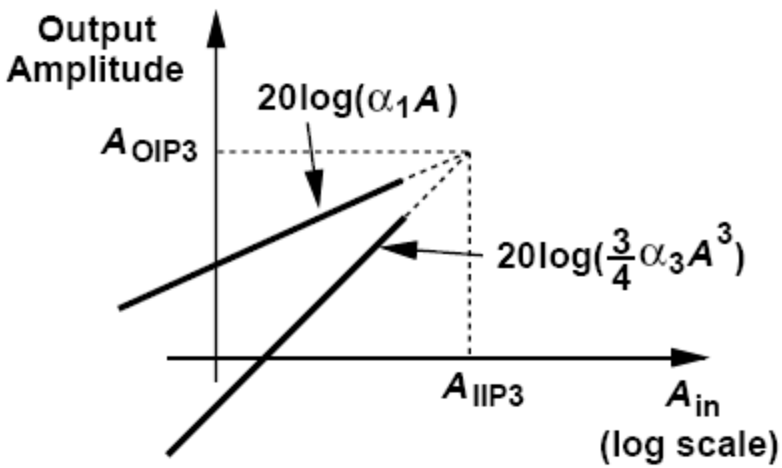
Intermodulation – Intercept Point

$OIP3[dBm]=IIP3[dBm] + G[dB]$

$OIP3[dBm]=P_1[dBm] + 0.5\Delta P[dB]$



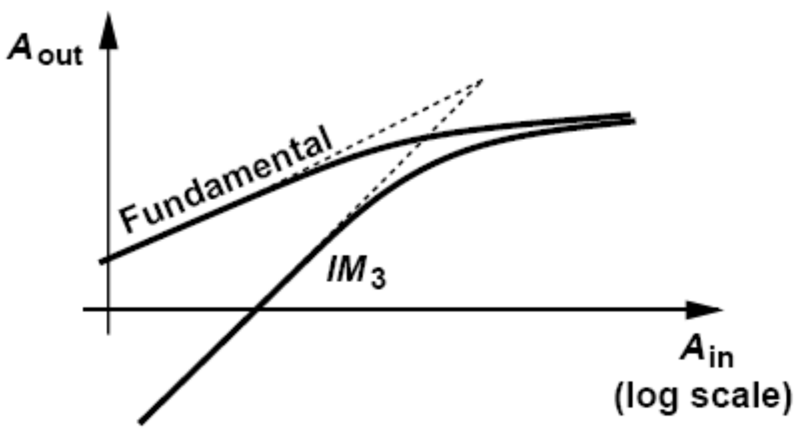
Intermodulation – Intercept Point

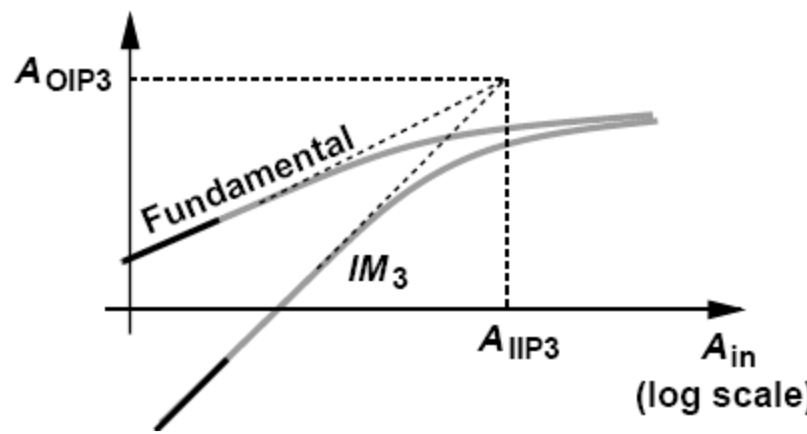


$$|\alpha_1 A_{IIP3}| = \left|\frac{3}{4}\alpha_3 A_{IIP3}^3\right|$$

$$A_{IIP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}.$$

$$\frac{A_{IIP3}}{A_{1dB}} = \sqrt{\frac{4}{0.435}} \approx 9.6 \text{ dB}.$$





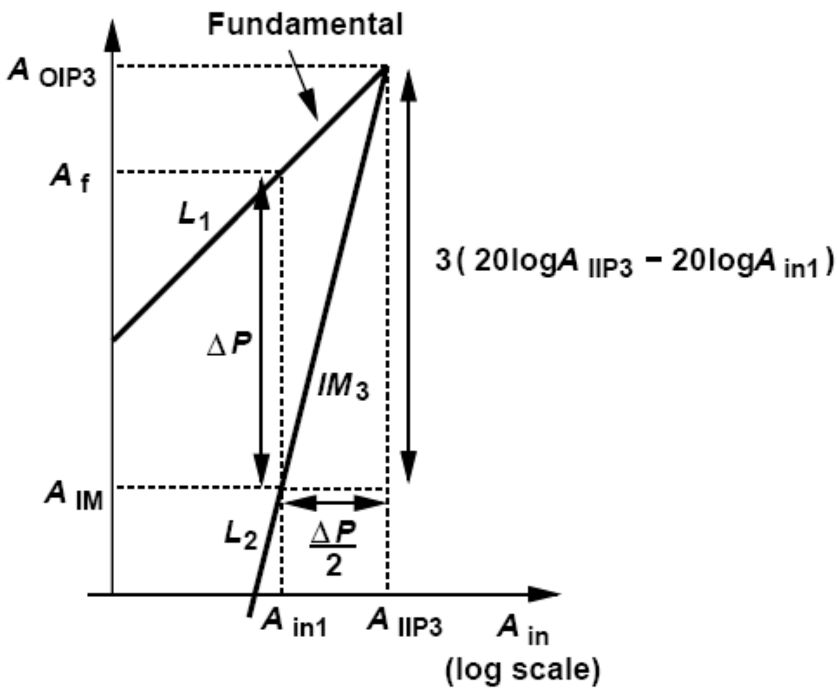
Intermodulation: IP_3 and P_{1dB}

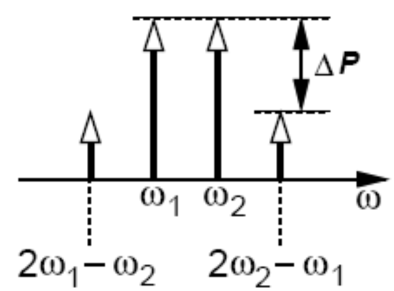
$$\frac{A_{IIP3}}{A_{1dB}} = \sqrt{\frac{4}{0.435}} \approx 9.6 \text{ dB}.$$

- Under ideal conditions, the IP_3 is 9.6 dB higher than the P_1dB point (compression).
- So if you know one of these number, you can estimate the other!

Intermodulation – Intercept Point

For a given input level (well below P_1dB), the IIP_3 can be calculated by halving the difference between the output fundamental and IM levels and adding the result to the input level, where all values are expressed as logarithmic quantities.



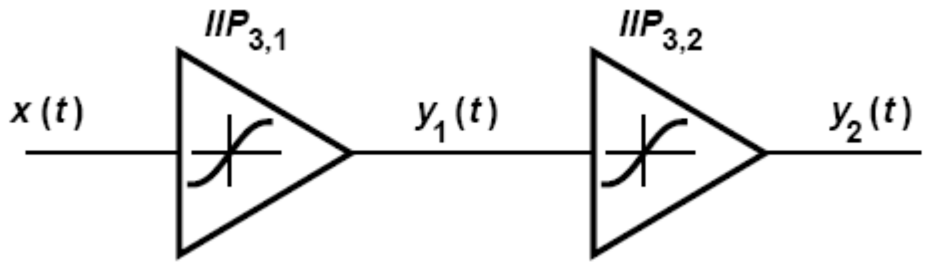


$$IIP_3\big|_{dBm} = \frac{\Delta P\big|_{dB}}{2} + P_{in}\big|_{dBm}$$

Nonlinearity Issues

- Definition
- Modeling Nonlinearities
- Effects of Nonlinearity
  - Harmonic Distortion
  - Gain Compression
  - Intermodulation
- Characterization of Nonlinearities
- **Cascaded Nonlinear Stages (2.2.5)**

Cascaded Nonlinear Stages



$$y_1(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

$$y_2(t) = \beta_1 y_1(t) + \beta_2 y_1^2(t) + \beta_3 y_1^3(t)$$

$$\begin{aligned} y_2(t) &= \beta_1[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)] + \beta_2[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^2 \\ &+ \beta_3[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^3. \end{aligned}$$

Cascaded Nonlinear Stages

- Considering only the first- and third-order terms, we have:

$$y_2(t) = \alpha_1\beta_1 x(t) + (\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3)x^3(t) + \cdots$$

- Thus: (Eq. 2.47) $A_{IIP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}.$

$$A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1\beta_1}{\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3}\right|}.$$

Cascaded Nonlinear Stages

- A more intuitive view: square and invert the two sides of the equation

$A_{IP3,1}$ and $AI_{P3,2}$ represent the input IP3s of the first and second stages

$$\frac{1}{A_{IP3}^2} = \frac{3}{4}\left|\frac{\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3}{\alpha_1\beta_1}\right|$$
$$= \frac{3}{4}\left|\frac{\alpha_3}{\alpha_1} + \frac{2\alpha_2\beta_2}{\beta_1} + \frac{\alpha_1^2\beta_3}{\beta_1}\right|$$
$$= \left|\frac{1}{A_{IP3,1}^2} + \frac{3\alpha_2\beta_2}{2\beta_1} + \frac{\alpha_1^2}{A_{IP3,2}^2}\right|$$

=> The higher gain of the first stage, the more nonlinearity is contributed by the second stage.

Cascaded Nonlinear Stages

- For more stages: $$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2} + \frac{\alpha_1^2\beta_1^2}{A_{IP3,3}^2} + \cdots$$
- If each stage in a cascade has a gain greater than unity, the nonlinearity of the latter stages becomes increasingly more critical because the IP_3 of each stage is equivalently scaled down by the total gain preceding that stage.

$$\frac{1}{IIP3_{total}} = \frac{1}{IIP3_A} + \frac{G_A}{IIP3_B} + \frac{G_A G_B}{IIP3_C}$$

- The higher gain of the first stage, the more nonlinearity is contributed by the second stage.
- Note that IIP3 and OIP3 are related through gain.

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