## Exam in Statistical Methods, 2014-12-17

Time allowed:
Allowed aids:
Assisting teacher:
Grades:
kl: 8-12
Calculator. One handwritten A4 paper (both sides) with the students own notes.
Lotta Hallberg
$A=19-20$ points, $B=17-18 p, C=14-16 p, D=12-13 p, E=10-11 p$

Provide a detailed report that shows motivation of the results.

## 1

Let $f(y \mid \alpha, \beta)=\frac{1}{\alpha} e^{\frac{-(y-\beta)}{\alpha}}, \quad \beta<y<\infty, \quad 0<\alpha<\infty$ be density function to the random variable $Y$, $\alpha$ and $\beta$ are parameters
a) Show that $f(y \mid \alpha, \beta)$ is a density function. 2 p
b) Determine the distribution function $1 p$
c) Calculate the probability $P(\beta+1<Y \leq \beta+2)$ when $\alpha=2$ 1p

## 2

Let the bivariate random variable $(X, Y)$ have density function:
$f(x, y)=k(x+2 y)$ where $0<2 y<x<2$.
a) Determine $k$. 2p
b) Calculate $E[X \mid Y=1 / 2]$ 3p

## 3

A company's management want to investigate the stress level of the employees. Therefore they check with 40 randomly selected employees and ask if they feel stress at work and 8 of them answered yes. Assume that the total number of employees is very large.

Estimate the proportion $p$ of stressed employees in the company using:
a) Method of moments. $1 p$
b) Maximum Likelihood method. $2 p$
c) Bayes method. Use the conjugate beta prior, beta(2,4) $2 p$
d) Test the hypothesis $H_{0}: p=0,15$ against $H_{a}: p>0,15$ using the observation above. Use large sample theory. $10 \%$ significance level.
$2 p$

LH

## 4

The following data are measured on 7 female runners.
Step = average number of steps per second
$\mathrm{m} / \mathrm{s}=$ running speed, meters per second.
A runner are assumed to be good if the number of steps per second increase with the speed.

| Step $=\mathbf{Y}$ | $\mathrm{m} / \mathrm{s}=\mathrm{x}$ |
| :--- | :--- |
| 3.05 | 4.76 |
| 3.12 | 5.06 |
| 3.17 | 5.25 |
| 3.25 | 5.59 |
| 3.36 | 5.99 |
| 3.46 | 6.32 |
| 3.55 | 6.63 |

a) Set up the simple linear regression model and estimate the regression parameters $\beta_{0}$ and $\beta_{1}$.
b) Test if the slope is zero. You may use without showing any calculations that $\mathrm{SSE}=0,00043$. Use 5\% significance level. Interpret your result. 2p

