LINKÖPINGS UNIVERSITET

Department of Science and Technology
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Exam TEN1 for TNE041, **Modern Physics**, 31 August 2024, 8.00 – 12.00.

Allowed examination material: Physics handbook (Studentlitteratur)
calculator (with no wifi)
additional formulae (attached)
one hand-written sheet (A4, not copied, with notes on one side)

Define all quantities you use and give a clear answer, including unit if a numerical value is given. No points are given if only the answer is submitted, with the exception of true/false questions. The maximum score is 24 points (6x4). The limits for different grades given below are with bonus included. The solutions may be given in English or in Swedish.

The following limits for grades apply:

Grade 3	$\geq$ 10 points
Grade 4	$\geq$ 15 points
Grade 5	$\geq$ 19 points

Questions are answered by Michael Hörnquist who is available on phone during the whole exam. Answers and short solutions will be available at Studieinfo at 3 pm at the latest. Results will be reported not later than 15 working days after the exam.

Good luck!

1. (a) Are the following statements true or false?
   i. The photoelectric effect shows that light is a particle phenomenom.
   ii. The function $\Psi(x,t) = A\cos(kx - \omega t)$ is a solution to the time-dependent Schrödinger equation for a free particle if $\omega = \hbar k^2/(2m)$.
   iii. The transmission coefficient T for a free electron with energy 10 eV incident on a potential barrier of height 5 eV and width 2 nm is $T = 1$.
   iv. The Pauli principle applies to both electrons and protons.

   Only the answers (true/false) are required. (2p)

   (b) Consider an electron accelerated from rest by a potential of 1 GV. How much less than the speed of light is it travelling after the acceleration? Answer in meters per second. (2p)

2. A Bragg diffraction experiment is conducted using a beam of electrons accelerated through a 1,0 kV potential difference.

   (a) If the spacing between the atomic planes in the crystal is 0,1 nm, what are the values of lhe least two positive angles with respect to the planes where diffraction maxima can be observed? (2p)

   (b) If a beam of X-rays produces diffraction maxima at the same angles as the electron beam, what is the X-ray photon energy? (2p)

3. Consider a QM-particle described by the wave function ψ given as

$$\psi(x) = \begin{cases} 0 & \text{if } x < 0 \\ 2a^{3/2}xe^{-ax} & \text{if } x \geq 0 \end{cases}$$

   with $a = 1,0\ (\text{nm})^{-1}$. The wave function is normalized.

   (a) Determine the expectation value of the position of the particle. (2p)

   (b) Calculate the uncertainty in the position of the particle, i.e., determine Δx. (2p)

4. An electron is in the $n = 2, \ell = 1, m_\ell = 1, m_s = -1/2$ state of a hydrogen atom.

   (a) What is its energy? (1p)

   (b) Which of these quantum numbers are consequences of the Schrödinger equation? (1p)

   (c) What is the absolute value of the angular momentum (expressed in $\hbar$) and what is its angle to the positive z-axis? (2p)

5. The rms-value ("root-mean-square") of a quantity x is given by $x_{rms} = \sqrt{\overline{x^2}}$, where the bar indicates the expectation value.

   Determine the rms-value for the speed v of a classical gas molecule, assuming non-interactions among the molecules, temperature T, mass m and no effects of gravity. (4p)

6. Some semiconductors are not transparent to visible light, but to "light" of larger wavelength (infrared). This is the case for, e.g., Indium phosphide (InP).

   Explain how this can be, using the concept of band gaps. Necessary data might be taken from Physics Handbook. (4p)

ADDITIONAL FORMULAE TNE041 MODERN PHYSICS

Special relativity:

Momentum $\mathbf{p} = \gamma_u m\mathbf{u}$ where $\gamma_u = \dfrac{1}{\sqrt{1-\dfrac{u^2}{c^2}}}$

Energy $E = \gamma_u mc^2$ $E^2 = p^2c^2 + m^2c^4$

Internal (rest) energy $E_{int} = mc^2$ Kinetic energy $E_{kin} = (\gamma_u - 1)mc^2$

Quantum mechanics:

Penetration depth $\delta = \dfrac{\hbar}{\sqrt{2m(U_0 - E)}}$ Gaussian wave packet $\psi(x) = Ae^{-(x/2\varepsilon)^2}e^{ik_0x}$

If $\delta << L$ (barrier width) then the transmission coefficient can be approximated as

$$T \approx 16\frac{E}{U_0}(1-\frac{E}{U_0})e^{-2(\sqrt{2m(U_0-E)}/\hbar)L}$$

Solutions of the time independent Schrödinger equation for a particle with mass m in an infinite well, side lengths L_x, L_y, L_z:

$$\psi_{n_x,n_y,n_z}(x,y,z) = A\sin\frac{n_x\pi x}{L_x}\sin\frac{n_y\pi y}{L_y}\sin\frac{n_z\pi z}{L_z} \text{ and } E_{n_x,n_y,n_z} = \left(\frac{n_x^2}{L_x^2}+\frac{n_y^2}{L_y^2}+\frac{n_z^2}{L_z^2}\right)\frac{\pi^2\hbar^2}{2m}$$

where $n_x, n_y, n_z = 1,2,\ldots$

Statistical mechanics:

Distribution functions (the probability that a state with energy E (E_i) is occupied)

Maxwell-Boltzmann: $N(E) = Ae^{-E/k_BT}$ (continuous) or $N(E_i) = \dfrac{g_i}{Z}e^{-E_i/k_BT}$ (discrete) ,

g_i: degree of degeneracy for energy level E_i ,partition function $Z = \sum_i g_i e^{-E_i/k_BT}$

Fermi-Dirac: N(E) = $\frac{1}{e^{(E-E_F)/k_B T}+1}$ Bose-Einstein: N(E) = $\frac{1}{e^{\alpha+E/k_B T}-1}$

Average values

Discrete $\overline{Q}=\frac{\sum_n Q_n N(E_n)}{\sum_n N(E_n)}$ Continuous $\overline{Q}=\frac{\int Q(E)N(E)D(E)dE}{\int N(E)D(E)dE}$ where *D(E)* is the density of states.

Solid state physics, some crystal lattices: