# Tutorial - 1 (problems)

Modulation, Tx Architectures and linearity.

1.1) In the problem, $BW = f_2 - f_1$

$$= -(600\,Hz - 3000\,Hz)$$
$$= 2400\,Hz$$

By Shannon's channel capacity theorem, bit rate c

$$C = B \log_2\left(1 + \frac{S}{n_0 B}\right) = B \log_2 (1 + SNR)$$

where $n_0$ is the single sided power spectral density. Since SNR = 30 dB.

$$C = (2400) \log_2 (1 + 1000)$$
$$\simeq 24\,kbps$$

NOTES: a) Explain and refresh Shannon's channel capacity theorem.

b) Remember that SNR is not in dB scale but linear scale.

c) Overall, a simple problem.

d) Note the $\log_2$.

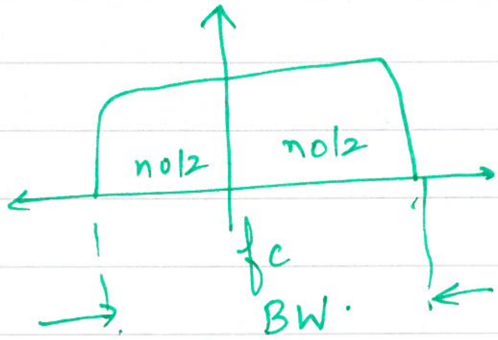
1.2) Similar problem as 1.1 but;

$B = 30\,kHz$, $S = -60\,dBm = \underline{\underline{10^{-9}\,W}}$, $\frac{n_0}{2} = 10^{-18}\,W/Hz$.

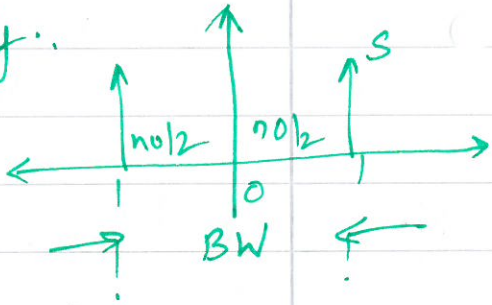
a) At this point, stress on conversion from $dBm \rightarrow W$.

b) $\frac{n_0}{2}$ is the double sided power spectral density.

To explain this,




or in case of 0 frequency, $n_0/2$ is used to denote the noise on the positive/negative frequency.




So, $S = 10^{-9}\,W$

Total noise power $= n_0 = 2 \times 10^{-18}\,W/Hz \times B$

$= 6 \times 10^{-14}\,W$

$\Rightarrow SNR = \underline{\underline{1.67 \times 10^4}}$ ($\simeq 42\,dB$ (not relevant though))

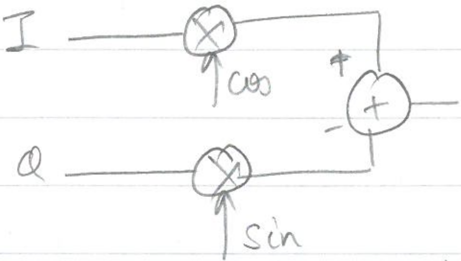
$C = B \log_2\left(1 + \frac{S}{n_0 B}\right)$

$= 30 \times 10^3 \log(1.67 \times 10^4) = \underline{\underline{420\ kbps}}$

1.3) (a) Constellation Diagrams.

Note : Quick introduction of constellation diagrams to be given here. For eg. choose BPSK v/s QPSK to explain that BPSK needs one basis function while QPSK has two. Introduce QAM too.

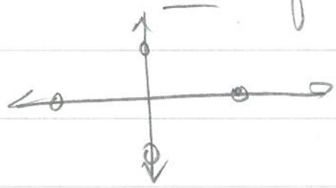

BPSK



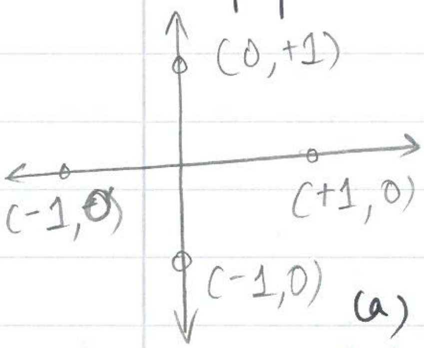

QPSK | 16 QAM | 64 QAM.

one basis function

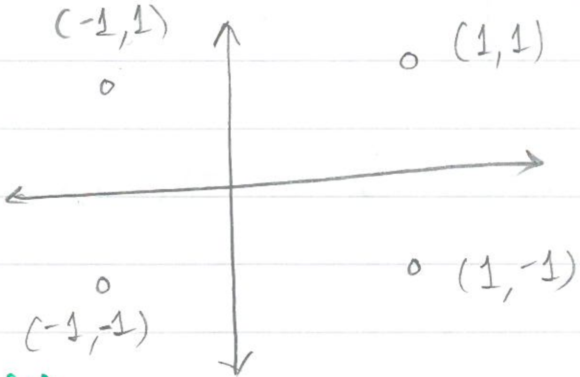
2-basis functions



QPSK (2 popular choices)



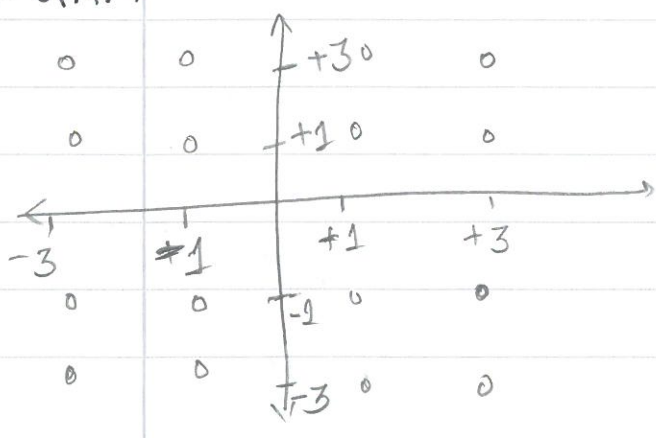

(a)




(b)

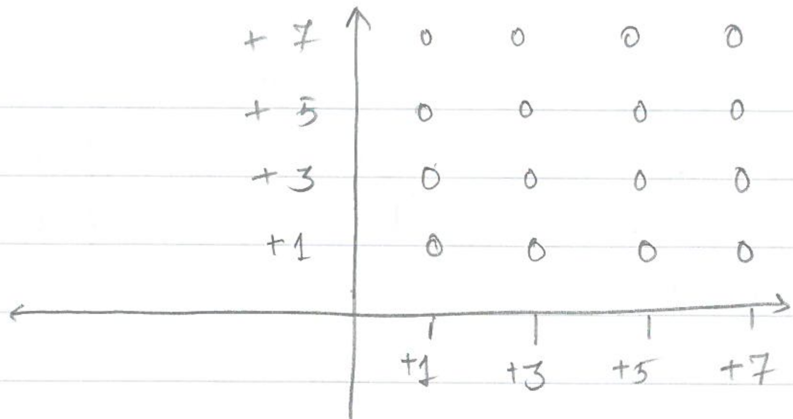
Note : (b) better than (a)
Detecting b is easier. (b) has two levels only for detection (±1), (a) requires even a detection of 0.

16-QAM




Of course different 16-QAM exist. This is the most popular one.

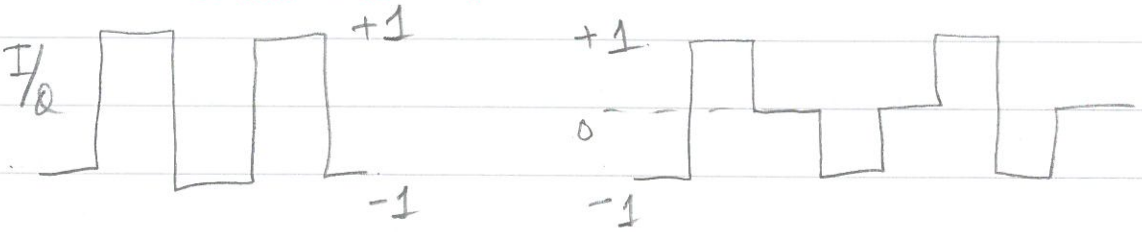
64 QAM




Note: +1 as the starting point to ensure that 2 is the distance between adjacent points.

(b) Time domain waveforms.

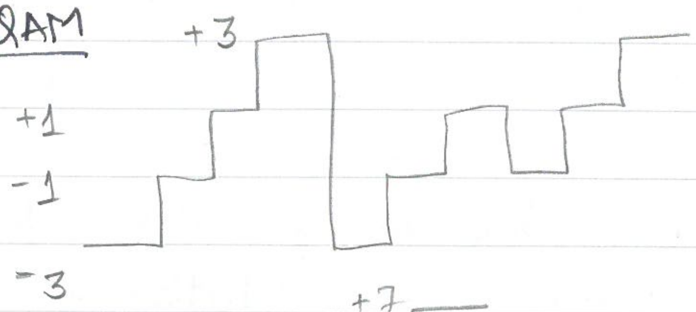
QPSK. 2-levels (1-bit)




3 levels (2-bits)

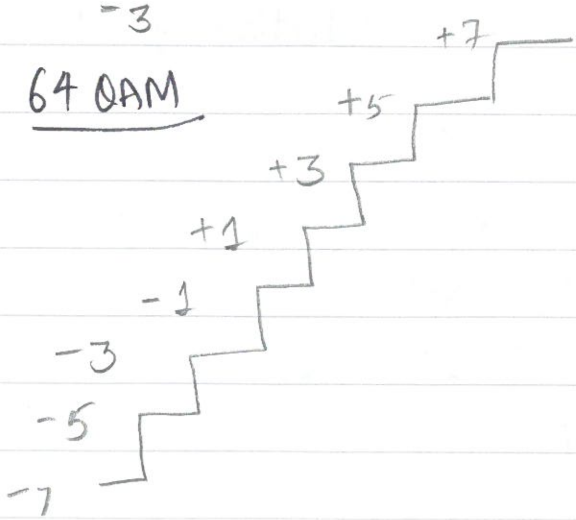
Note: Stress is on the number of levels.

16 QAM




4-levels. (2-bits)

64 QAM




8 levels (3-bits)

c) Bandwidth efficiency

Assume, $R_p$ pulses/s are transmitted.

QPSK ~~sa~~ transmits 2-bits / pulse $\Rightarrow$ $2R_p$ data rate.

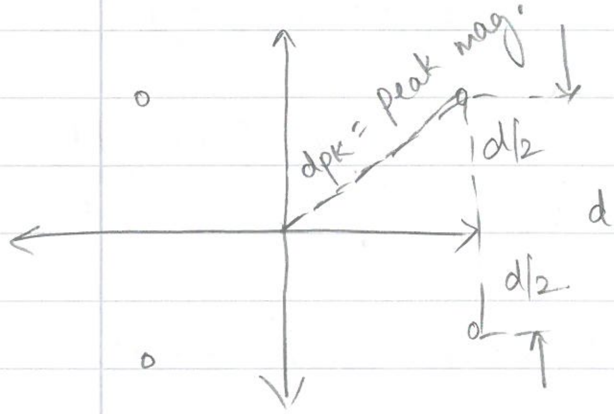
16-QAM 4 - bits / pulses $\Rightarrow$ $4R_p$ data rate

64-QAM 6 - bits / ~~sym~~ pulse $\Rightarrow$ $6R_p$ data rate

Note: All the three schemes have the same bandwidth but 16-QAM has twice the data rates and 64-QAM thrice QPSK.
More number of bits the closer they get and more difficult to detect.

d) Peak power to have the same error probability.

Note: In order to have the same error probability across different modulation schemes, their minimum or "Euclidean" distance should be same.




From this figure;

$$d_{pk} = \sqrt{2}\cdot\frac{d}{2} = \frac{d}{\sqrt{2}}$$

Now, the peak magnitude is $d_{pk}$.

QPSK

$\therefore$ We know, $P \propto V^2$.

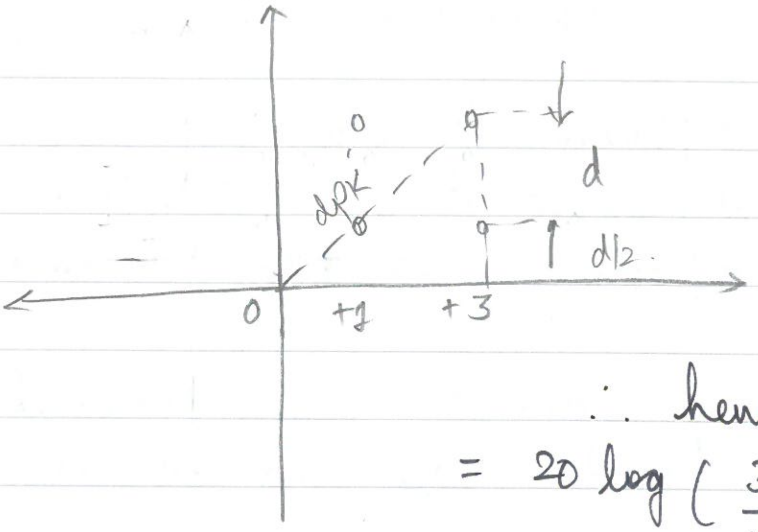
$\therefore$ So peak power is $10 \log (P_{pk})$

$= 20 \log_{10} (V_{pk})$

$= 20 \log_{10} (d/\sqrt{2})$

$= 20 \log d - 3$ dB - (1)

Similarly, consider 16-QAM




As before,

$$d_{pk} = \cancel{3}\left(d + \frac{d}{2}\right)\sqrt{2}$$

$$d_{pk} = \frac{3d}{2}\sqrt{2} = \frac{3d}{\sqrt{2}}$$

$\therefore$ hence peak power

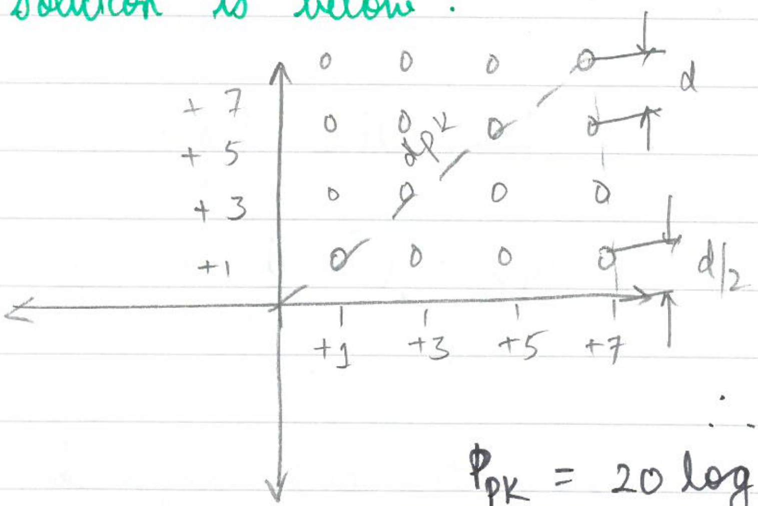
$$= 20 \log\left(\frac{3d}{\sqrt{2}}\right)$$

$$= 20 \log(d) + 20 \log(3/\sqrt{2})$$

$$= 20 \log(d) + 6.53 \quad -(2)$$

$\therefore$ Eq. (1) and (2) $\Rightarrow$ 16-QAM needs ~~8 3 dB~~ 9.5 dB more signal power peak or 9.5 dB higher peak SNR.

No need to solve again for 64-QAM, but the solution is below:




$$d_{pk} = \left(3d + \frac{d}{2}\right)\sqrt{2}$$

$$= \left(\frac{7d}{2}\right)\sqrt{2} = \frac{7d}{\sqrt{2}}.$$

$\therefore$ hence peak power

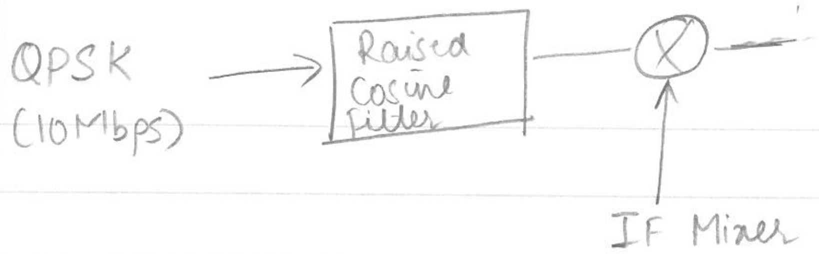
$$P_{pk} = 20 \log\left(\frac{7d}{\sqrt{2}}\right)$$

or $$P_{pk} = 20 \log(d) + 20 \log\left(\frac{7}{\sqrt{2}}\right)$$

$$= 20 \log(d) + 13.89 \text{ dB}$$

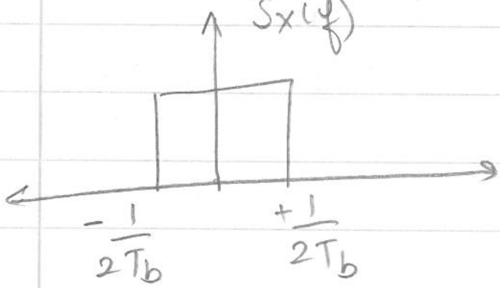
$\therefore$ 64 QAM requires 16.9 dB more peak power than QPSK and 7.4 dB more than 16-QAM.

1.4)



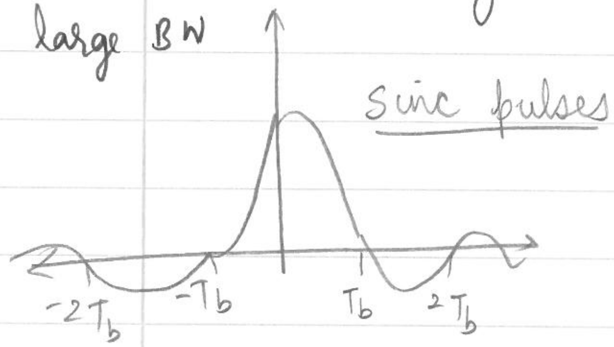

Firstly, the QPSK data rate is 10Mbps.
But we know that QPSK transmits 2-bits/symbol i.e. its occupied BW = $\frac{\text{Data Rate}}{2}$ = 5 MHz bandwidth.

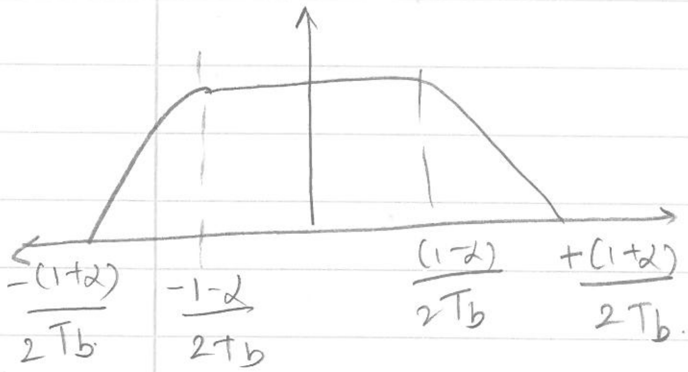
So, in the frequency domain this looks like




where $T_b = \frac{1}{5 MHz}$

In order to normally transmit this requires sinc pulses and large BW which are difficult to generate. So, RCF is used to limit the BW.



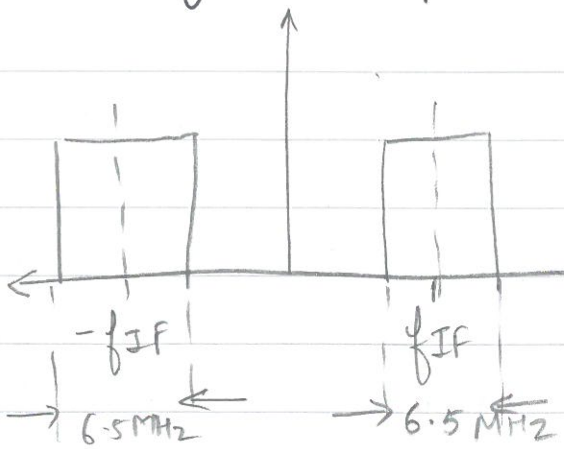

∴ Without RCF,
BW = 5 MHz.

With RCF, the new BW becomes

$$\frac{(1+\alpha)}{T_b} = (1.3)(5\text{ MHz}) = \underline{\underline{6.5\text{ MHz}}}.$$

→ Signal BW = $\underline{6.5\text{ MHz}}$

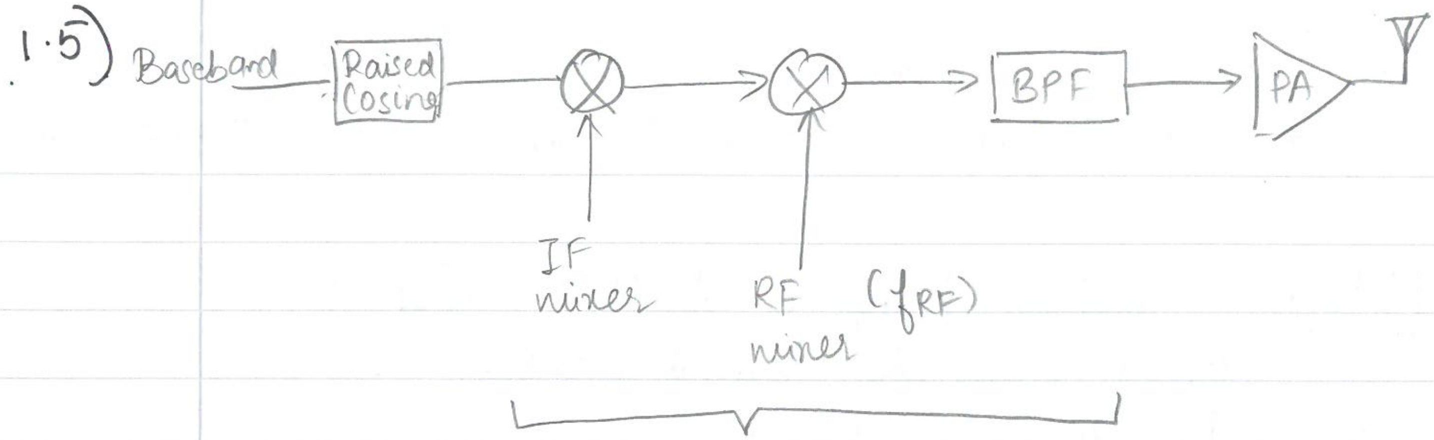
After IF Mixing, the spectrum becomes.




Say, if $f_{IF} <$ 6.5/2 MHz, the two spectra will alias which will corrupt the data.

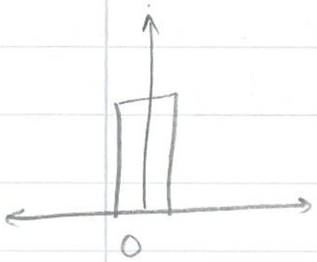

Hence, minimum IF $\geq$ 3.25 MHz

1.5)



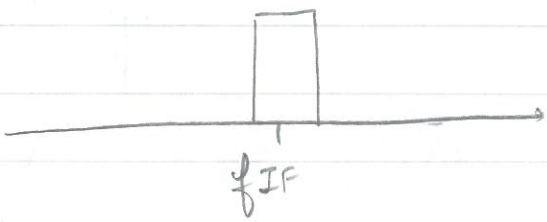

**Note:** Key concepts here are image rejection and filter losses.

First draw the various spectrums.

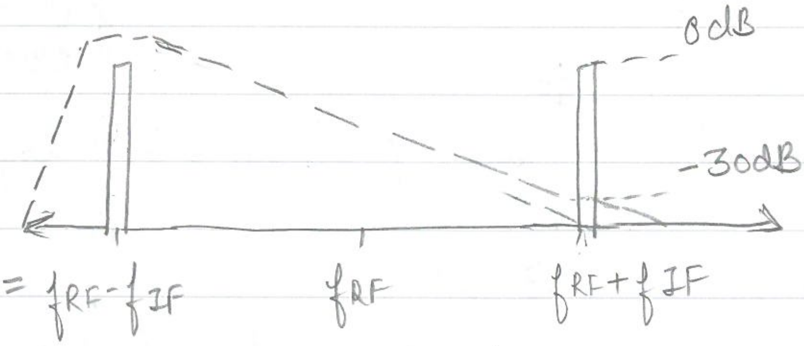



Baseband

Before IF mixing




After IF mixing




Assume higher sideband removal by the bandpass filter. The carrier frequency $f_c = f_{RF} - f_{IF}$.

For lower sideband removal. $f_c = f_{RF} + f_{IF}$

The filter characteristics are given. We need to find what filter order will give 30 dB attenuation.

Note: Undesired image is ALWAYS $2f_{IF}$ away from the desired image
$\Rightarrow$ centre frequency is $f_c$
undesired image at $f_c + 2f_{If}$.
we need 30 dB attenuation.

$\therefore$ Remember this is decade. $1 \rightarrow 100 \rightarrow 1000 \rightarrow$, etc
To calculate the image frequency in decades.

$\therefore \quad \log_{10}\left(\frac{f_2}{f_1}\right), \quad \underline{\underline{f_2 > f_1}}$.

$$\log_{10}\left(\frac{f_c + 2f_{If}}{f_c}\right)$$

Note: Same formula for lower sideband rejection as BPF is centred at $f_c$.

$f_c = 0.35$ GHz, $f_{IF} = 0.15$ GHz.

$\Rightarrow$ decades $= \log_{10}(0.65 / 0.35) = \log(1.86)$

$\therefore \quad (20N)(0.27) \geq 30$

$\therefore N = 5.6 \simeq 6^{th}$ order filter is needed

(b) $\therefore$ We see higher order filter results in power loss in the desired channel.

$\therefore$ Power Loss $= 6 \times (.2)$

$= \underline{\underline{1.2 \text{ dB}}}$

(c) The filter order is limited to 5.

$$\Rightarrow (20)(5) \log\left(\frac{f_c + 2f_{IF}}{f_c}\right) = 30$$

$\therefore$ You have 1 choices, modify $f_{IF}$ since $f_c$ is fixed for a given radio standard.

Solving for $f_{IF}$.

$$\frac{fc + 2f_{If}}{fc} = 10^{(.3)}$$

fc = fRF - fIF

$$\Rightarrow f_{If} = \underline{\underline{166 \text{ MHz}}}.$$

Important Conclusion.

Tradeoff between $f_{IF}$ and filter order

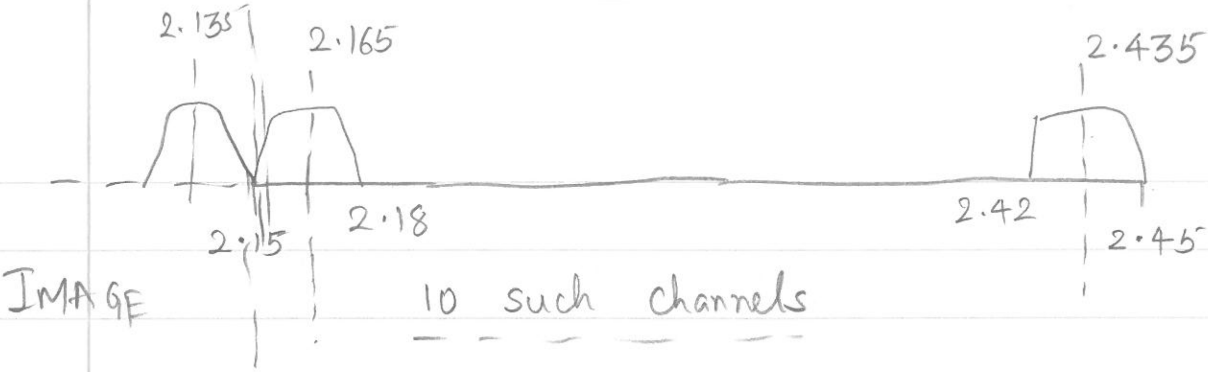
high $f_{IF}$, lower filter order.
low $f_{IF}$, higher filter order

Increasing $f_{IF}$ bring IF more closer to $f_{RF}$ and reduces $f_{RF}$., which may become difficult for implementation.

This is frequency planning

Increasing $f_c$ also makes this worse.
$f_c = 1$ GHz increases filter order to $\underline{\underline{13}}$.
$\therefore$ more selectivity or Q for the filter.

1.6)




From previous example we know that image frequency is located $2f_{IF}$ away. To avoid aliasing between image and desired frequency.


Image should be $> .3\ GHz$ away.

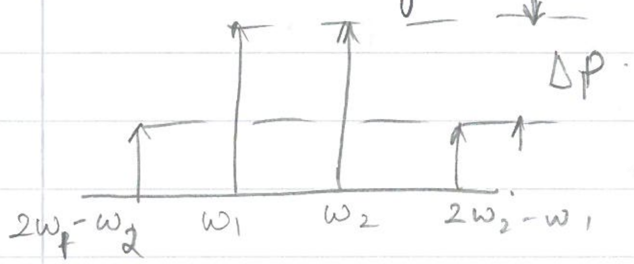
$\Rightarrow$ Image $> 2f_{If}$ away

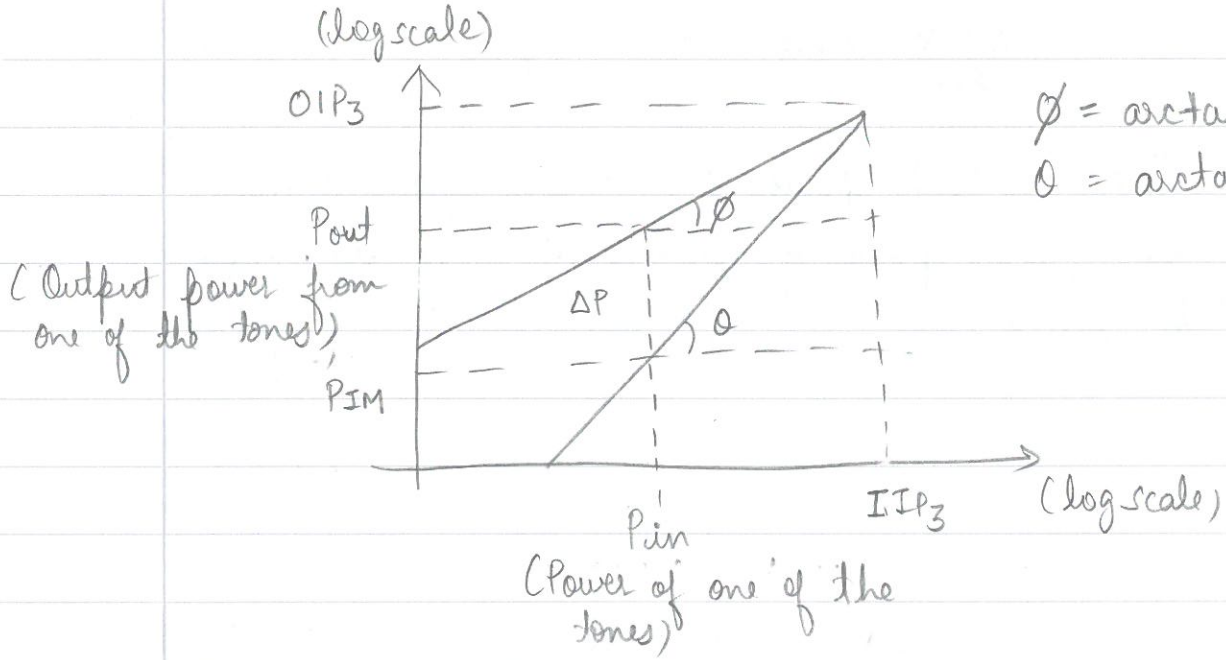
$\Rightarrow f_{IF} > \underline{\underline{150\ MHz}}$

1.7) Before starting problems 1.7 – 1.9, check with the students if a derivation of IP3 v/s IM3 is required. Explain and derive this if necessary.

This is covered in 2.2.2. of the book.

For a non-linear system shown with two tones,




$\phi = \arctan(1) = 45^\circ$

$\theta = \arctan(3)$

Looking at this figure we have,

$$OIP_3 - P_{IM} = (IIP_3 - P_{in}) \tan\theta$$

$$OIP_3 - P_{IM} = 3(IIP_3 - P_{in}) \quad -(1)$$

Also $$OIP_3 - P_{IM} = (IIP_3 - P_{in}) \tan\phi + \Delta P$$

$$= \Delta P + (IIP_3 - P_{in}) \quad -(2)$$

$\therefore$ From (1) and (2)

$$\Delta P = 2(IIP_3 - P_{in})$$

$$\Rightarrow IIP_3 = P_{in} + \frac{\Delta P}{2} \quad -(3)$$

If the gain G, we can transport this equation to the output

$$OIP_3 - G = P_{out} - G + \frac{\Delta P}{2}.$$

$$\Rightarrow \quad OIP_3 = \frac{\Delta P}{2} + P_{out} \quad -(4)$$

Use this equation in problem (1·7) directly

$$OIP_3 = \frac{+65}{2} + 17 \text{ dBm}$$

$$OIP_3 = \cancel{\text{dBm}} \; 49.5 \text{ dBm}$$

**Note:** Total output power = 20 dBm from two tones., the power of individual tones is 17 dBm (3 dB lower)

1.8) $OIP_3 = 33\ dBm$

The amplifier works in a 6 dB back off mode [As an addition, explain what back off mode is]. [Sometimes the peak power to average power is too high, hence, the amplifier is used below its maximum potential for linearity purpose]. Let the gain be G.

$\Rightarrow \quad IIP_3 = OIP_3 - G.$

$\Rightarrow \quad P_{1dB} = OIP_3 - G - 9.6\ dB.$

The amplifier is used 6dB below its 1 dB compression point

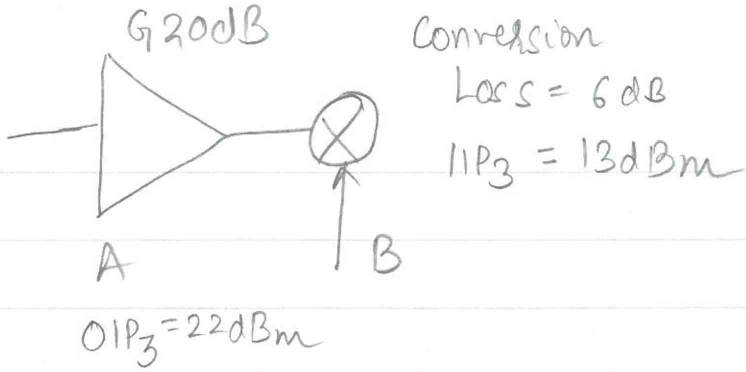
$\Rightarrow \quad P_{in} = OIP_3 - G - 9.6\,dB - 6\,dB.$

or $P_{out\,max} = OIP_3 - G - 9.6\,dB - 6\,dB + G$

$= 33 - 15.6$

$= \underline{\underline{17.4\ dBm}}$

1.9)




Note : Remember the conversion loss.
(Mixers can have some gain loss)
(may be for improved linearity)

We have

$$G_A = 20\,dB = 100$$

$$OIP_{3_A} = 22\,dBm = 10^{(22/10)}\,mW = 158.5\,mW$$

Similarly,

$$G_B = -6\,dB = 10^{(-6/10)} = .25$$

$$IIP_3 = 13\,dBm \Rightarrow OIP_3 = 7\,dBm = 5\,mW$$

We know that for a cascaded system,

$$\frac{1}{OIP_{3\,total}} = \frac{1}{G_B\, OIP_{3A}} + \frac{1}{OIP_{3B}}.$$

Substitute and solve, we get

$$OIP_{3\,total} = 6.48\,dBm$$

Similarly, recalculate the $OIP_3$ by exchanging the location of the mixer and amplifier.

$$\frac{1}{OIP_{3total}} = \left(\frac{1}{(5)(100)} + \frac{1}{(158.5)}\right) = \underline{\underline{20.8\ dBm}}$$

Important Observation: The $IP_3$ of the final stage contributes more to the $IP_3$ of the overall system. This can be seen here. (similar to resistors in || where the smallest one is the major contributor)