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Modern Physics, TNE041

Exam, 31 August 2024, Answers and short solutions

1. (a) True, False, False, True.
(b) Clearly relativistic mechanics is neccessary since $K = eV >> mc^2$ Kinetic energy $K = (\gamma - 1)mc^2$ gives $\gamma = 1 + K/(mc^2) \approx K/(mc^2)$, from which we solve for electron speed u. $u = c\sqrt{1-(mc^2/K)^2} \approx c(1-\frac{1}{2}(mc^2/K)^2)$ [Taylor expansion]. The speed difference is obtained as $c - u = \frac{1}{2}(mc^2/K)^2 c = 39$ m/s.

2. (a) Bragg's law in the simplest case is $2d\sin\theta = m\lambda$, where m is a positive integer ($m = 0$ corresponds to $\theta = 0$ and is of no interest). The wavelength is $\lambda = h/p = h/\sqrt{2m_e eV} = 0,03885$ nm. Plugging into Bragg's law and solving for θ yields $\theta = \arcsin(m\lambda/(2d) = \arcsin(mh/(2d\sqrt{2m_e eV})) = \arcsin(m \cdot 0,007766)$. The least values are given by $m = 1$ and $m = 2$ and become $\theta = 11,1^\circ$ and $\theta = 22,8^\circ$.
(b) To obtain the same positions for the maxima of X-rays (photons) we need the same wavelength $\lambda = 0,03885$ nm. $E = pc = hc/\lambda = 32$ eV.

3. (a) $\overline{x} = \int_{-\infty}^{\infty} x|\psi(x)|^2\,dx = 3/(2a) = 1,5$ nm
(b) $\Delta x = \sqrt{\overline{x^2} - \overline{x}^2} = \sqrt{3/a^2 - (3/(2a))^2} = \sqrt{3/4}/a = 0,866$ nm.

4. (a) $E_2 = -13,6 \text{ eV}/2^2 = -3,4$ eV.
(b) n, ℓ, m_l have their origin from the mathematical solution of the Schrödinger equation.
(c) $|\mathbf{L}| = \sqrt{\ell(\ell+1)}\hbar = \sqrt{2}\hbar$, $L_z = m_\ell\hbar = \hbar$. One obtains $\arccos(L_z/|\mathbf{L}|) = 45^\circ$.

5. The classical contiunuous case is treated by the continuous (integral) forms of MB-statistics. $\overline{v^2} = \int\int\int(v_x^2 + v_y^2 + v_z^2)N(E)dv_x dv_y dv_z / \int\int\int N(E)dv_x dv_y dv_z$ with $N(E) = A\exp(-E/k_BT)$ and $E = \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2)$. The integrals are straightforward in polar coordinates, but somewhat tedious; Physics handbook might be helpful. $v_{rms} = \sqrt{\overline{v^2}} = \sqrt{3k_BT/m}$.
The $\overline{v^2}$ value can also be found from the Maxwell speed distribution, with essentially the same effort.

6. Visible light has photon energies in the range from 1,7 to 3,0 eV. From PH T-8.5 we have the band gap of InP as 1,35 eV, that is, all photons in visible light has the possibility to excite electrons across the gap, hence being absorbed, and the material is not transparent.

   Larger wavelength means less energy, and when the photon energies get lower than 1,35 eV, the possibility for being absorbed tends to zero since the energy is not enough to excite an electron over the band gap. Hence the material becomes transparent to IR-light.