# TSDT14 Signal Theory

## Lecture 10

## Reconstruction in CD Players

## Exam Problems

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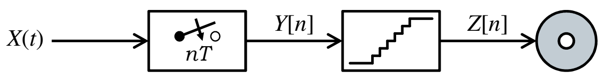
## CDs at Recording 1(2)




Band-limited input: $R_X(f) \approx 0, \quad |f| \geq B = 20\text{ kHz}$

Sampling frequency: $f_s = 44.1\text{ kHz}$

Uniform quantization: $16\text{ bits} \quad \Rightarrow \quad N = 2^{16}\text{ steps}$

Saturation level: $A$

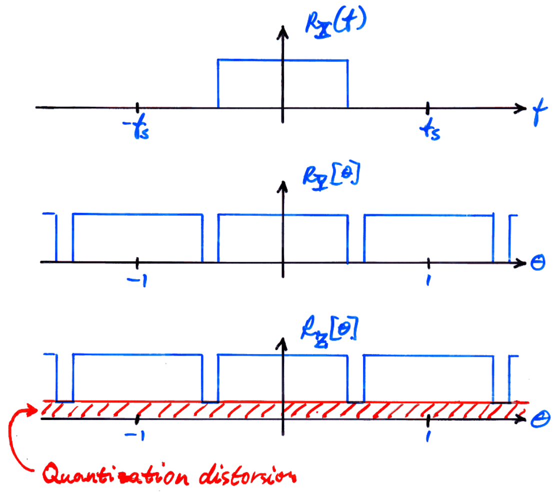
Quantization step height: $\Delta$

$$\frac{\Delta^2}{12} = \frac{A^2}{2^{32}\cdot 3}$$

Signal-to-Distorsion Ratio: $\text{SDR}_{\max} = 10\log_{10}(2^{32}) \approx 96\text{ dB}$



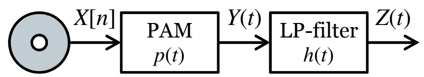
## CDs at Recording 2(2)

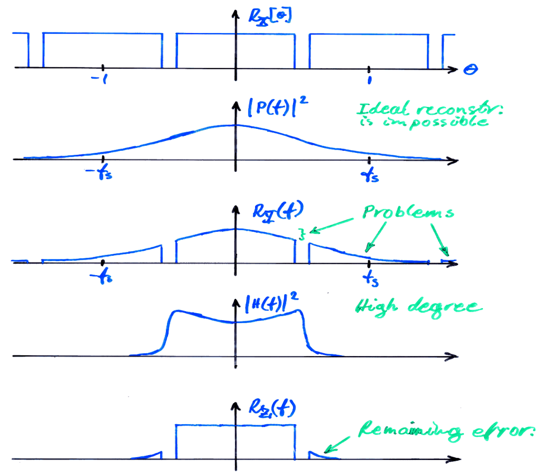





## Listening to CDs – 1

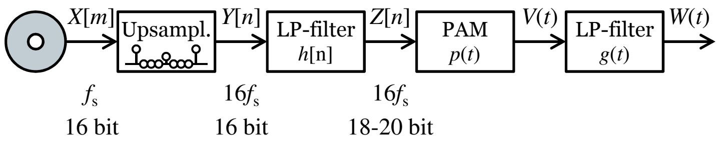
Direct PAM



# Listening to CDs – 2

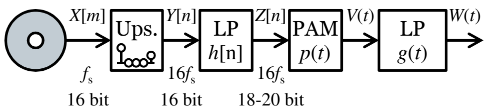
Oversampling 1(2)

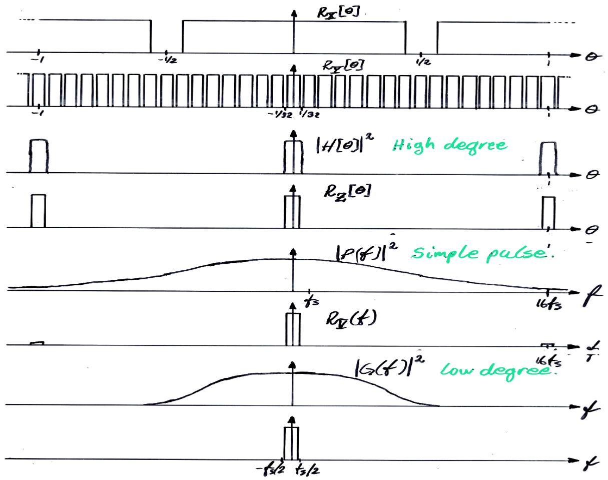





# Listening to CDs – 2

Oversampling 2(2)

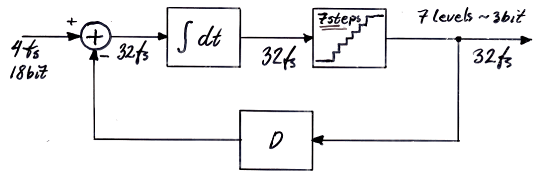


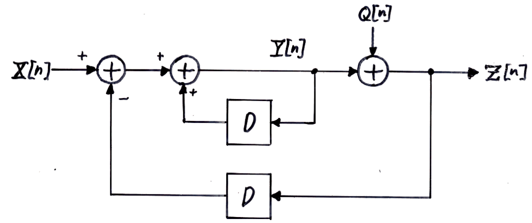






# Listening to CDs – 3

Noise Shaping 1(4) – Principle of First-Order Noise Shaper

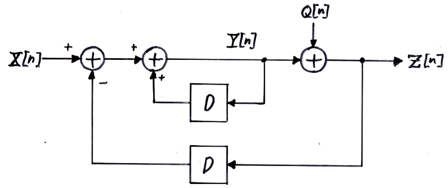





# Listening to CDs – 3

Noise Shaping 2(4) – Analysis 1




If no $Q[n]$:

$$Z[n] = Y[n] = X[n] + Y[n-1] - Z[n-1] = X[n]$$

Error: $S[n] = Z[n] - X[n]$

Fig $\Rightarrow$

$$\left.\begin{aligned} Y[n] &= X[n] + Y[n-1] - Z[n-1] \\ Z[n] &= Y[n] + Q[n] \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \quad S[n] = \left(X[n] + \underbrace{Y[n-1] - Z[n-1]}_{-Q[n-1]}\right) + Q[n] - X[n]$$

$$= Q[n] - Q[n-1]$$

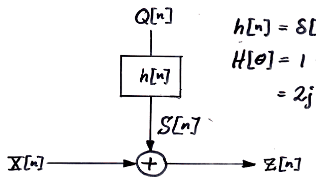
Interpretation:
The quantization noise is filtered.

$$h[n] = \delta[n] - \delta[n-1]$$

$$H[\theta] = 1 - e^{-j2\pi\theta} = 2j e^{-j\pi\theta} \sin(\pi\theta)$$

# Listening to CDs – 3

Noise Shaping 3(4) – Analysis 2




$h[n] = \delta[n] - \delta[n-1]$

$H[\theta] = 1 - e^{-j2\pi\theta}$

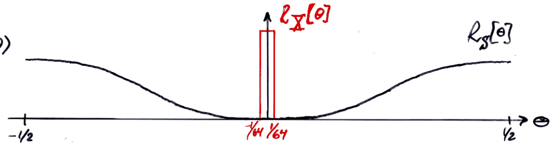
$= 2je^{-j\pi\theta}\sin(\pi\theta)$

Quantization error: $Q[n]$:

$$\frac{\Delta^2}{12} = \frac{A^2}{7^2}\cdot\frac{1}{3} = \frac{A^2}{147} \Rightarrow r_Q[k] = \frac{A^2}{147}\delta[k], \quad R_Q[\theta] = \frac{A^2}{147}$$

Error: $S[n]$:

$$R_S[\theta] = |H[\theta]|^2 R_Q[\theta] = \frac{4A^2}{147}\sin^2(\pi\theta)$$




Error power in audio range (sin x ≈ x for small x)

$$\int_{-1/64}^{1/64} R_S[\theta]\,d\theta = 2\int_0^{1/64}\frac{4A^2}{147}\sin^2(\pi\theta)\,d\theta \approx \frac{A^2}{20}\int_0^{1/64}(\pi\theta)^2\,d\theta$$

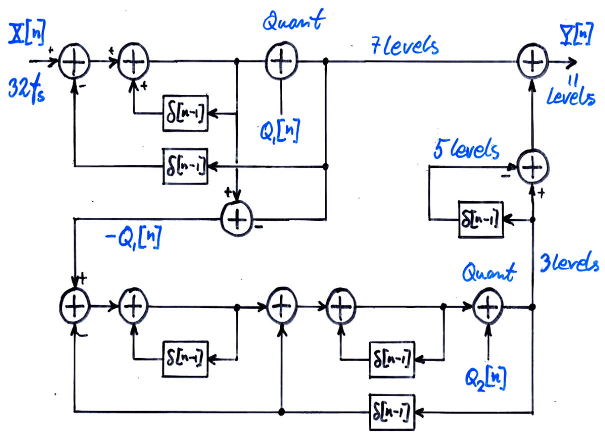
$$\approx \frac{A^2}{2}\int_0^{1/64}\theta^2\,d\theta = \frac{A^2}{2}\left[\frac{\theta^3}{3}\right]_0^{1/64} = \frac{A^2}{2^{19}\cdot 3} \approx 57\,dB$$

Approx. 9.5 bits



# Listening to CDs – 3

Noise Shaping 4(4) – Implementation of Third-Order Noise Shaper




Error PSD:

$$R_S[\theta] = \frac{64A^2}{147}\cdot\sin^6(\pi\theta)$$

Error Power in Audio Range:

$$2\int_0^{1/64}\frac{64A^2}{147}(\pi\theta)^6\,d\theta = \dots \approx$$

$$\approx \frac{A^2}{2^{33}\cdot 3} \approx 100\,dB$$

Corresponds to:

Approx. 16.5 bits

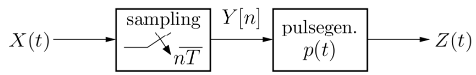


# 2010-08-28 – Problem 4

The time-continuous process $X(t)$ is bandlimited white noise with mean $m_X = 0$ and PSD

$$R_X(f) = \begin{cases} 1, & |f| \le W, \\ 0, & \text{elsewhere.} \end{cases}$$

The signal $X(t)$ is sampled and pulse-amplitude-modulated according to the figure below.



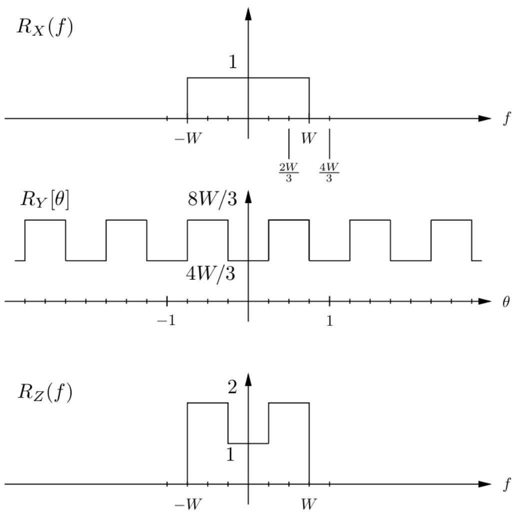

The sampling frequency is $f_s = \frac{1}{T} = \frac{4W}{3}$ and the pulse shape of the PAM is $p(t) = \frac{3\sin(2\pi Wt)}{4\pi Wt}$.

a) Determine and draw the PSDs $R_Y[\theta]$ and $R_Z(f)$. (3 p)

b) Determine the reconstruction error $\varepsilon^2 = E\{(Z(t) - X(t))^2\}$. (2 p)

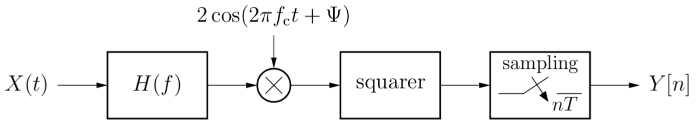


# 2010-08-28 – Problem 4 – PSDs



# 2009-10-23 – Problem 3

The input $X(t)$ to the system below is a strictly stationary white process with PSD $R_0$, and the stochastic variable $\Psi$ is as usual uniformly distributed on $[0, 2\pi)$ and independent of $X(t)$. The input is Gaussian with mean 0.




The initial filter has frequency response

$$H(f) = \begin{cases} 2, & |f| < f_0, \\ 0, & \text{elsewhere.} \end{cases}$$

**Note: This is one of the tasks from Tutorial 8.**

The carrier frequency is $f_c = 2f_0$, while the sampling frequency is $f_s = 3f_0$.

Calculate the power $P_Y$ of the output $Y[n]$. (5 p)

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