LINKÖPINGS UNIVERSITET

Exam in TSDT14 Signal Theory

Exam code:	TEN1
Date:	2019-01-08 **Time:** 8.00–12.00
Place:	
Teacher:	Mikael Olofsson, tel: 281343
Visiting exam:	9 and 11
Administrator:	Carina Lindström, 013-284423, carina.e.lindstrom@liu.se
Department:	ISY
Allowed aids:	Olofsson: *Tables and Formulas for Signal Theory* A German 10-Mark note of the fourth series (1991-2001). Pocket calculators of all kinds with empty memory.
Number of tasks:	6
Grading:	Task one is not graded in terms of points. At least two of its three sub-tasks have to be treated correctly as partial fulfillment to pass the exam. Tasks 2-6 yield at most 5 points each. Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unresonable answers. Totally, you can get 25 points at most. For grade three you need 10 points, for grade four 15 points and for grade five 20 points.
Solutions:	Will be published in the course room no later than three days after the exam, under Previous exams.
Result:	You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.
Exam return:	In the student office of ISY, Building B, between entrances 27-29, ground floor, Corridor D, right next to Café Java, starting about two weeks after the exam.

1 These sub-tasks have to be treated correctly as partial fulfillment to pass the exam:

a. What is meant by an *exactly predictable* process?

b. The time-discrete signal $Y[n]$ is created by sampling the signal $X(t)$, with PSD

$$R_X(f) = \begin{cases} 1, & |f| < 5 \text{ Hz}, \\ 0, & \text{elsewhere.} \end{cases}$$

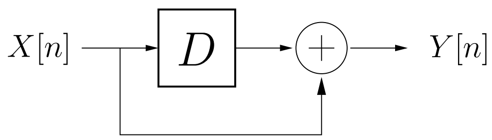
The sampling frequency is 8 Hz. Scetch the PSD $R_Y[\theta]$. Do not forget to grade the axes.

c. The process $Y[n] = X^3[n]$ is created from a time-discrete Gaussian process, $X[n]$, with mean zero and ACF

$$r_X[k] = \left(\frac{1}{3}\right)^{|k|}.$$

Determine the ACF of the resulting process.

2 Let $X[n]$ in the system below be a sequence of independent random variables, uniformly distributed on the interval $[-1, 1]$.



Determine

a. the ACF $r_Y[k]$. (3 p)
b. the PSD $R_Y[\theta]$. (1 p)
c. the probability $\Pr\{Y[n] > 3/2\}$. (1 p)

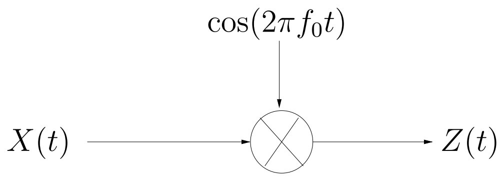
3 Consider a system that generates the output $Y(t) = X^3(t)$. According to the Tables and Formulas booklet, page 15, we have

$$r_Y(\tau) = 9r_X^2(0)r_X(\tau) + 6r_X^3(\tau)$$

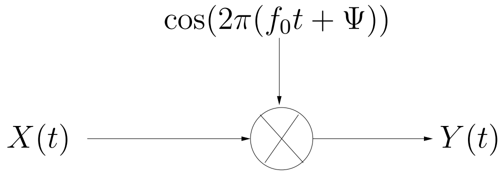
if $X(t)$ is a stationary Gaussian Process with mean zero.

Use Prices theorem to prove that. (5 p)

4 The time-continuous WSS signal $X(t)$ with ACF $r_X(\tau)$ is amplitude-modulated by the carrier $\cos(2\pi f_0 t)$ according to the figure below.



Show that the output $Z(t) = X(t)\cos(2\pi f_0 t)$ is not WSS if we have $r_X(1/f_0) \neq 0$. This signal can be made WSS if we introduce a stochastic variable Ψ so that the output is given by the following figure.



Then we have $Y(t) = X(t)\cos(2\pi(f_0 t + \Psi))$. Determine the ACF $r_Y(\tau)$ of $Y(t)$ if Ψ is uniformly distributed on the interval $(0, 1]$, independent of the input. (5 p)

5 Consider the process

$$X(t) = A + B(t),$$

where A is a Gaussian variable with non-zero variance σ_A^2, and where $B(t)$ is a WSS Gaussian process with mean zero and ACF $r_B(\tau) = R_0 \operatorname{sinc}(20\tau)$. Finally, A and $B(t)$ are independent for all time instances t.

Determine the following, at least by giving arguments supporting the answers. (1 p per subtask)

a. Is $B(t)$ SSS?
b. Is $B(t)$ ergodic?
c. Is $X(t)$ WSS?
d. Is $X(t)$ SSS?
e. Is $X(t)$ ergodic?

If any of those questions cannot be answered with Yes or No, then give arguments why the given information is not enough.

6 The input to a space-discrete LSI-system is

$$x[n_1, n_2] = \begin{cases} 1, & (n_1, n_2) \in \{(1,0), (0,1)\}, \\ 0, & \text{elsewhere.} \end{cases}$$

The impulse response of the system is

$$h[n_1, n_2] = \begin{cases} 1, & |n_1| \leq 1 \text{ and } n_2 = 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Determine the output $y[n_1, n_2]$ of the system

a. with calculations in the space-domain. (3 p)

b. with calculations in the frequency-domain. (2 p)

Formulas on Complex Processes

Complex process: $X(t)$, where $X_1(t)$ is the real part, and $X_2(t)$ is the imaginary part.

ACF: $r_X(\tau) = \mathrm{E}\{X(t+\tau)X^*(t)\}$
$\quad = r_{X_1}(\tau) + r_{X_2}(\tau) + j\big(r_{X_1,X_2}(-\tau) - r_{X_1,X_2}(\tau)\big)$

PSD: $R_X(f) = R_{X_1}(f) + R_{X_2}(f) + j\big(R^*_{X_1,X_2}(f) - R_{X_1,X_2}(f)\big)$

Formulas on Baseband Representation - Deterministic Input

$\tilde{s}(t)$ is the message and $s(t)$ is the modulated signal, with bandwidth of $\tilde{s}(t)$ less than f_c.

Passband: $s(t) = \mathrm{Re}\Big\{\tilde{s}(t)e^{j2\pi f_\mathrm{c} t}\Big\} = s_\mathrm{I}(t)\cos(2\pi f_\mathrm{c} t) - s_\mathrm{Q}(t)\sin(2\pi f_\mathrm{c} t)$

Baseband: $\tilde{s}(t) = s_\mathrm{I}(t) + js_\mathrm{Q}(t)$

Spectrum: $S(f) = \frac{1}{2}\big(\tilde{S}(f-f_\mathrm{c}) + \tilde{S}^*(-f-f_\mathrm{c})\big)$, $\qquad \tilde{S}(f) = 2S(f+f_\mathrm{c})u(f+f_\mathrm{c})$

Filtering: $y(t) = (x*h)(t) \quad \Leftrightarrow \quad \tilde{y}(t) = \frac{1}{2}(\tilde{x}*\tilde{h})(t)$
where $\tilde{x}(t)$, $\tilde{h}(t)$ and $\tilde{y}(t)$ are the complex baseband representations of $x(t)$, $h(t)$ and $y(t)$, respectively.

Formulas on Baseband Representation - Stochastic Input

$\tilde{S}(t)$ is the message with $m_{\tilde{S}} = 0$ and $S(t)$ is the modulated signal. The bandwidth of $\tilde{S}(t)$ must be less than f_c.

Passband: $S(t) = S_\mathrm{I}(t)\cos(2\pi f_\mathrm{c} t + \Psi) - S_\mathrm{Q}(t)\sin(2\pi f_\mathrm{c} t + \Psi)$
with Ψ uniformly distributed on $[0, 2\pi)$, independent of $\tilde{S}(t)$.

Baseband: $\tilde{S}(t) = S_\mathrm{I}(t) + jS_\mathrm{Q}(t)$

Mean: $m_S = 0$

ACF: $r_S(\tau) = \frac{r_{S_\mathrm{I}}(\tau) + r_{S_\mathrm{Q}}(\tau)}{2}\cos(2\pi f_\mathrm{c}\tau) - \frac{r_{S_\mathrm{I},S_\mathrm{Q}}(-\tau) - r_{S_\mathrm{I},S_\mathrm{Q}}(\tau)}{2}\sin(2\pi f_\mathrm{c}\tau)$

PSDs: $R_S(f) = \frac{1}{4}\big(R_{\tilde{S}}(f-f_\mathrm{c}) + R_{\tilde{S}}(-f-f_\mathrm{c})\big)$, $\qquad R_{\tilde{S}}(f) = 4R_S(f+f_\mathrm{c})u(f+f_\mathrm{c})$