TSKS02 Telecommunication

Lecture 11
Source Coding
Synchronization

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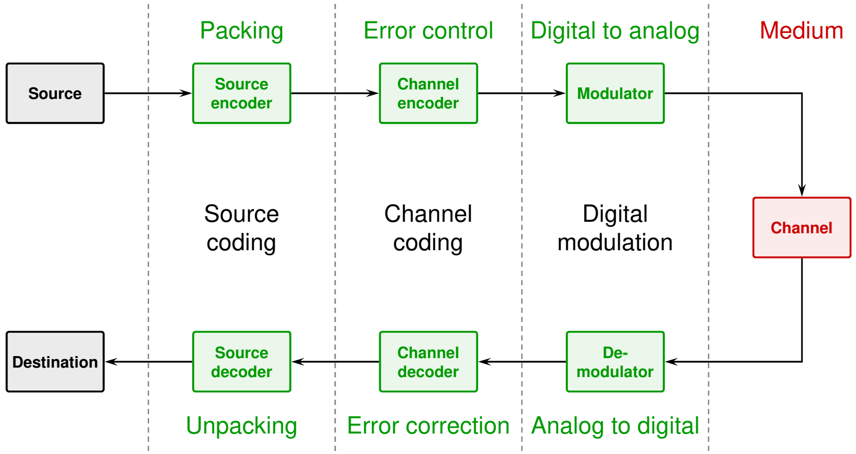
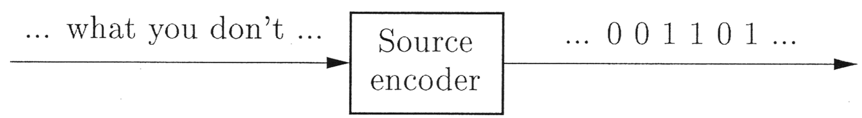
A One-way Telecommunication System



Task of a Source Encoder

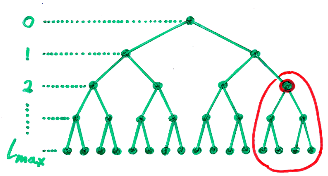
A source encoder translates messages into bits.



Kraft's Inequality 1(2)

There exists a tree code with lengths $l_1, \dots, l_N$ iff $\sum_{i=1}^{N} 2^{-l_i} \leq 1$ holds.

Proof: Start with "only if". Assume that we have a tree code. Define $l_{max} = \max\{l_1, \dots, l_N\}$. Complete binary tree of depth l_{max}:



A codeword of length l_i (depth l_i) has $2^{l_{max}-l_i}$ leafs below itself on depth l_{max}. Totally $2^{l_{max}}$ leafs. If no codeword is unnecessarily long:

$$\sum_{i=1}^{N} 2^{l_{max}-l_i} = 2^{l_{max}}$$

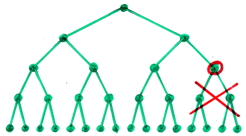
Codewords can be unnecessarily long. Some leafs are in that case not below any codeword:

$$\sum_{i=1}^{N} 2^{l_{max}-l_i} \leq 2^{l_{max}} \quad \Rightarrow \quad \sum_{i=1}^{N} 2^{-l_i} \leq 1.$$

Kraft's Inequality 2(2) (Continued proof)

Now the "if" part. Assume that $\sum_{i=1}^{N} 2^{-l_i} \leq 1$ holds. Sort lengths: $l_1 \leq l_2 \leq \ldots \leq l_N$.

The same tree:



Algorithm to construct a code:

For $i = 1, 2, \ldots, N$:

Choose a point on depth l_i.

Remove the sub-tree below that point.

Question: Will there be leafs left for all codewords?

In step j there will be this many leafs left:

$$\underbrace{2^{l_{max}} - \sum_{i=1}^{j-1} 2^{l_{max}-l_i}}_{\text{Integer}} = 2^{l_{max}} \cdot \Big(1 - \underbrace{\sum_{i=1}^{j-1} 2^{-l_i}}_{<1}\Big) > 0$$

$\Rightarrow$ There are leafs left in each step.

$\Rightarrow$ We can construct a tree code with lengths $l_1, l_2, \ldots, l_N$.

Entropy

Source statistics: $p_i = Pr\{A = a_i\}$ Entropy of A: $H(A) = -\sum_{i=1}^{N} p_i \cdot \log_2(p_i)$

Lengths: $L = l_i \Leftrightarrow A = a_i$ Theorem: $m_L \geq H(A)$

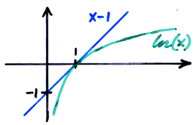
Proof: Consider $H(A) - m_L$. Should be ≤ 0.

$$H(A) - m_L = -\sum_{i=1}^{N} p_i \cdot \log_2(p_i) - \sum_{i=1}^{N} p_i \cdot l_i = \sum_{i=1}^{N} p_i \cdot (-\log_2(p_i) - l_i)$$

$$= \sum_{i=1}^{N} p_i \cdot \left(\log_2(2^{-l_i}) - \log_2(p_i)\right) = \sum_{i=1}^{N} p_i \cdot \log_2\left(\frac{2^{-l_i}}{p_i}\right) = \Big/ \log_2(x) = \frac{\ln(x)}{\ln(2)} \Big/$$

$$= \frac{1}{\ln(2)} \sum_{i=1}^{N} p_i \cdot \ln\left(\frac{2^{-l_i}}{p_i}\right) \leq \Big/ \ln(x) \leq x - 1 \;\; \forall x \Big/$$

$$\leq \frac{1}{\ln(2)} \sum_{i=1}^{N} p_i \cdot \left(\frac{2^{-l_i}}{p_i} - 1\right) = \frac{1}{\ln(2)} \Big(\underbrace{\sum_{i=1}^{N} 2^{-l_i}}_{\text{Kraft: } \leq 1} - \underbrace{\sum_{i=1}^{N} p_i}_{=1}\Big)$$

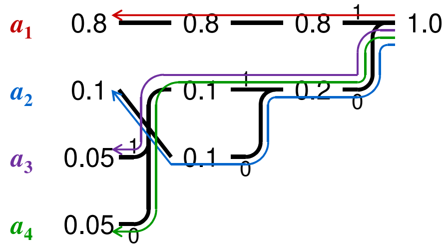


$$\leq \frac{1}{\ln(2)} \cdot (1 - 1) = 0$$ Done!

Equality? Exactly here! That is $\ln\left(\frac{2^{-l_i}}{p_i}\right) = 0 \Rightarrow l_i = -\log_2(p_i)$

Note: $\Rightarrow 2^{-l_i} = 2^{\log_2(p_i)} = p_i \Rightarrow \sum_{i=1}^{N} 2^{-l_i} = \sum_{i=1}^{N} p_i = 1.$

Huffman Coding



$c^{(1)} = 1$

$c^{(2)} = 00$

$c^{(3)} = 011$

$c^{(4)} = 010$

p_i	$c^{(i)}$	l_i	$p_i l_i$
0.8	1	1	0.8
0.1	00	2	0.2
0.05	011	3	0.15
0.05	010	3	0.15

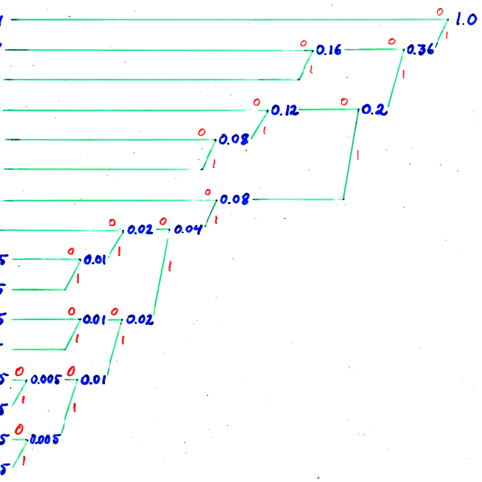
$m_L = 1.3$

Entropy: $H(A) = -\sum_{i=1}^{N} p_i \log_2 p_i \approx 1.022$

Redundancy: $m_L - H(A) \approx 0.278$

Compr. ratio: $\frac{\log_2 N}{m_L} \approx 1.54$

Simplified Huffman Code for an Extended Source



a_1a_1	0.64	0
a_1a_2	0.08	100
a_2a_1	0.08	101
a_1a_3	0.04	1100
a_1a_4	0.04	11010
a_3a_1	0.04	11011
a_4a_1	0.04	1110
a_2a_2	0.01	111100
a_2a_3	0.005	1111010
a_2a_4	0.005	1111011
a_3a_2	0.005	1111100
a_4a_2	0.005	1111101
a_3a_3	0.0025	11111100
a_3a_4	0.0025	11111101
a_4a_3	0.0025	11111110
a_4a_4	0.0025	11111111

Result of Source Coding

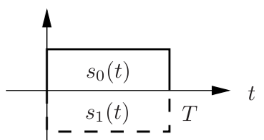
Almost equally probable
almost uncorrelated bits.

The closer we get to entropy, the closer we get to almost equally probable and almost uncorrelated bits.

Eye Patterns Again 1(3) – Signals

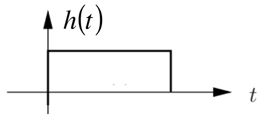
Signals:

$$s_1(t) = -s_0(t)$$



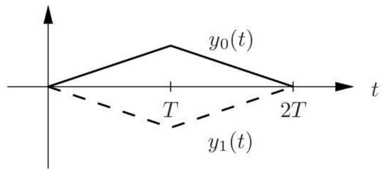
Matched filter:

$$h(t) \propto s_0(T-t)$$



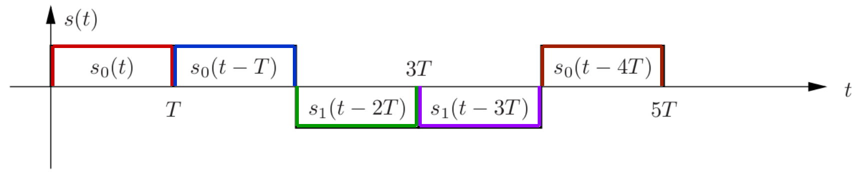
Outputs:

$$y_i(t) = (s_i * h)(t)$$

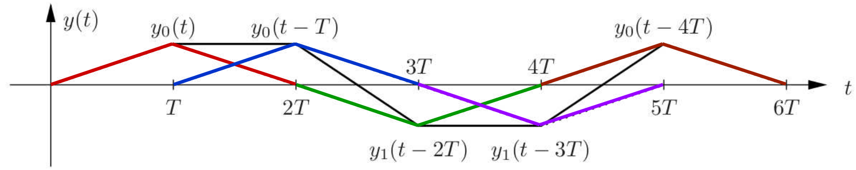


Eye Patterns Again 2(3) – Sequence

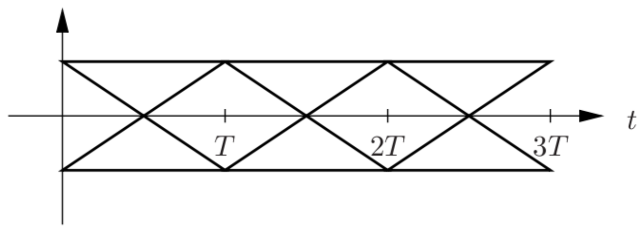
Input sequence:



Corresponding output:

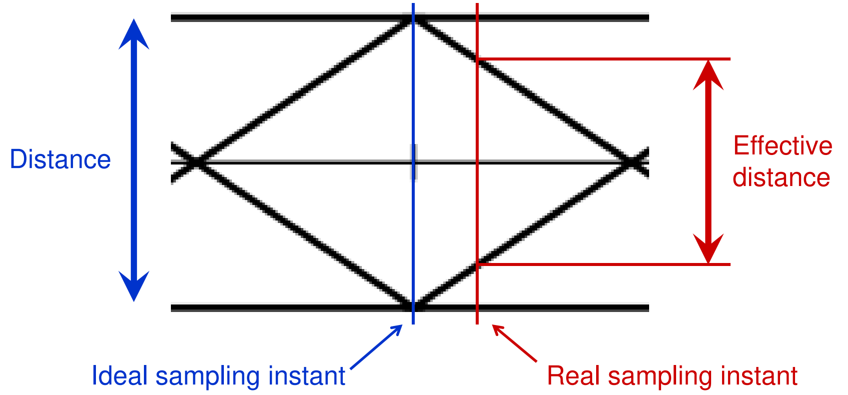


Eye Patterns Again 3(3) – Result



All possible output transitions during an interval of duration T, repeated a number of intervals.

Impact of Synchronization Errors

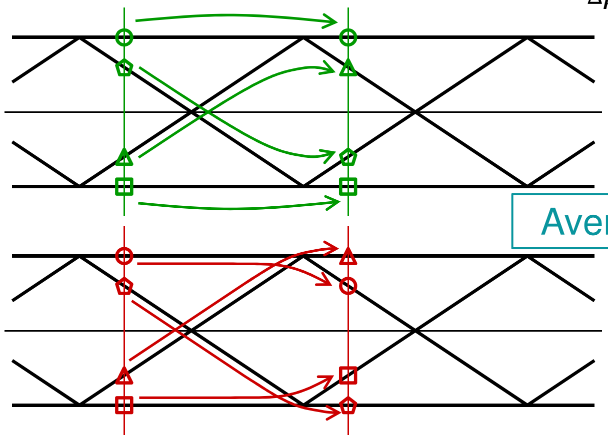


Timing Recovery – Mueller & Müller

Case: Sampling too late

x_k : Sample a_k : Detected

$\Delta_k = x_k a_{k-1} - x_{k-1} a_k$



$\Delta_k = 0$

Average: $\Delta_k < 0$

$\Delta_k < 0$

TSKS02 Telecommunication – Exam

Two parts:	Question part:	3×5p	At least 5 points.
	Problem part:	3×5p	At least 5 points.
Grades:	Grade 3 (ECTS C):	12p	
	Grade 4 (ECTS B):	17p	Max: 30p
	Grade 5 (ECTS A):	22p	
Allowed aids:	Pocket calculator with empty memory.		
	Language dictionary to-from English.		

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