TSKS02 Telecommunication

Lecture 5

Demodulation of Angle Modulation

Pulse Modulation Methods

– Sampling, PAM

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Part 1:

Angle Modulation

PM – Phase Modulation

Momentary phase: $\phi\{m(t)\} = a \cdot m(t),$

Signal: $x(t) = A \cdot \cos\left(2\pi f_c t + a \cdot m(t)\right).$

Momentary frequency: $f_{\text{mom}}(t) = \frac{1}{2\pi} \cdot \frac{d}{dt}\left(2\pi f_c t + a \cdot m(t)\right) = f_c + \frac{a}{2\pi} \cdot \frac{d}{dt} m(t),$

Frequency deviation: $f_d(t) = \frac{a}{2\pi} \cdot \frac{d}{dt} m(t).$

Peak frequency dev.: $f_{d,\max} = \frac{a}{2\pi} \cdot \max\left|\frac{d}{dt} m(t)\right|$

Frequency mod. idx: $\mu_f = \frac{a}{2\pi B} \cdot \max\left|\frac{d}{dt} m(t)\right|$

FM – Frequency Modulation

Momentary phase: $\phi\{m(t)\} = a \int m(t)\, dt,$ $\phi\{m(t)\} = a \int_{t_0}^{t} m(\tau)\, d\tau,$

Signal: $x(t) = \cos\left(2\pi f_c t + a \int m(t)\, dt\right)$

Momentary frequency: $f_{\text{mom}}(t) = \frac{1}{2\pi} \cdot \frac{d}{dt}\left(2\pi f_c t + a \int m(t)\, dt\right) = f_c + \frac{a}{2\pi} \cdot m(t).$

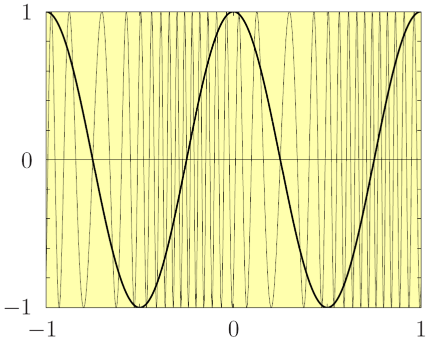
Frequency deviation: $f_d(t) = \frac{a}{2\pi} \cdot m(t).$

Peak frequency dev.: $f_{d,\max} = \frac{a}{2\pi} \cdot \max|m(t)|$

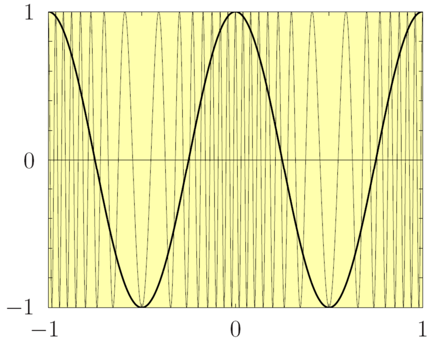
Frequency mod. idx: $\mu_f = \frac{a}{2\pi B} \cdot \max|m(t)|$

Angle Modulation in the Time Domain

Phase Modulation



Frequency Modulation



Demodulating Phase Modulation

Sent signal: $x(t) = A \cdot \cos\left(2\pi f_c t + \phi\{m(t)\}\right)$

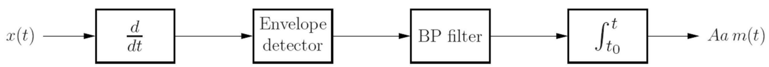
Derivative: $\frac{d}{dt}x(t) = -A\left(2\pi f_c + \frac{d}{dt}\phi\{m(t)\}\right)\sin\left(2\pi f_c t + \phi\{m(t)\}\right)$

Envelope detector gives us: $A\left(2\pi f_c + \frac{d}{dt}\phi\{m(t)\}\right)$

BP or HP filter gives us: $A\frac{d}{dt}\phi\{m(t)\}$

For PM: $\phi\{m(t)\} = am(t).$

Integrate: $\int_{t_0}^{t} A\frac{d}{d\tau}\phi\{m(\tau)\}\,d\tau = Aa\left(m(t) - m(t_0)\right)$ Filter!



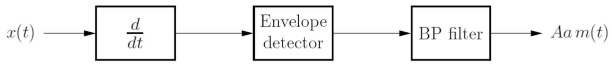
Demodulating Frequency Modulation

As for PM: Derivative, envelope detection and HP/BP filter gives us:

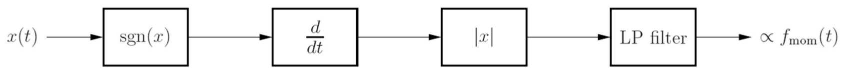
$A\frac{d}{dt}\phi\{m(t)\}$

For FM: $\phi\{m(t)\} = a\int m(t)\,dt.$

Then: $A\frac{d}{dt}\phi\{m(t)\} = A\frac{d}{dt}a\int m(t)\,dt = Aa\,m(t)$ Done!



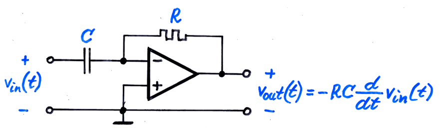
Alternative Demodulation



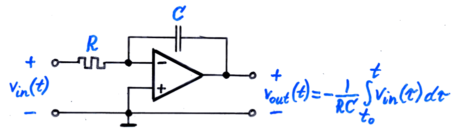
$$\mathrm{sgn}(x) = \begin{cases} 1, & x > 0, \\ 0, & x = 0, \\ \text{-}1, & x < 0. \end{cases}$$

Building Blocks

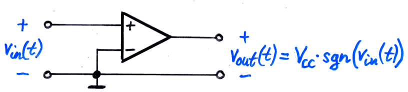
Derivative



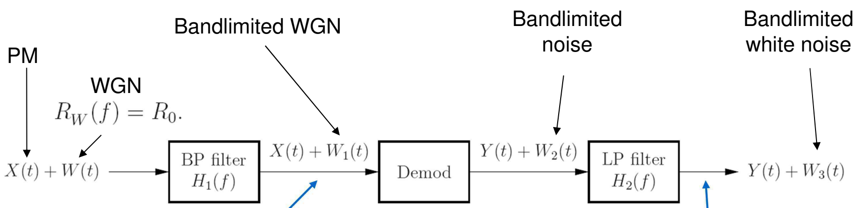
Integration



The Sign function



Noise Impact on Phase Modulation



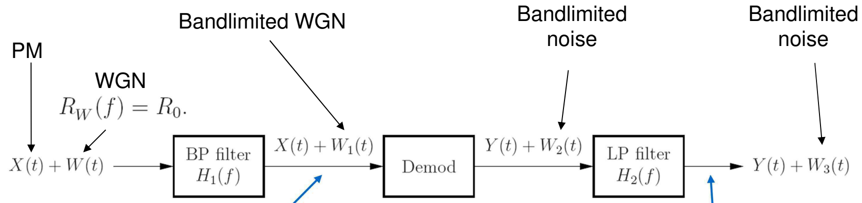
Input SNR

$$\frac{P_X}{P_{W_1}} = \frac{A^2}{4B_X R_0}$$

Output SNR

$$\frac{P_Y}{P_{W_3}} = \frac{A^2 a^2 P_M}{4BR_0}$$

Noise Impact on Frequency Modulation



Input SNR

$$\frac{P_X}{P_{W_1}} = \frac{A^2}{4B_X R_0}$$

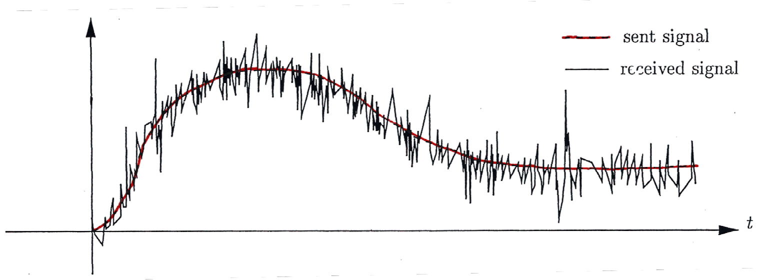
Output SNR

$$\frac{P_Y}{P_{W_3}} = \frac{3A^2 a^2 P_M}{4B^3 R_0}$$

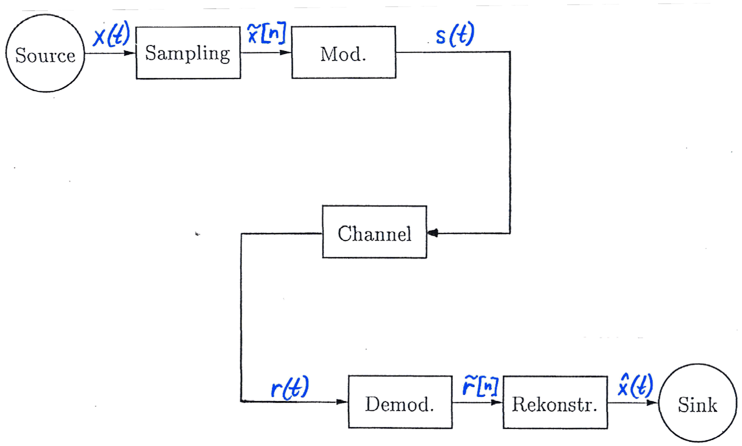
Part 2:

Pulse Modulation Methods

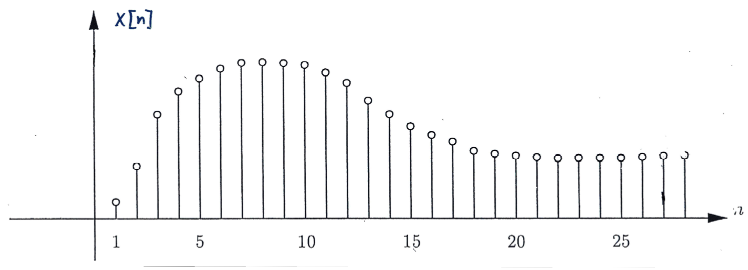
A Noisy Channel



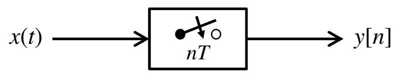
A Communication System



A Time-Discrete Signal



Sampling



Sampling frequency: $f_s = 1/T$

Time domain: $y[n] = x(nT)$

Frequency domain: $$\tilde{X}[\theta] = \frac{1}{T}\sum_m X\left(\frac{\theta - m}{T}\right) = f_s \sum_m X\big((\theta - m) f_s\big)$$

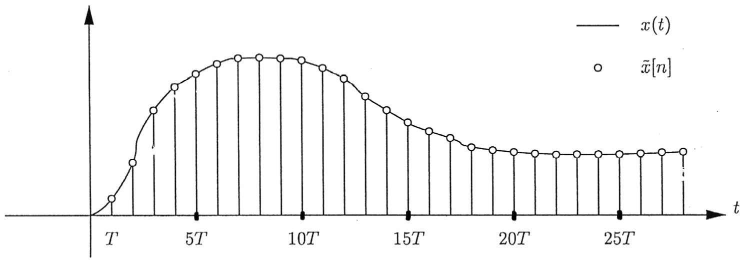
If $X(f) = 0$ for $|f| \geq f_s/2$:

$$\tilde{X}[\theta] = f_s X(\theta f_s) \quad \text{for} \quad |\theta| < 1/2$$

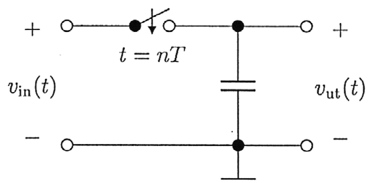
$$\tilde{X}[\theta] = \tilde{X}[\theta - k] \quad \text{for} \quad k \text{ integer}$$

Sampling

$\tilde{x}[n] = x(nT)$

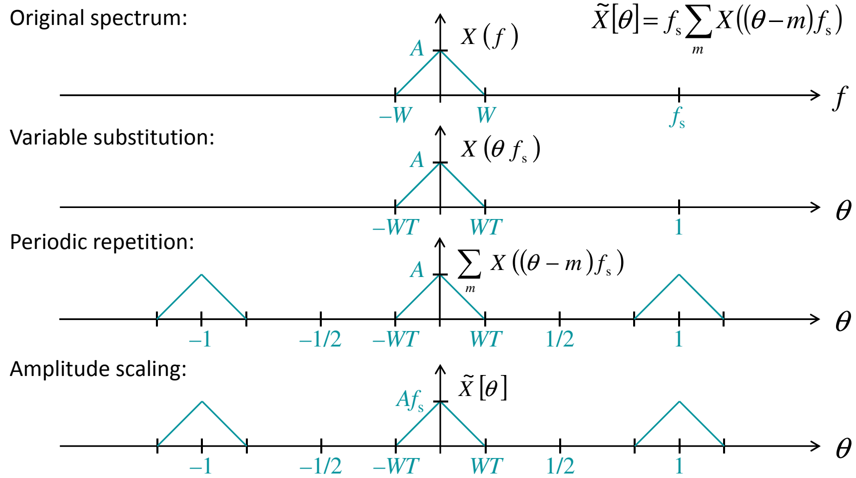


A Simple Sample-and-Hold Circuit



Sampling – Frequency Domain

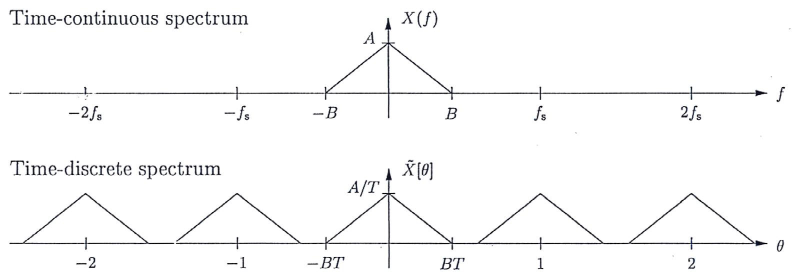
$$\tilde{X}[\theta] = f_s \sum_m X((\theta - m) f_s)$$



Sampling

Poisson's summation formula: $\tilde{X}[\theta] = \frac{1}{T} \sum_{k=-\infty}^{\infty} X\left(\frac{\theta - k}{T}\right)$

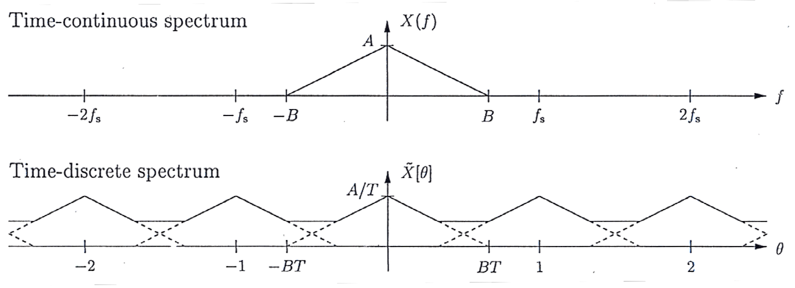
Low bandwidth: $B < f_s/2$



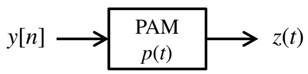
Sampling

Poissons summation formula : $\tilde{X}[\theta] = \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(\frac{\theta-k}{T}\right)$

High bandwidth : $B > f_s/2$



PAM

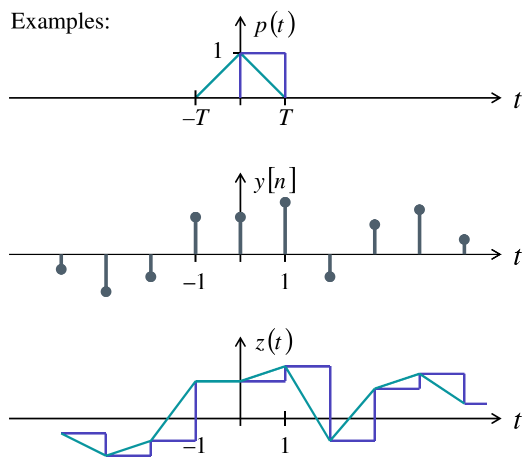


Time domain:

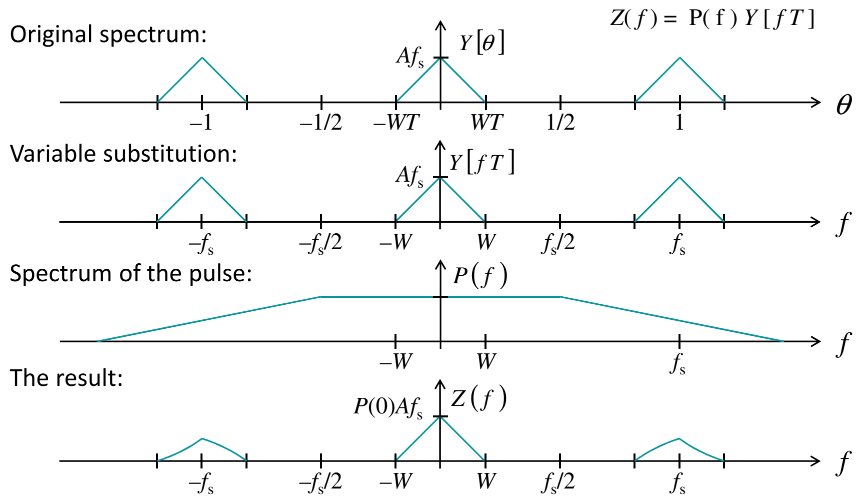
$$z(t) = \sum_n y[n]\, p(t-nT)$$

Frequency domain:

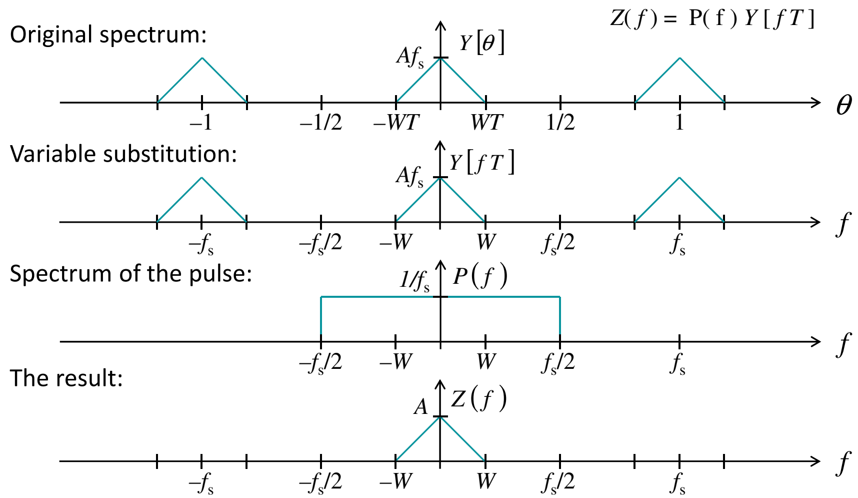
$$Z(f) = P(f)\, Y[fT]$$



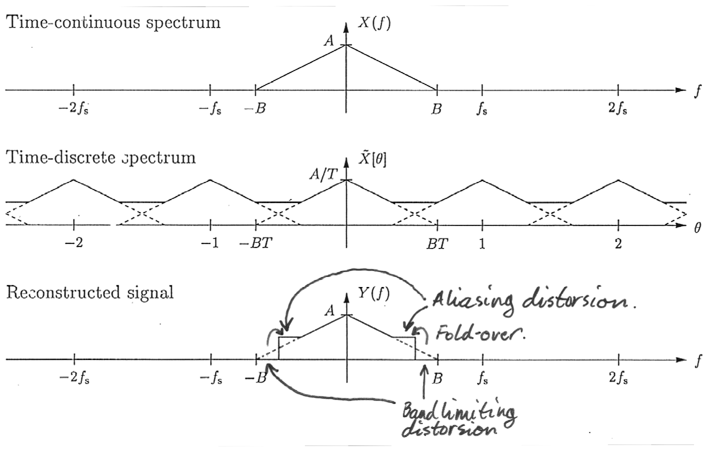
PAM – Frequency Domain 1(2) – General case



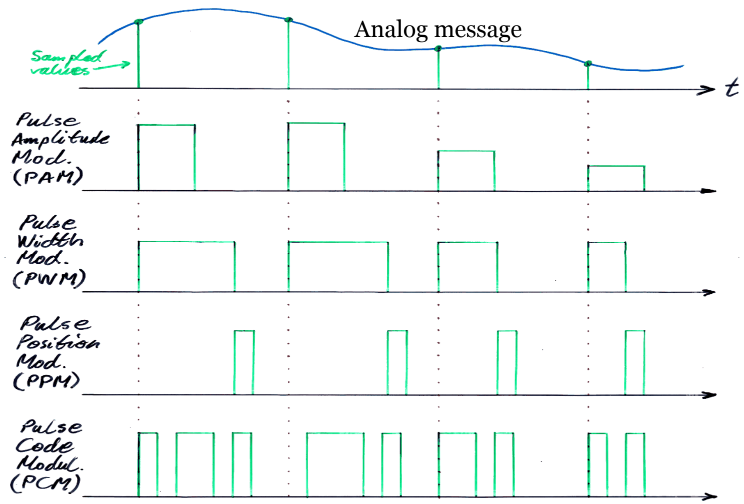
PAM – Frequency Domain 1(2) – Ideal Reconstruction



Aliasing



Pulse Modulation Techniques



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