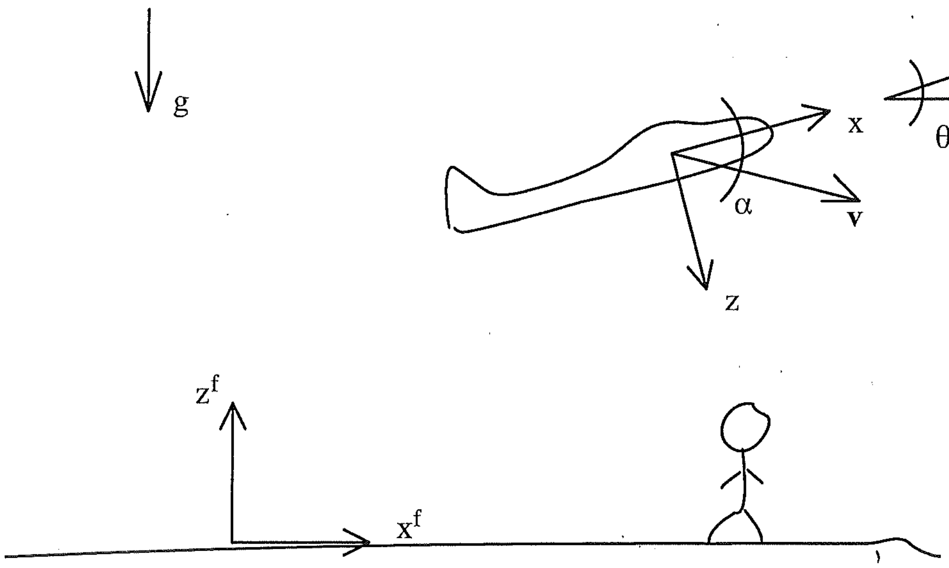
Computer assignment I

The motion of an aeroplane moving in the xz–plane is to be studied, i.e. the so–called longitudinal degrees of freedom are included. We introduce one coordinate system Oxyz attached to the aeroplane and one coordinate system $Ox^f y^f z^f$ fixed to the ground and assumed to be inertial.



Equations of motion and kinematical equations

The longitudinal equations of motion for an aeroplane, together with kinematical relations for the orientation and position of the aeroplane can be written

$$\dot{u} = -qw - g\sin\theta + X/m$$

$$\dot{w} = qu + g\cos\theta + Z/m$$

$$\dot{q} = M/I_{yy}$$

$$\dot{\theta} = q$$

$$\dot{x}^f = u\cos\theta + w\sin\theta$$

$$\dot{z}^f = (-u\sin\theta + w\cos\theta)(-1) \qquad (1)$$

where X, Z and M are forces and moments on the aeroplane, not including the force of gravity, i.e. aerodynamical forces and moment and forces and moment from the engine. The factor (–1) in the last equation serves to rotate the ground–fixed system so that the z^f–axis points upwards. The two last equations are the relations between the aeroplane velocity in moving (aeroplane fixed) and ground fixed coordinates respectively. The velocity vector is always the time derivative relative to a fixed reference of the position vector relative to a fixed reference, but can be represented in either fixed or moving coordinates. The first two equations are the equations of motion expressed in the aeroplane fixed coordinate system so that u and w are the components of the velocity vector in a rotating coordinate system and $\dot{u}$ and $\dot{w}$ are components of the derivative relative to a rotating reference of the velocity vector. The acceleration of the aeroplane is the time derivative relative to a fixed reference of the velocity vector; its components in the rotating system are $\dot{u} + qw$ and $\dot{w} - qu$.

The above six equations shall be solved numerically, at first using the following initial conditions:

$$u_i = V$$

$$w_i = 0$$

$$q_i = 0$$

$$\theta_i = 0$$

$$x_i^f = 0$$

$$z_i^f = H \qquad (2)$$

where V and H are the velocity and height of the flight condition of your data. If your data is for sea level flight, the height is set to H=100 m to avoid underground flight.

Assignment I:a

Implement eqs. (1) above in MATLAB with X=Z=M=0. Calculate the motion for 100 s using the initial conditions (2) above and plot z^f as a function of x^f. In the absence of aerodynamical forces and engine forces the correct solution is, of course, a parabola. Verify that the endpoint of your numerical solution agrees with an analytical solution.

Reference state

We will use a force model where the aerodynamical forces are linearized about a reference state. We assume that this reference state is constant velocity flight along a straight line parallell to the ground with angular velocity zero. The reference state is, thus, trimmed with all forces and moments in equilibrium. Further, the x–axis is in the direction of the velocity in the reference state, so that the pitch angle θ is also zero in the reference state. Let index "0" denote the reference state and put:

$$u_0 = V$$

$$w_0 = 0$$

$$q_0 = 0$$

$$\theta_0 = 0$$

$$x_0^f = 0$$

$$z_0^f = H \tag{3}$$

The flight condition of your dataset is thus assumed to be a trimmed state, which we take as the reference condition of our model. By inserting (3) into the first three of equations (1) we can compute X, Z and M, i.e. the forces except gravitation, in the reference state:

$$X/m = g\sin\theta_0 = 0$$

$$Z/m = -g\cos\theta_0 = -g$$

$$M/I_{yy} = 0 \tag{4}$$

where the assumption that flight is parallell to the ground in the reference state is introduced as θ_0=0.

Assignment I:b

Implement the forces (4) and perform a simulation with the initial conditions (2). Since this choice of initial conditions means that the calculation is started from the (trimmed) reference state the motion should be along a straight line parallell to the ground. Verify this.

Force model

We assume, with a drastic simplification, that the aerodynamic forces on the aeroplane can be written as

$$X/m = g\sin\theta_0 + X_u(u - u_0) + X_w(w - w_0)$$

$$Z/m = -g\cos\theta_0 + Z_u(u - u_0) + Z_w(w - w_0)$$

$$M/I_{yy} = M_w(w - w_0) + M_q(q - q_0) \tag{5}$$

This is a linear model, which is assumed to be valid close to the reference state defined by equations (3).

Note that (5) does not contain any terms for changes in throttle or elevator settings, which are, thus, assumed to be constant.

Also note that (5:3) would normally contain a term $M_{\dot{w}}(\dot{w} - \dot{w}_0)$ which has been dropped here to simplify the implementation.

Finally, note the difference between *initial conditions* and *reference state*. The initial conditions defines the state from which we start the numerical solution in a particular simulation whereas the reference state is the state about which the force model has been linearized. If we had had a better force model, there wouldn't have been a reference state.

We will now study the aeroplane motion for the initial conditions

$$u_i = V$$

$$w_i = 0$$

$$q_i = 0$$

$$\theta_i = 0.1 \text{ rad}$$

$$x_i^f = 0$$

$$z_i^f = H \qquad (6)$$

The aeroplane is thus trimmed with the stick fixed when suddenly (at time t=0) there is a disturbance in pitch angle.

Assignment I:c

Implement the force model (5). Perform a simulation with the initial conditions (6) for 100 s and plot the six phase variables as functions of time as well as $z^f(x^f)$ and the angle of attack as a function of time. If the curves tremble suspiciously, then increase the precision in the MATLAB function `ode45`.

The computed motion should consist of a oscillation with a period of the order of tenths of seconds, which is called the phugoid mode. This motion is normally slightly damped, but can be slightly growing.

Find the period time of the motion by measuring in one of the plots. Compare with the rough approximation

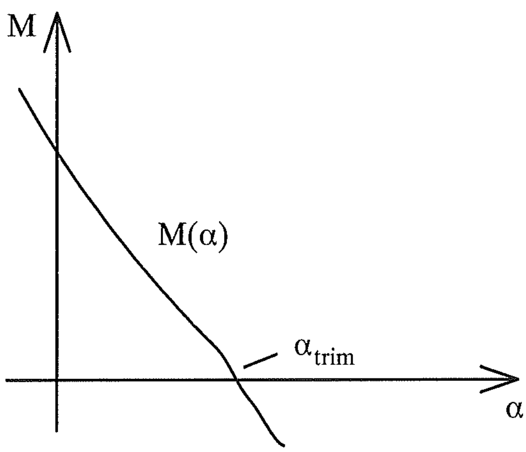
$$\tau_{phugoid} = \sqrt{2}\pi u_0/g$$

In the phugoid motion u usually reaches its maximum about 90 degrees before θ. Measure this phase difference in your plots. Don't answer 90 degrees. Explain how the angle was obtained.

Finally, try to figure out what the aeroplane motion through the air looks like and describe it in words. Draw the aeroplane orientation at a few places in the $z^f(x^f)$ plot. Lead: compare the magnitude of the angle of attack with that of the pitch angle.

Static stability

The concept "static stability" means that the aerodynamic forces should give a restoring moment if there is a disturbance in the angle of attack. This gives a rough idea of the stability of the aeroplane. The pitching moment as a function of angle of attack should look something like this for static stability:



Moment curve for a statically stable aeroplane.

The condition for static stability is:

$$\frac{dM}{d\alpha} < 0$$

Assignment I:d

Modify M_w by putting $C_{m_\alpha} = 0.1$ in the expression in Appendix A. Perform a computation with the initial conditions (6) for 100 s, and plot the six phase variables as functions of time as well as $z^f(x^f)$ and the angle of attack as a function of time. Comment on the result. It might be necessary to increase the coefficient more, to $C_{m_\alpha} = 0.5$ say, to obtain the desired effect for large and heavy transports.

The short–period mode

The longitudinal motion of an aeroplane typically has two different modes, the phugoid mode studied in assignment I:c above and the short period mode. The short period mode has a much shorter period than the phugoid mode and is also strongly damped, which is a desired property since an aeroplane with a growing short period mode is completely uncontrollable. This strong damping, however, makes the mode difficult to see in the plots.

Assignment I:e

Restore the static stability that was sabotaged in assignment I:d. Instead, sabotage the damping of the short period mode by putting $M_q = Z_w = 0$ in the force model.

Change the initial conditions (6) by putting $\theta_i = 0$ and instead put $q_i = 0.1$ rad/s. Perform a calculation with these initial conditions for a time corresponding to about one fifth of the phugoid oscillation period, and plot the six phase variables as functions of time as well as $z^f(x^f)$ and the angle of attack as a function of time. Compare the angle of attack with the pitch angle for both this oscillation and the phugoid oscillation and comment on the difference. Describe the motion in the short–period–mode.