TSKS21 Signaler, information & bilder

Föreläsning 8

DFT och fönstring

Sampling och rekonstruktion

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DFT – Diskret fouriertransform

Tidsdiskret signal med begränsad tidsutbredning:

$$x[k]=0 \quad \text{för} \quad k \notin \{0,1,\ldots,N-1\}$$

Fouriertransform: $X(\Omega)=\sum_{k=-\infty}^{\infty} x[k]e^{-jk\Omega}=\sum_{k=0}^{N-1} x[k]e^{-jk\Omega}$ Kont. m. period 2π.

DFT av längd L: $X_L(n)=X\left(2\pi\frac{n}{L}\right)=\sum_{k=0}^{N-1} x[k]e^{-j2\pi\frac{n}{L}k}$

för $n \in \{0,1,\ldots,L-1\}$

IDFT – Inversen till diskret fouriertransform

IDFT: $x_L[k]=\frac{1}{L}\sum_{n=0}^{L-1} X_L(n)e^{j2\pi\frac{n}{L}k} \quad \Rightarrow \quad x_L[k+L]=x_L[k]$

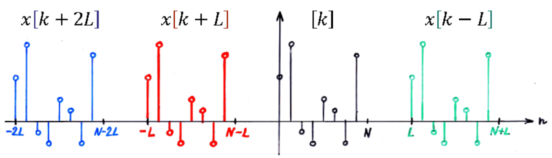
Diskret med period L.

Förhållande till $x[k]$:

$$x_L[k]=\frac{1}{L}\sum_{n=0}^{L-1}\sum_{m=0}^{N-1} x[m]e^{-j2\pi\frac{n}{L}m}\, e^{j2\pi\frac{n}{L}k}=\sum_{m=0}^{N-1} x[m]\underbrace{\frac{1}{L}\sum_{n=0}^{L-1} e^{-j2\pi(m-k)\frac{n}{L}}}_{=\begin{cases} L, & m=k \mod L \\ 0, & \text{för övrigt}\end{cases}} =\sum_{i--\infty}^{\infty} x[k-iL]$$

Om $L<N$ så får vi överlapp mellan de olika kopiorna. Detta kallas vikning.

Därför: Krav $L \geq N$



DFT – Periodisk faltning

Vi är vana vid: $Y(\Omega)=X(\Omega)H(\Omega) \quad \Leftrightarrow \quad y[k]=(x*h)[k]$

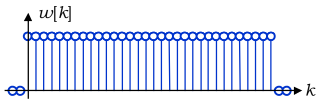
Men vi har: $y[k]=x[k]h[k] \quad \Leftrightarrow \quad Y(\Omega)=\frac{1}{2\pi}\int_0^{2\pi} X(\Phi)H(\Omega-\Phi)d\Phi$

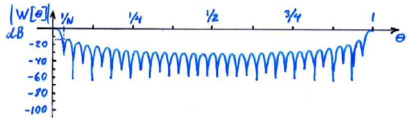
Med DFT: $Y_L(n)=X_L(n)H_L(n) \quad \Leftrightarrow \quad y_L[k]=\sum_{m=0}^{L-1} x_L[m]h_L[k-m]$

Men också: $y_L[k]=x_L[k]h_L[k] \quad \Leftrightarrow \quad Y_L(n)=\frac{1}{L}\sum_{m-0}^{L-1} X_L(m)H_L(n-m)$

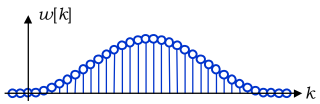
Exempel på fönster, N=32

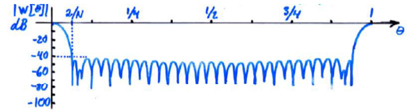
Rektangulärfönster: $w[k] = 1, \quad k \in \{0, 1, \ldots, N-1\}$





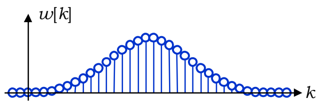
Hammingfönster: $w[k] = 0.54 - 0.46\cos(\frac{2\pi k}{N-1}), \quad k \in \{0, 1, \ldots, N-1\}$

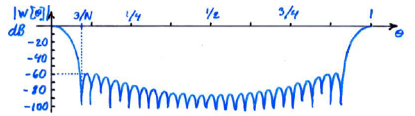




Blackmanfönster: $w[k] = 0.42 - 0.5\cos(\frac{2\pi k}{N-1}) + 0.08\cos(\frac{4\pi k}{N-1}),$

$k \in \{0, 1, \ldots, N-1\}$

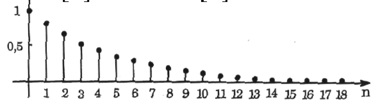


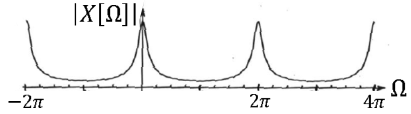


Att använda fönster

Signal:

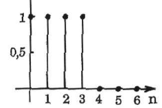
$[k] = e^{-k/5}u[k]$

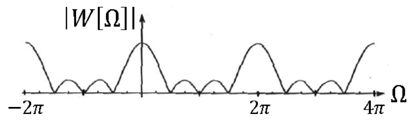




Rektangulärfönster:

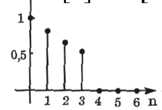
$[k] = 1, 0 \leq k < 4$

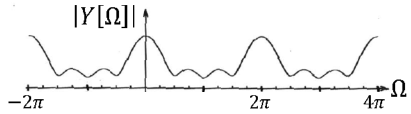




Resultat:

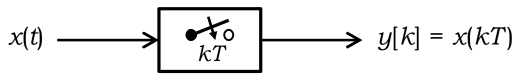
$[k] = x[k]w[k]$



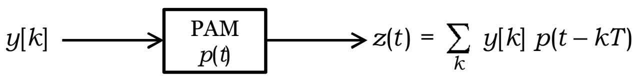


Linjära avbildningar

Sampling:

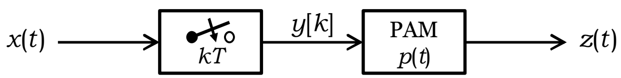


Pulsamplitudmodulering:
(PAM)

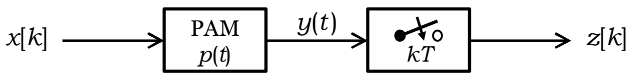


Rekonstruktion:

$t \rightarrow k \rightarrow t$:



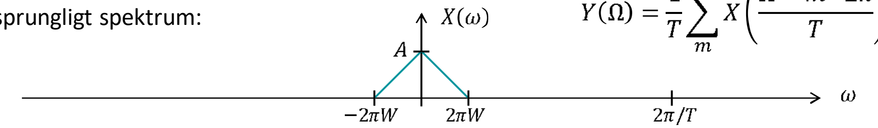
$k \rightarrow t \rightarrow k$:



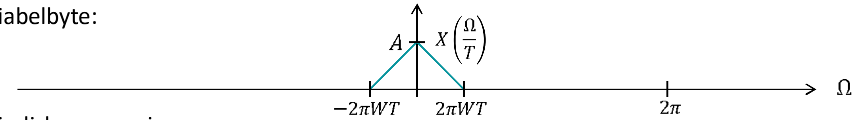
Sampling – Frekvensdomänen

$$Y(\Omega) = \frac{1}{T}\sum_m X\left(\frac{\Omega - m\cdot 2\pi}{T}\right)$$

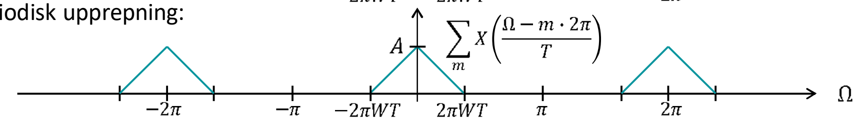
Ursprungligt spektrum:



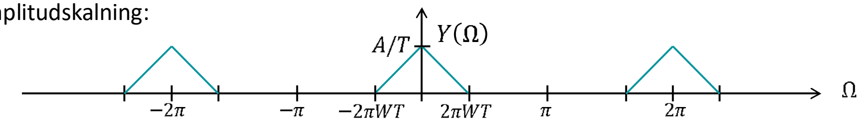
Variabelbyte:



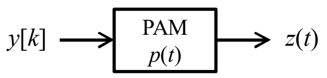
Periodisk upprepning:



Amplitudskalning:



PAM

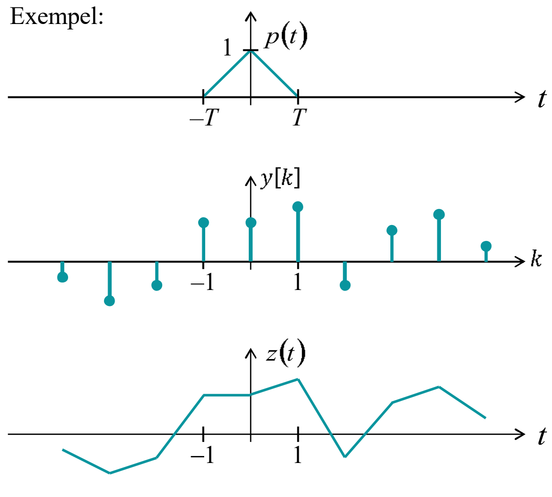


Tidsdomänen:

$$z(t) = \sum_k y[k]\, p(t - kT)$$

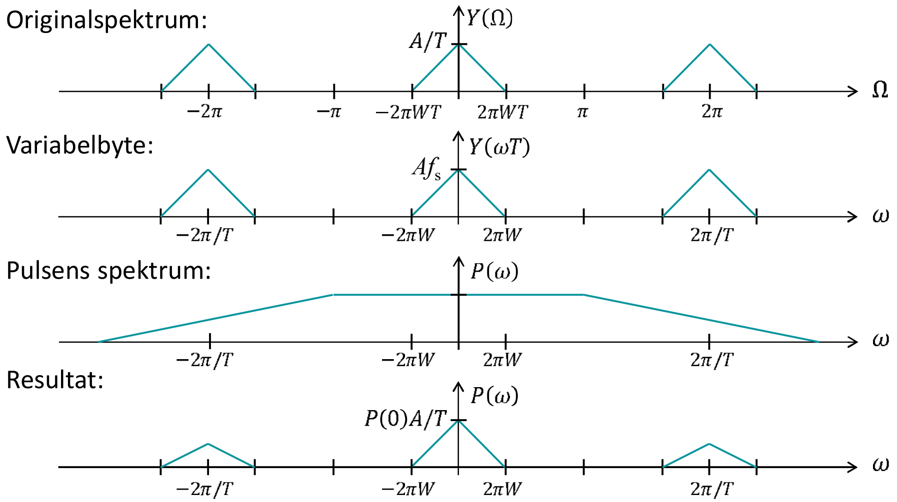
Frekvensdomänen:

$$Z(\omega) = P(\omega)Y(\omega T)$$



PAM – Frekvensdomänen

$$Z(\omega) = P(\omega)Y[\omega T]$$



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