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Modern Physics, TNE046

Exam, 26 August 2023, Answers and short solutions

1. (a) All statements given are false.

   (b) The accelerating potential V gives the proton a kinetic energy of the same order as its rest energy, hence relativistic mechanics is necessary. The kinetic energy is $eV = (\gamma_u - 1)m_p c^2$ from which one obtains after some algebra $u = [\sqrt{(eV)^2 + 2m_p c^2 eV}/(m_p c^2 + eV)]c = 0,875c$.

2. Let right be the positiv direction, primed entities be the ones after the collision.
   (a) The wavelength for a free particle is h/p, which relativistically becomes $h/(\gamma mv) = h/(\gamma_{0,8c} m0,8c)$ $= 1,8$ pm.
   (b) With λ as wavelength for the photon, conservation of energy becomes $hc/\lambda + \gamma mc^2 = hc/\lambda' + \gamma' mc^2$ and conservation of momentum $h/\lambda - \gamma mv = -h/\lambda' + \gamma' mv'$ (all entities taken positive and with sign explicitly written). Dividing the energy consevation by c and then adding the two equations yields $2h/\lambda + \gamma m(c - v) = 0 + \gamma' m(c + v')$. With $v = 0,8c$ and $v' = 0,6c$ one obtains $\lambda = 6h/(5mc) = 2,9$ pm.

3. (a) $\int_0^{1/a} |\psi(x)|^2\,dx = 1 - e^{-3} = 0,323$
   (b) $\Delta x = \sqrt{\bar{x^2} - \bar{x}^2} = \sqrt{3/a^2 - (3/(2a))^2} = \sqrt{3/4}/a = 0,866$ nm.

4. (a) $E_3 = -13,6 \text{ eV}/3^2 = -1,5$ eV.
   (b) The magnuitude of the angular momentum, $|\mathbf{L}|$, and the z-component of the same, L_z.
   (c) $|\mathbf{L}| = \sqrt{\ell(\ell+1)}\hbar$ with $\ell = 0, 1, 2$. $L_z = m_\ell \hbar$ with m_ℓ any integer between (and including) $-\ell$ and ℓ.

5. From PH: Electron concentration $n = N/V = 8,45 \cdot 10^{28}$ m^{-3}, resistivity $\rho = 1,678 \cdot 10^{-8}\,\Omega$m.
   (a) $E_F = \hbar^2/(2m)(3\pi^2 n)^{2/3} = 7,03$ eV
   (b) For a metal $\sigma = \rho^{-1} = ne^2\tau/m$. Energy of conduction electrons is estimated by the Fermi energy, hence their speed $E_F = mv_F^2/2$. The mean distance between collisions become $s = \tau v_F = m/(\rho e^2 n)\sqrt{2E_F/m} = 39$ nm.

6. (a) $1s^2 2^2 2p^6 3s^2 3p^3$ (Harris 8.4)
   (b) Visible light has wavelengths between 390 nm and 730 nm, corresponding to photon energies between 1,7 eV and 3,2 eV. Such photons do not have enough energy to excite electrons across the band gap of diamond, hence they pass through. On the other hand, all photons in the visible region have enough energy to excite electron across the band gap of silicon, hence many of them are absorbed in the material, which becomes opaque.