LINKÖPINGS UNIVERSITET

Exam in TSDT14 Signal Theory

Exam code:	TEN1
Date:	2020-01-08 **Time:** 8.00–12.00
Place:	
Teacher:	Mikael Olofsson, tel: 281343
Visiting exam:	9 and 11
Administrator:	Carina Lindström, 013-284423, carina.e.lindstrom@liu.se
Department:	ISY
Allowed aids:	Olofsson: *Tables and Formulas for Signal Theory* A German 10-Mark note of the fourth series (1991-2001). Pocket calculators of all kinds with empty memory.
Number of tasks:	6
Grading:	Task one is not graded in terms of points. At least two of its three sub-tasks have to be treated correctly as partial fulfillment to pass the exam. Tasks 2-6 yield at most 5 points each. Sloppy solutions and solutions that are hard to read are subject to hard judgement, as are unresonable answers. Totally, you can get 25 points at most. For grade three you need 10 points, for grade four 15 points and for grade five 20 points.
Solutions:	Will be published in the course room no later than three days after the exam, under Previous exams.
Result:	You get a message about your result via an automatic email from Ladok. Note that we cannot file your result if you are not registered on the course. That also means that you will not get an automated email about your result if you are not registered on the course.
Exam return:	In the student office of ISY, Building B, between entrances 27-29, ground floor, Corridor D, right next to Café Java, starting about two weeks after the exam.
Important:	**Solutions and answers must be given in English.**

1 These sub-tasks have to be treated correctly as partial fulfillment to pass the exam:

a. Define the concept *wide-sense stationarity* for a time-discrete process.

b. A time-continuous filter has frequency response

$$H(f) = \begin{cases} 4, & 1 < |f| < 2, \\ 0, & \text{elsewhere} \end{cases}$$

The input $X(t)$ to this filter has PSD $R_X(f) = R_0$. Determine the PSD of the output.

c. We have a stochastic variable that is uniformly distributed on the interval $[-A, A]$. It is quantized uniformly with 1024 quantization steps, and the saturation levels are $\pm A$. Determine the quantization distortion which - as usual - is defined as the expected quadratic error. The usual assumptions apply.

2 Let $X[n]$ be a WSS Gaussian process with mean m_X and ACF

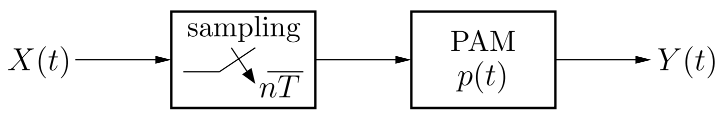
$$r_X[k] = R_0 \delta[k] + m_X^2.$$

Now consider the process $Y[n] = X[n]X[n-1]$.

a. Determine the mean and ACF of $Y[n]$. (4 p)

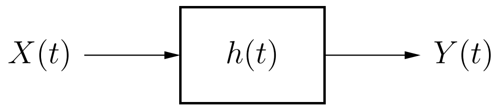
b. Is $Y[n]$ WSS? (1 p)

3 The ACF of the input in the figure below is $r_X(\tau) = \text{sinc}(\tau/T)$, and the pulse-shape of the pulse-amplitude modulator is $p(t) = \text{sinc}^2(t/T)$, where T is the sampling period.



Determine the signal-to-noise ratio P_X/ϵ^2 expressed in dB, where the distorsion ϵ^2 as usual is the power of the reconstruction error. (5 p)

4 The LTI system in the figure below has impulse response $h(t) = a \cdot e^{-t} \cdot u(t)$.



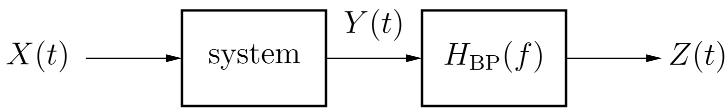
The input is bandlimited white Gaussian noise with mean zero and PSD

$$R_X(f) = \begin{cases} R_0, & |f| < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a. Determine the factor a such that the power of the output equals the power of the input. (4 p)

b. Determine the probability $\Pr\{Y(t) = 2\}$. (1 p)

5 Let $X(t)$ be a zero-mean Gaussian process with ACF $r_X(\tau) = \mathrm{sinc}(2B\tau)$. Consider the following situation:



The system labelled $H_{\mathrm{BP}}(f)$ is an ideal BP filter with frequency response

$$H_{\mathrm{BP}}(f) = \begin{cases} 1, & B < |f| < 2B, \\ 0, & \text{elsewhere.} \end{cases}$$

The system labelled *system* is an AM-SC modulator with output

$$Y(t) = A \cdot X(t) \cdot \cos(5\pi B t + \Psi),$$

where Ψ is uniformly distributed over $[0, 2\pi)$ and independent of $X(t)$.

Determine the power P_Z.

6 The input to a space-discrete LSI-system is

$$x[n_1, n_2] = \begin{cases} 1, & n_1 = n_2 = \pm 1, \\ 1, & n_1 = n_2 = 0, \\ 0, & \text{elsewhere.} \end{cases}$$

The impulse response of the system is

$$h[n_1, n_2] = \begin{cases} 1, & n_1 = -n_2 = \pm 1, \\ 2, & n_1 = n_2 = 0, \\ 0, & \text{elsewhere.} \end{cases}$$

Determine the output $y[n_1, n_2]$ of the system

a. with calculations in the space-domain. (3 p)

b. with calculations in the frequency-domain. (2 p)

Formulas on Complex Processes

Complex process: $X(t)$, where $X_1(t)$ is the real part, and $X_2(t)$ is the imaginary part.

ACF: $r_X(\tau) = \mathrm{E}\{X(t+\tau)X^*(t)\}$
$= r_{X_1}(\tau) + r_{X_2}(\tau) + j\big(r_{X_1,X_2}(-\tau) - r_{X_1,X_2}(\tau)\big)$

PSD: $R_X(f) = R_{X_1}(f) + R_{X_2}(f) + j\big(R^*_{X_1,X_2}(f) - R_{X_1,X_2}(f)\big)$

Formulas on Baseband Representation - Deterministic Input

$\tilde{s}(t)$ is the message and $s(t)$ is the modulated signal, with bandwidth of $\tilde{s}(t)$ less than f_c.

Passband: $s(t) = \mathrm{Re}\Big\{\tilde{s}(t)e^{j2\pi f_\mathrm{c} t}\Big\} = s_\mathrm{I}(t)\cos(2\pi f_\mathrm{c} t) - s_\mathrm{Q}(t)\sin(2\pi f_\mathrm{c} t)$

Baseband: $\tilde{s}(t) = s_\mathrm{I}(t) + js_\mathrm{Q}(t)$

Spectrum: $S(f) = \frac{1}{2}\big(\tilde{S}(f-f_\mathrm{c}) + \tilde{S}^*(-f-f_\mathrm{c})\big)$, $\tilde{S}(f) = 2S(f+f_\mathrm{c})u(f+f_\mathrm{c})$

Filtering: $y(t) = (x*h)(t) \quad \Leftrightarrow \quad \tilde{y}(t) = \frac{1}{2}(\tilde{x}*\tilde{h})(t)$
where $\tilde{x}(t)$, $\tilde{h}(t)$ and $\tilde{y}(t)$ are the complex baseband representations of $x(t)$, $h(t)$ and $y(t)$, respectively.

Formulas on Baseband Representation - Stochastic Input

$\tilde{S}(t)$ is the message with $m_{\tilde{S}} = 0$ and $S(t)$ is the modulated signal. The bandwidth of $\tilde{S}(t)$ must be less than f_c.

Passband: $S(t) = S_\mathrm{I}(t)\cos(2\pi f_\mathrm{c} t + \Psi) - S_\mathrm{Q}(t)\sin(2\pi f_\mathrm{c} t + \Psi)$
with Ψ uniformly distributed on $[0, 2\pi)$, independent of $\tilde{S}(t)$.

Baseband: $\tilde{S}(t) = S_\mathrm{I}(t) + jS_\mathrm{Q}(t)$

Mean: $m_S = 0$

ACF: $r_S(\tau) = \frac{r_{S_\mathrm{I}}(\tau)+r_{S_\mathrm{Q}}(\tau)}{2}\cos(2\pi f_\mathrm{c}\tau) - \frac{r_{S_\mathrm{I},S_\mathrm{Q}}(-\tau)-r_{S_\mathrm{I},S_\mathrm{Q}}(\tau)}{2}\sin(2\pi f_\mathrm{c}\tau)$

PSDs: $R_S(f) = \frac{1}{4}\big(R_{\tilde{S}}(f-f_\mathrm{c}) + R_{\tilde{S}}(-f-f_\mathrm{c})\big)$, $R_{\tilde{S}}(f) = 4R_S(f+f_\mathrm{c})u(f+f_\mathrm{c})$