

# TSEK03: Radio Frequency Integrated Circuits (RFIC)

## Lecture 5-6: Mixers

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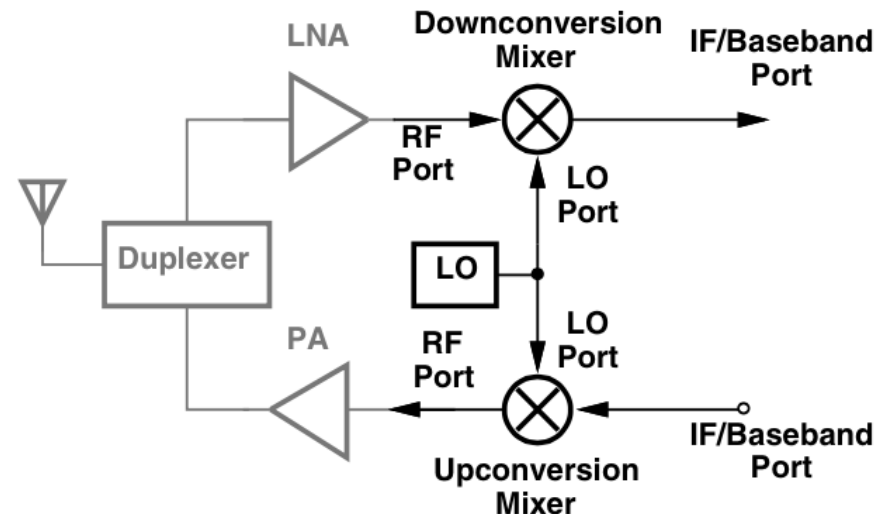
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# Overview

- Razavi: Chapter 6.1-6.3, pp. 343-398.
- Lee: Chapter 13.
  
- **6.1 Mixers general**
- 6.2 Passive downconversion mixers
- 6.3 Active downconversion mixers

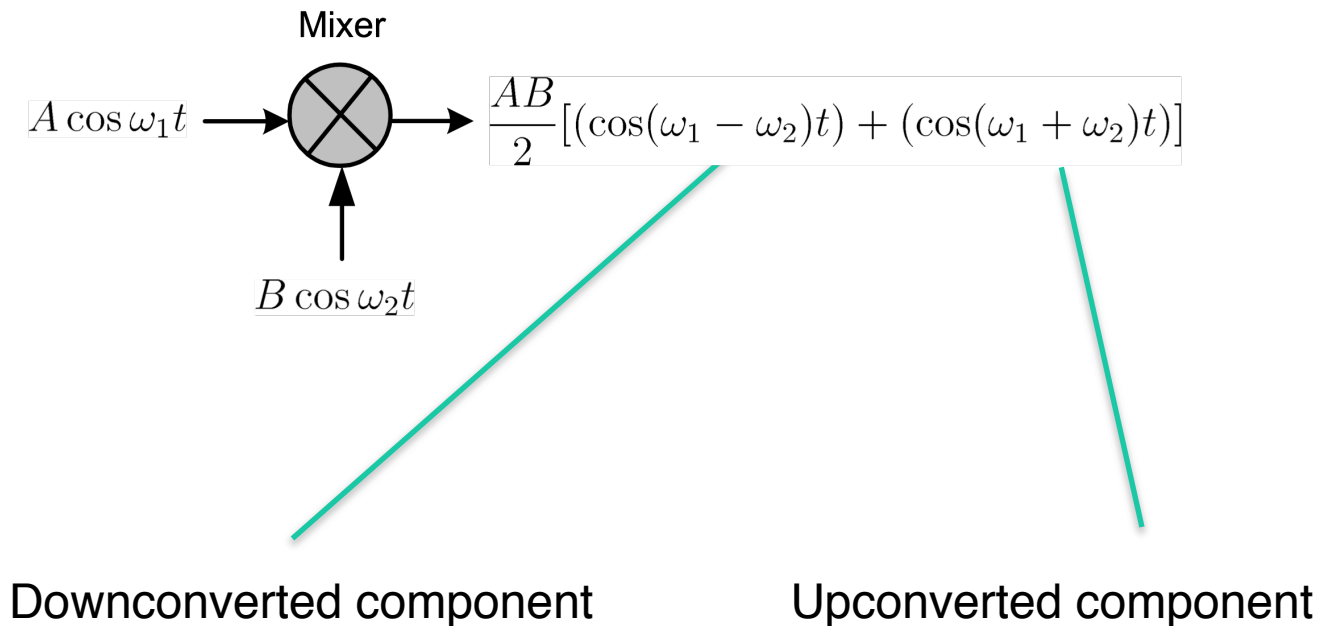
# 6.1 Mixers

- Mixers are used for frequency translation of signals.
- Instead of using several bandpass filters to tune a desired signal, the center frequency of a local oscillator is adjusted.
- Downconversion mixer: an RF signal is translated to a lower frequency known as intermediate frequency (IF).
- In an upconversion mixers, the IF signal is translated to a higher frequency (RF).



# Fundamental

- A mixer basically multiplies two signals in the time domain. Mixing can occur in any nonlinear device.
- $(A \cos \omega_1 t) * (B \cos \omega_2 t) = AB/2[(\cos(\omega_1 - \omega_2)t) + (\cos(\omega_1 + \omega_2)t)]$

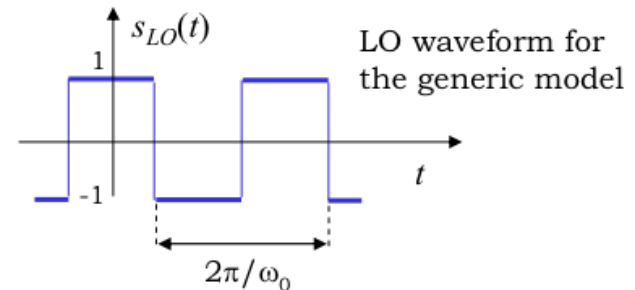


# Simple Mixer

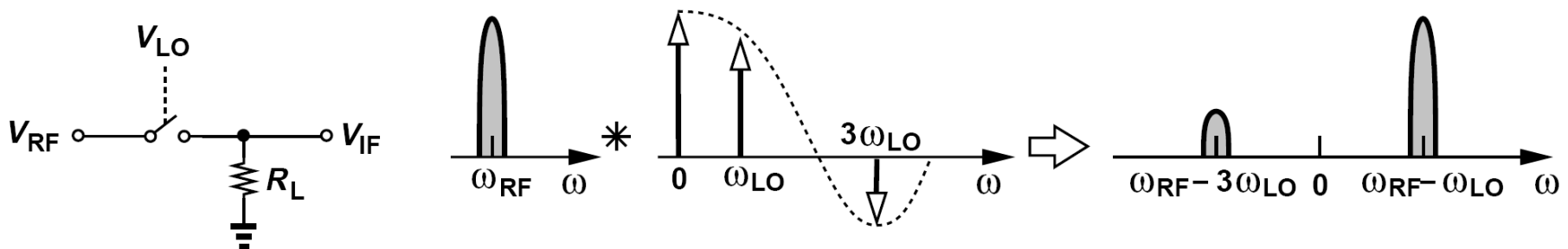
- A simple mixer is realized by a switch which is turned on (off) (abruptly) by the LO signal. It yields:

Switch is on:  $V_{IF} = V_{RF}$

Switch is off:  $V_{IF} = 0$



$$s_{LO}(t) = \frac{4}{\pi} \cos \omega_0 t - \frac{4}{3\pi} \cos 3\omega_0 t + \frac{4}{5\pi} \cos 5\omega_0 t - \dots$$



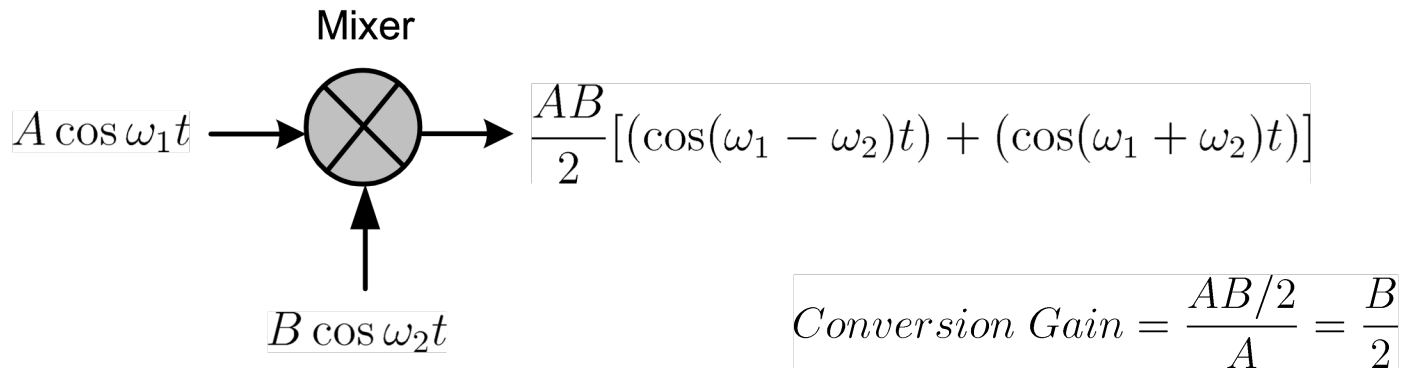
## 6.1.1 Performance Parameters

### Noise and Linearity

- The design of downconversion mixers is a compromise between the noise figure and the  $IP_3$  (or  $P_{1dB}$ ).
- In a receive chain, the input noise of the mixer following the LNA is divided by the LNA gain when referred to the RX input.
- Similarly, the  $IP_3$  of the mixer is scaled down by the LNA gain (different gains, see the LNA lecture)
- Mixer and LNA designs are usually linked.

# Conversion Gain

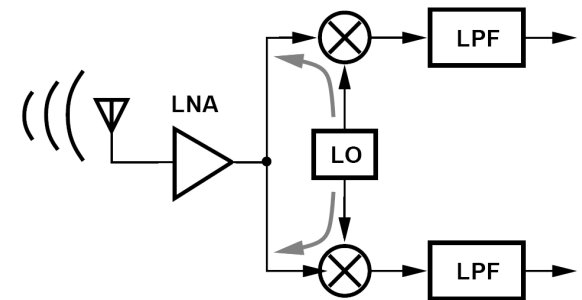
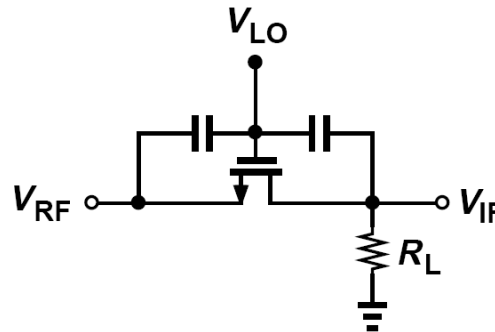
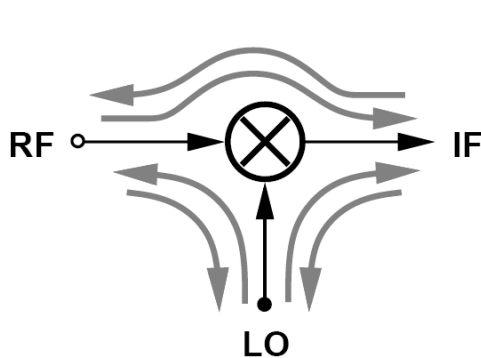
- The ratio of the desired IF output to the value of the RF input is called conversion gain (or loss).



- Mixer gain is critical in suppression of noise (of the following stages) while retaining linearity.
- Low supply voltages make it difficult to achieve a gain of more than roughly 10 dB (while keeping the linearity).

# Port-to-Port Feedthrough

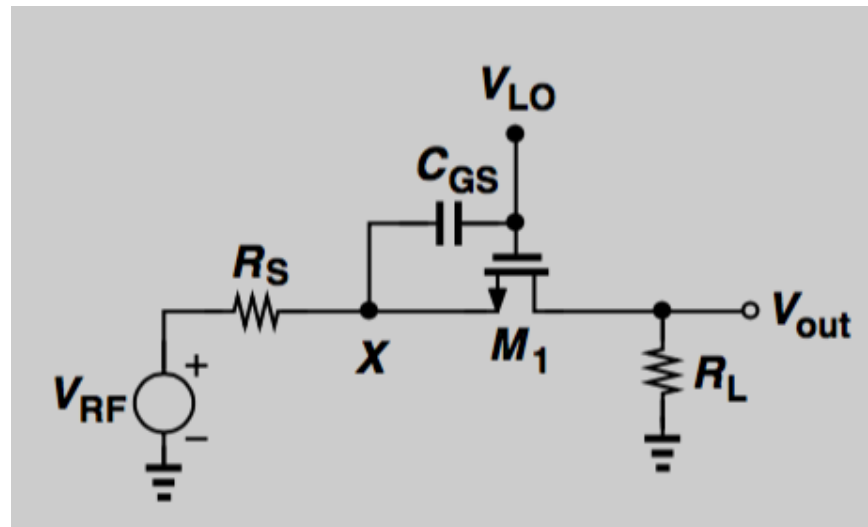
- Coupling due to device capacitances causes port-to-port feedthrough. The impact depends on architecture.



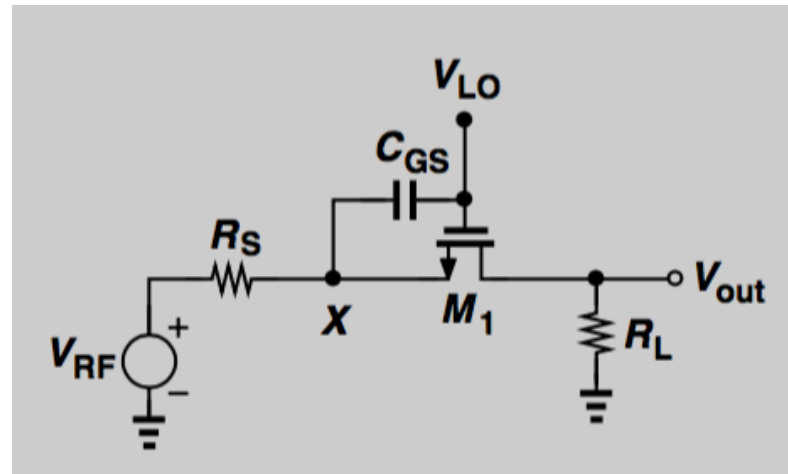


# Example 6.1

- Consider the mixer shown below, where  $V_{LO} = V_1 \cos \omega_{LO}t + V_0$  and  $C_{GS}$  denotes the gate-source overlap capacitance of  $M_1$ . Neglecting the on-resistance of  $M_1$  and assuming abrupt switching, determine the dc offset at the output for  $R_S = 0$  and  $R_S > 0$ . Assume  $R_L \gg R_S$ .



# Example 6.1



## Solution:

The LO leakage to node  $X$  is expressed as

$$V_X = \frac{R_S C_{GS}}{R_S C_{GS} + 1} V_{LO}, \quad (6.1)$$

because even when  $M_1$  is on, node  $X$  sees a resistance of approximately  $R_S$  to ground. With abrupt switching, this voltage is multiplied by a square wave toggling between 0 and 1. The output dc offset results from the mixing of  $V_X$  and the first harmonic of the square wave. Exhibiting a magnitude of  $2 \sin(\pi/2)/\pi = 2/\pi$ , this harmonic can be expressed as  $(2/\pi) \cos \omega_{LO} t$ , yielding

# Example 6.1

$$V_{out}(t) = V_X(t) \times \frac{2}{\pi} \cos \omega_{LO} t + \dots \quad (6.2)$$

$$= \frac{R_S C_{GS} \omega_{LO}}{\sqrt{R_S^2 C_{GS}^2 \omega_{LO}^2 + 1}} V_1 \cos(\omega_{LO} t + \phi) \times \frac{2}{\pi} \cos \omega_{LO} t + \dots, \quad (6.3)$$

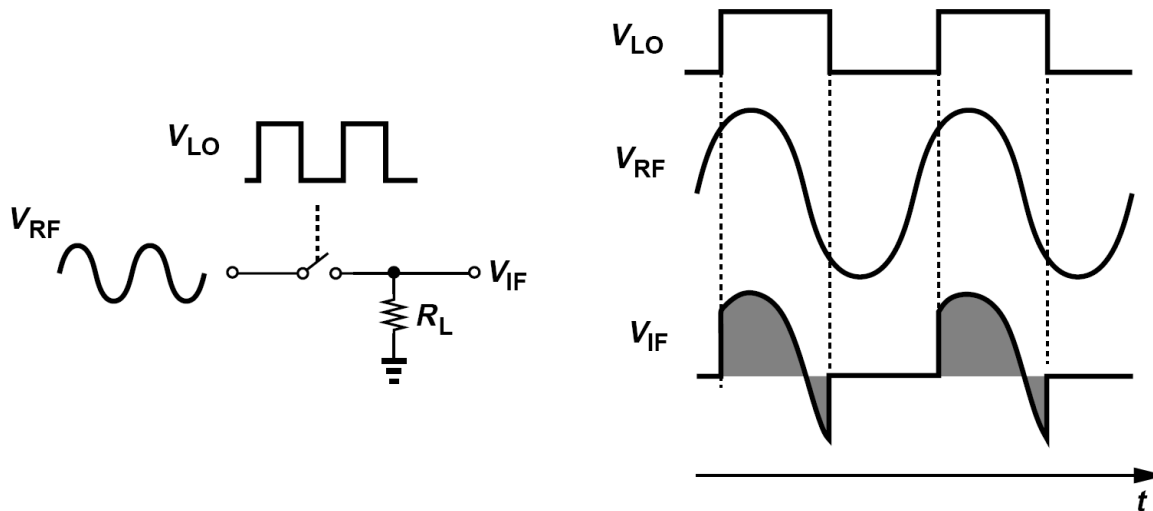
where  $\phi = (\pi/2) - \tan^{-1}(R_S C_{GS} \omega_{LO})$ . The dc component is therefore equal to

$$V_{dc} = \frac{V_1}{\pi} \frac{R_S C_{GS} \omega_{LO} \cos \phi}{\sqrt{R_S^2 C_{GS}^2 \omega_{LO}^2 + 1}}. \quad (6.4)$$

As expected, the output dc offset vanishes if  $R_S = 0$ .

# DC offset

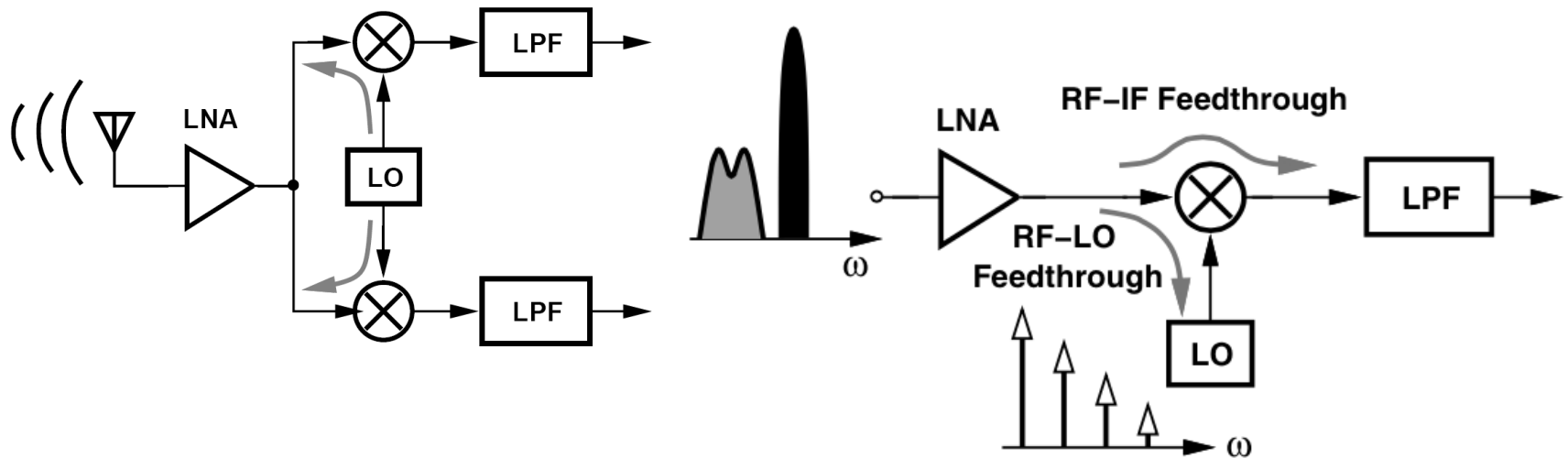
- Suppose the RF input is a sinusoid having the same frequency as the LO.



- Each time the switch turns on, the same portion of the input waveform appears at the output, producing a certain average.

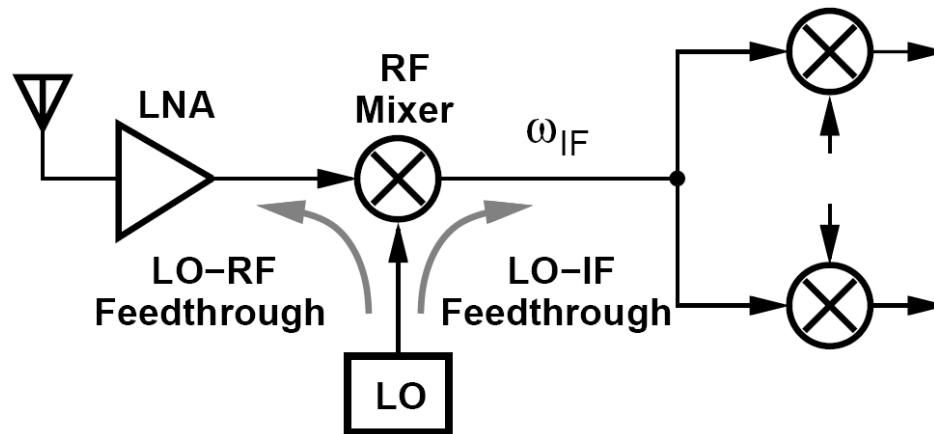
# Port-to-Port Feedthrough

- Direct-conversion:
  - LO-RF feedthrough determined by the symmetry of the mixer circuit and LO waveforms.
  - The LO-IF feedthrough is suppressed by the baseband low-pass filter.



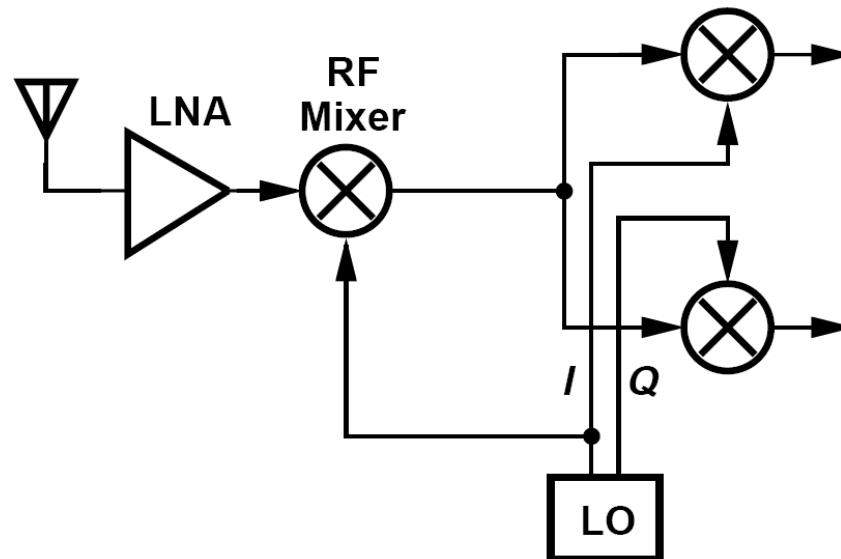
# Port-to-Port Feedthrough

- Heterodyne: LO-IF most serious if  $\omega_{IF}$  and  $\omega_{LO}$  close (filtering problems)

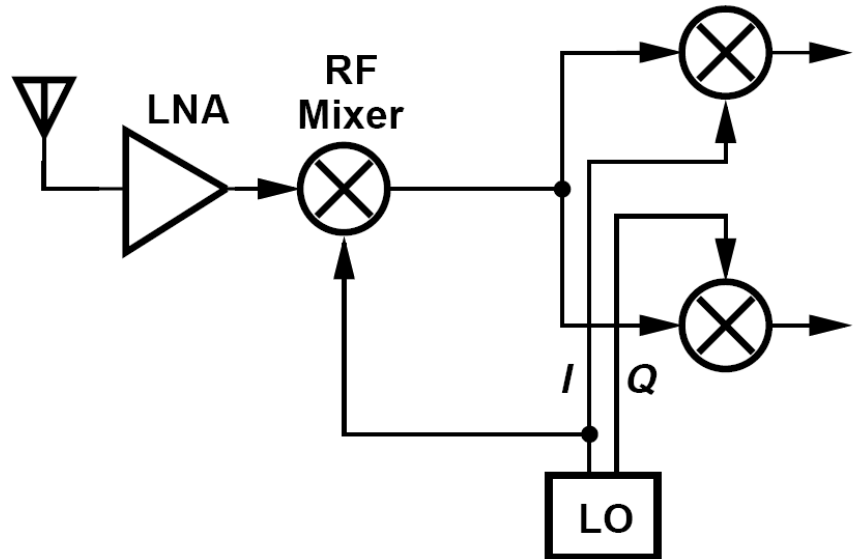


## Example 6.2

- Shown below is a receiver architecture wherein  $\omega_{LO} = \omega_{RF}/2$  so that the RF channel is translated to an IF of  $\omega_{RF} - \omega_{LO} = \omega_{LO}$  and subsequently to zero. Study the effect of port-to-port feedthroughs in this architecture.



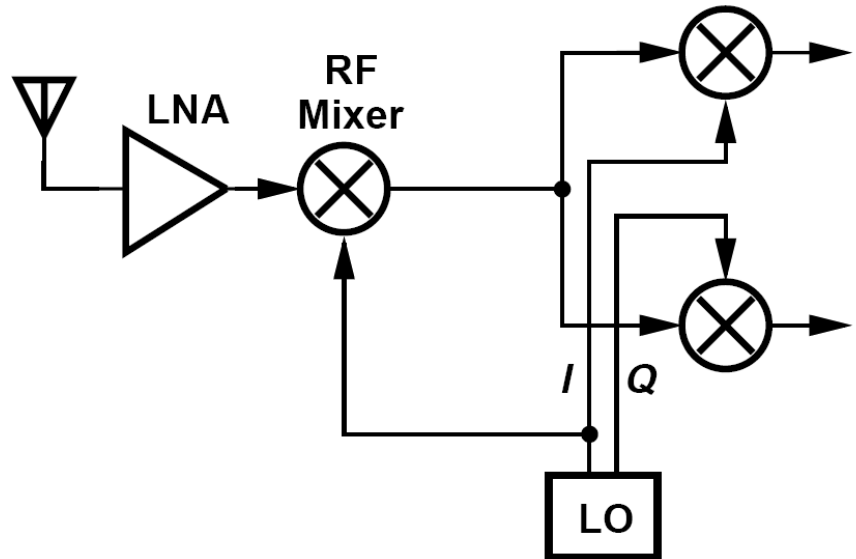
## Example 6.2



For the RF mixer, the LO-RF feedthrough is unimportant as it lies at  $\omega_{RF}/2$  and is suppressed. Also, the RF-LO feedthrough is not critical because in-band interferers are far from the LO frequency, creating little injection pulling. (Interferers near the LO frequency are attenuated by the front end before reaching the mixer.) The RF-IF feedthrough proves benign because low-frequency beat components appearing at the RF port can be removed by high-pass filtering.



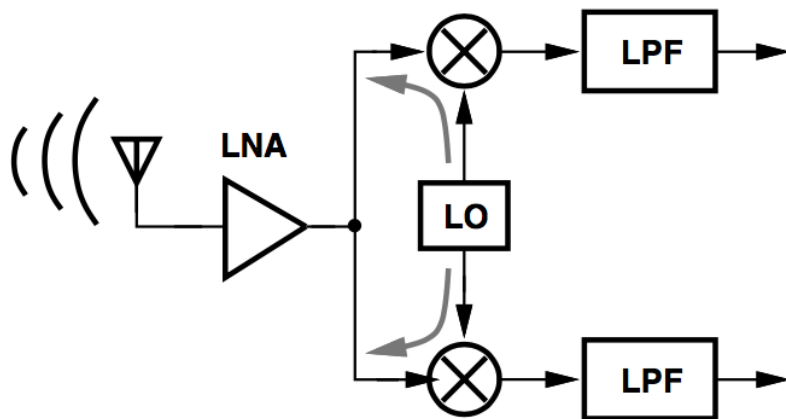
## Example 6.2



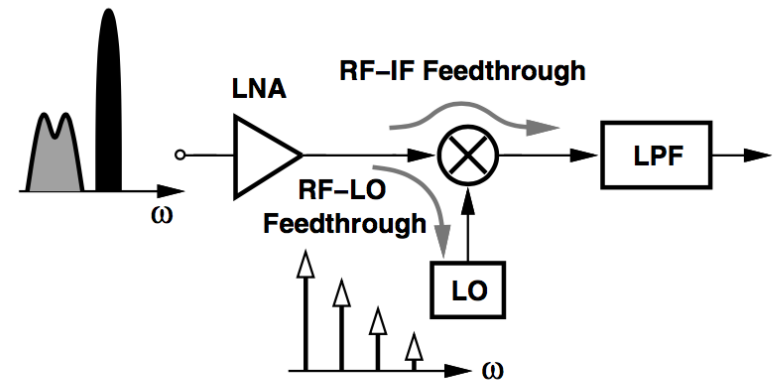
The most critical feedthrough in this architecture is that from the LO port to the IF port of the RF mixer. Since  $\omega_{IF} = \omega_{LO}$ , this leakage lies in the *center* of the IF channel, potentially **desensitizing the IF mixers** (and producing dc offsets in the baseband). Thus, the RF mixer must be designed for minimal LO-IF feedthrough (Section 6.1.3).

# Example 6.2

The IF mixers also suffer from port-to-port feedthroughs. Resembling a direct-conversion receiver, this section of the architecture follows the observations made for the topologies in Figs. 6.4 and 6.7.



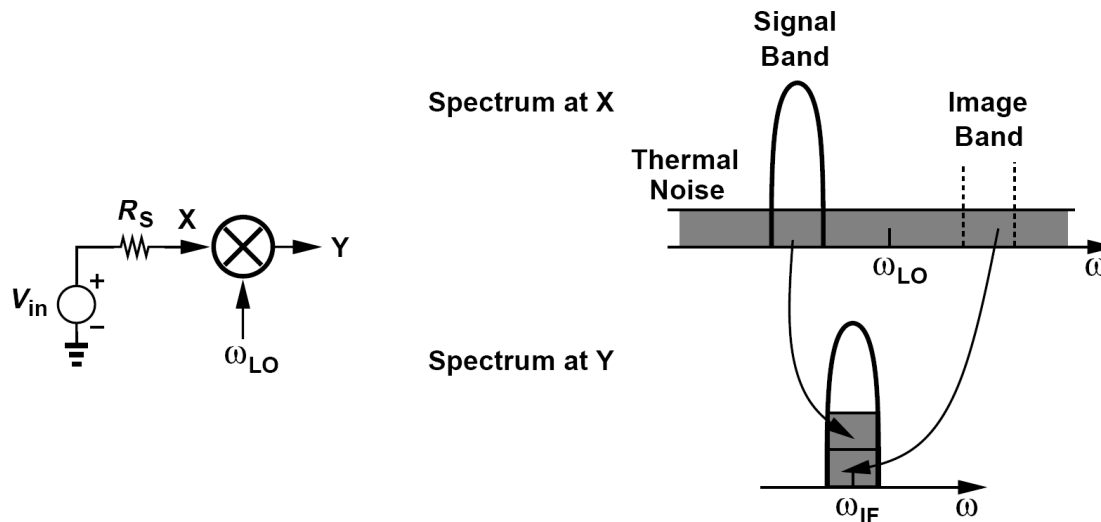
**Figure 6.4** *Effect of LO-RF feedthrough.*



**Figure 6.7** *Effect of RF-LO feedthrough in a direct-conversion receiver.*

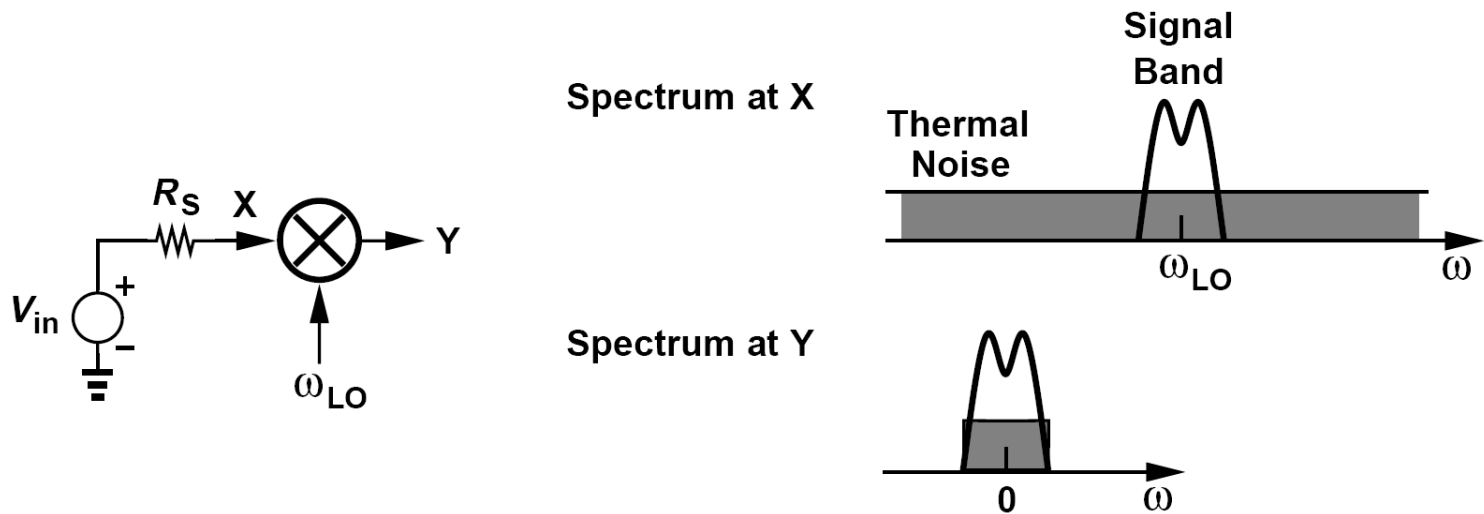
## 6.1.2 Mixer Noise Figure

- $NF = \text{SNR at RF input} \div \text{SNR at IF port}$
- Noiseless mixer: exhibits a flat frequency response at its input from the image band to the signal band.
- The noise figure of a noiseless mixer is 3 dB. This quantity is called the “single-sideband” (SSB) noise.



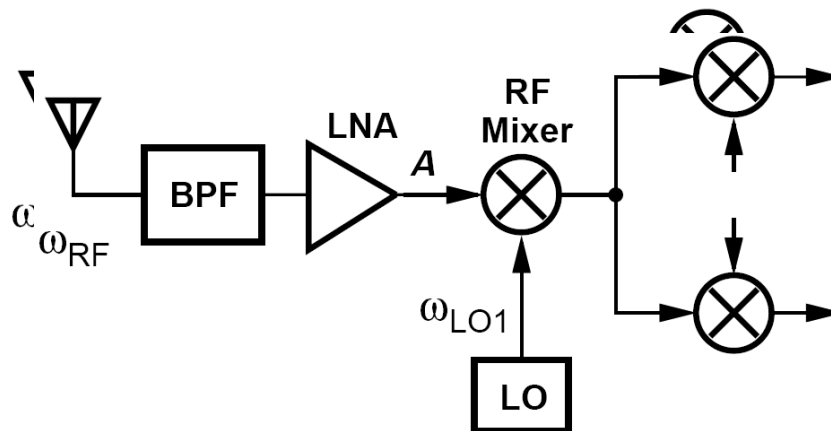
# DSB Noise Figure

- For a direct-conversion mixer, only the noise in the signal band is translated to the baseband.
- The noise figure is thus equal to 0 dB (if mixer is noiseless). This quantity is called the “double-sideband” (DSB) noise figure.



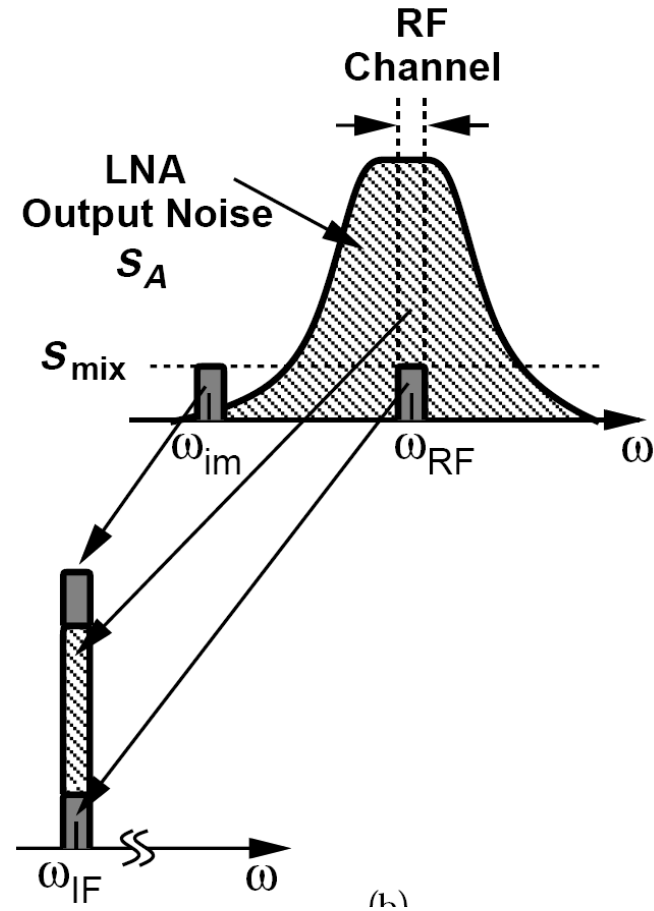
## Example 6.3

- A student designs the heterodyne receiver shown below for two cases: (1)  $\omega_{LO1}$  is far from  $\omega_{RF}$ , (2)  $\omega_{LO1}$  lies inside the band and so does the image. Study the noise behavior of the receiver in the two cases.



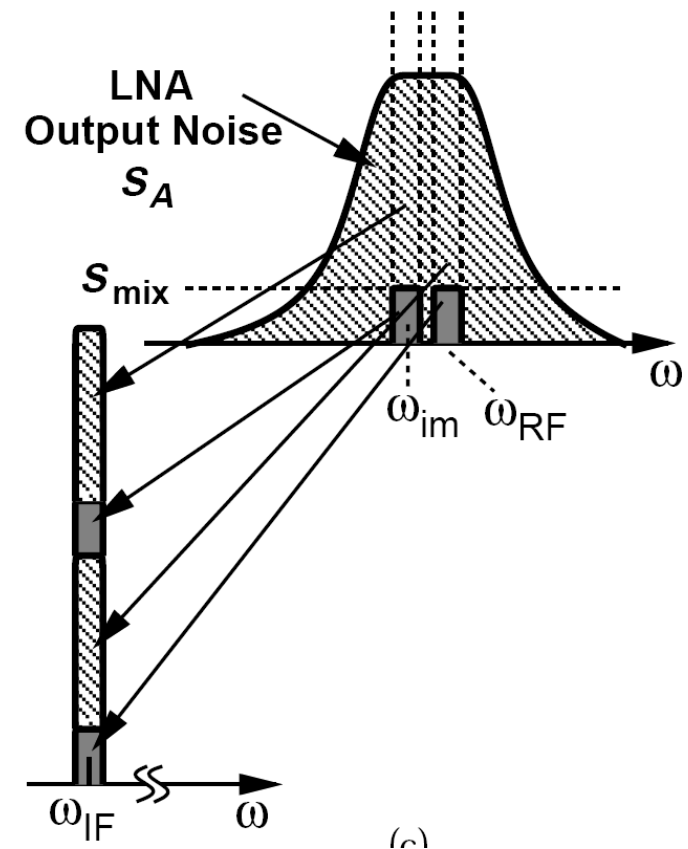
## Example 6.3

- In the first case, the selectivity of the antenna, the BPF, and the LNA suppresses the thermal noise in the image band. Of course, the RF mixer still folds its own noise. The overall behavior is illustrated below, where  $S_A$  denotes the noise spectrum at the output of the LNA and  $S_{\text{mix}}$  the noise in the input network of the mixer itself.
- Thus, the mixer downconverts three significant noise components to IF: the amplified noise of the antenna and the LNA around  $\omega_{\text{RF}}$ , its own noise around  $\omega_{\text{RF}}$ , and its image noise around  $\omega_{\text{im}}$ .



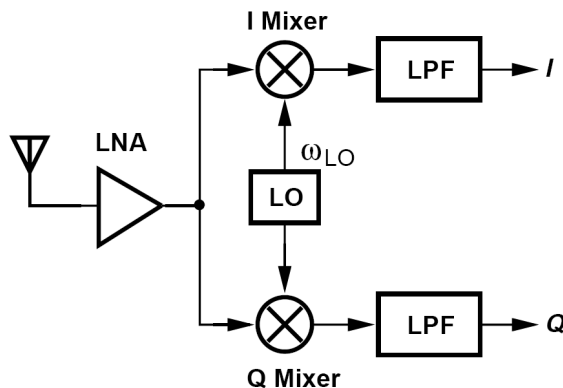
## Example 6.3

- In the second case, the noise produced by the antenna, the BPF, and the LNA exhibits a flat spectrum from the image frequency to the signal frequency. As shown on the right, the RF mixer now downconverts four significant noise components to IF: the output noise of the LNA around  $\omega_{RF}$  and  $\omega_{im}$ , and the input noise of the mixer around  $\omega_{RF}$  and  $\omega_{im}$ .
- We therefore conclude that the noise figure of the second frequency plan is substantially higher than that of the first. In fact, if the noise contributed by the mixer is much less than that contributed by the LNA, the noise figure penalty reaches 3 dB. Low-IF receivers do not suffer from this drawback because they employ image rejection.



# NF of Direct-Conversion Receivers

- It is difficult to define a noise figure for receivers that translate the signal to a zero IF.



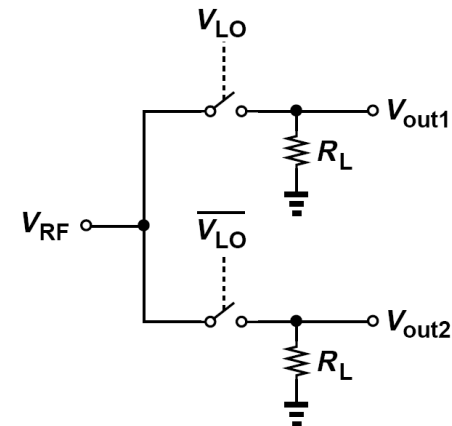
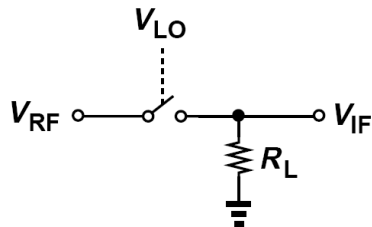
$$NF = \frac{SNR_{in}}{SNR_I} = \frac{SNR_{in}}{SNR_Q}$$

- This is the most common NF definition for direct-conversion receivers.
- The SNR in the final combined output would serve as a more accurate measure of the noise performance, but it depends on the modulation scheme.



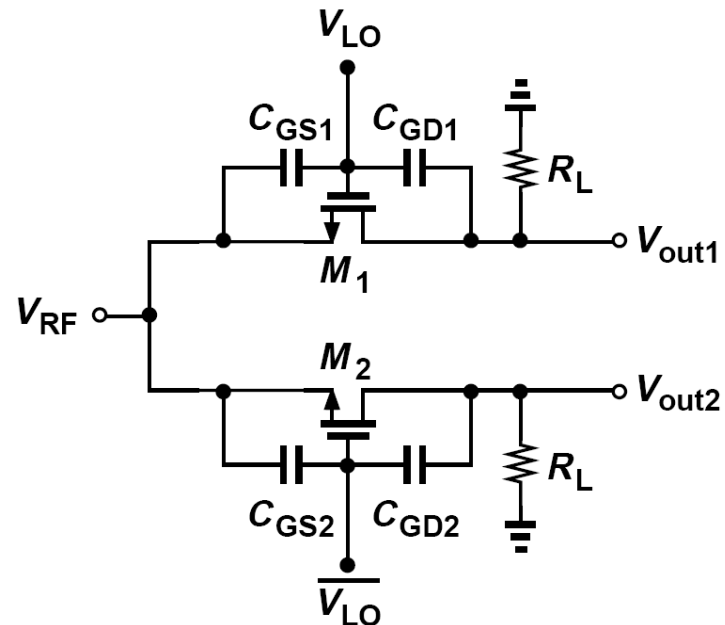
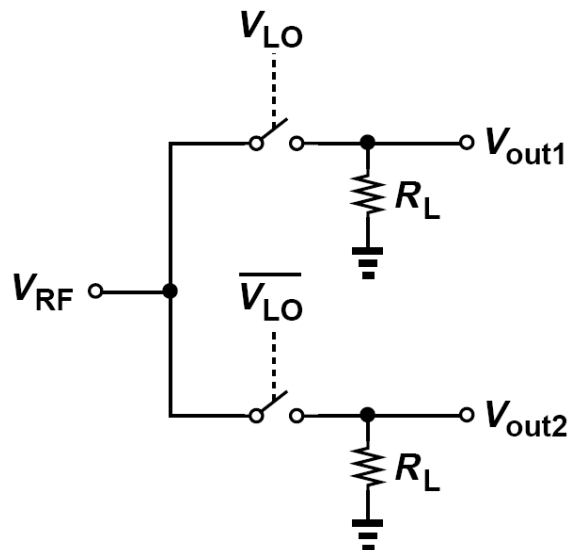
## 6.1.3 Single-Balanced and Double-Balanced Mixers

- The simple mixer previously discussed operate with a single-ended RF input and a single-ended LO, discarding the RF signal for half of the LO period.
- Figure right shows a more efficient approach whereby two switches are driven by differential LO phases, thus “commutating” the RF input to the two outputs. Called a “single-balanced” mixer.
- The conversion gain is doubled.



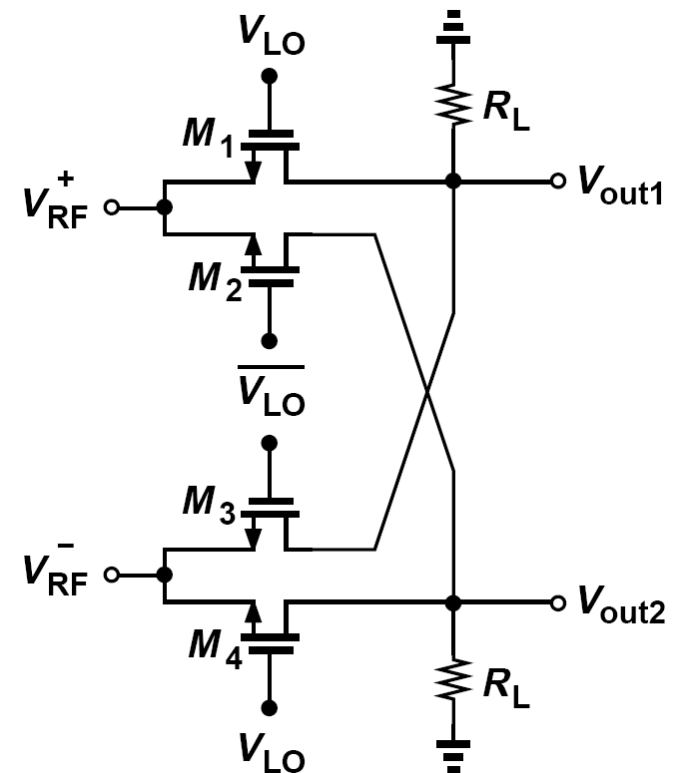
# Single-Balanced Mixers

- The LO-RF feedthrough at  $\omega_{LO}$  vanishes if the circuit is symmetric.
- Implementation with parasitics (feedthrough paths).



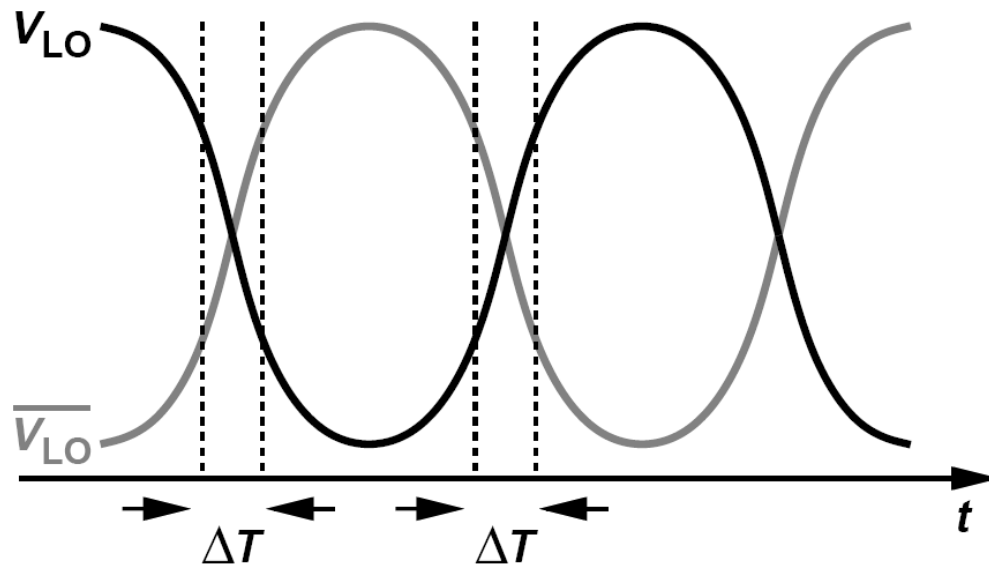
# Double-Balanced Mixers

- To reduce the LO-IF feedthrough of the single-balanced mixer, we can use the double-balanced mixer.
- We connect two single-balanced mixers such that their output LO feedthrough cancel but their output signals do not.
- If coupling of  $V_{LO}$  to  $V_{out}$  is  $aV_{LO}$  and from  $V_{LO}$  to  $V_{out}$  is  $-aV_{LO}$ , they will be cancelled in this configuration.



# Ideal LO Waveform

- The LO waveform must ideally be a square wave to ensure abrupt switching and hence maximum conversion gain. But at very high frequencies, the LO waveforms resemble sinusoids. This can raise the noise figure.

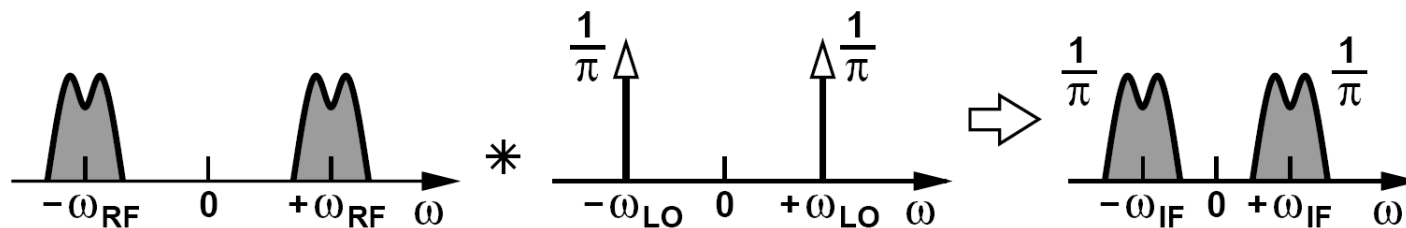
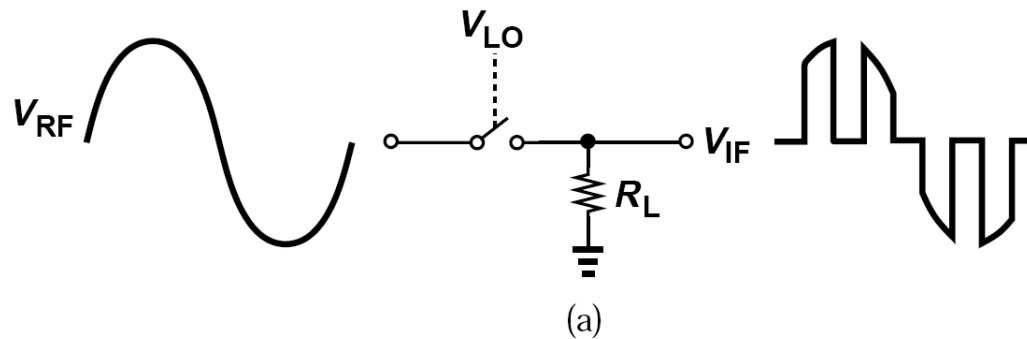


# Overview

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- Lee: Chapter 13.
  
- 6.1 Mixers general
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- 6.3 Active downconversion mixers

## 6.2 Passive Mixers

- Mixers can be “active” or “passive”. In passive mixers, the transistor does not operate as an amplifier. The conversion gain in the mixer below is equal to  $1/\pi$  ( $\approx -10$  dB) for abrupt LO switching.
- Called “return-to-zero” (RZ) mixer because the output falls to zero when the switch turns off.

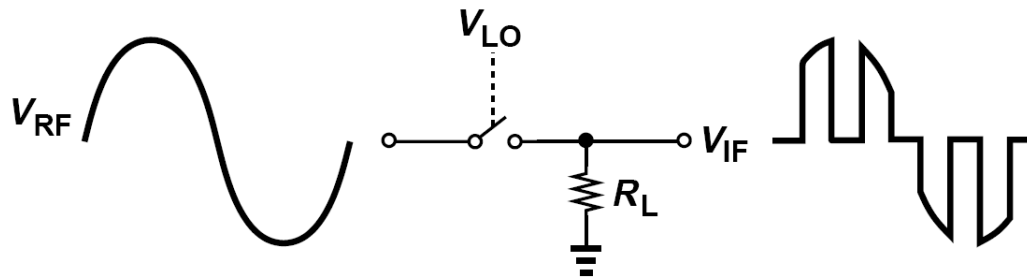


## Example 6.5

- Explain why that mixer is ill-suited to direct-conversion receivers.
- Since the square wave toggling between 0 and 1 carries an average of 0.5,  $V_{RF}$  itself also appears at the output with a conversion gain of 0.5. Thus, low-frequency beat components resulting from even-order distortion in the preceding stage directly go to the output, yielding a low  $IP_2$ .

## Example 6.6

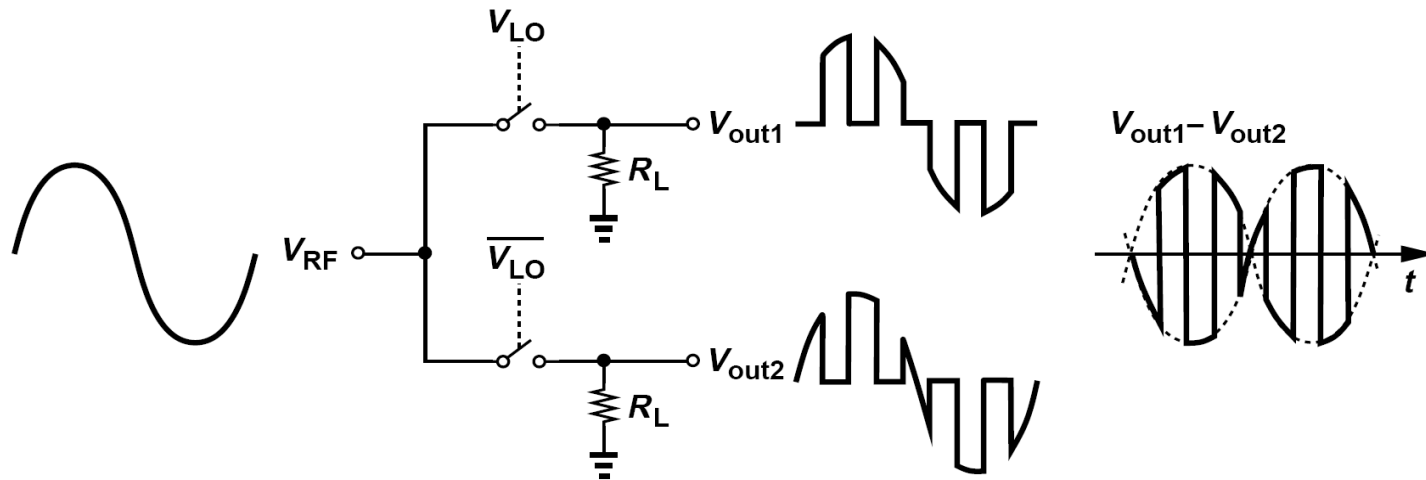
- Determine the conversion gain if this circuit is converted to a single-balanced topology.





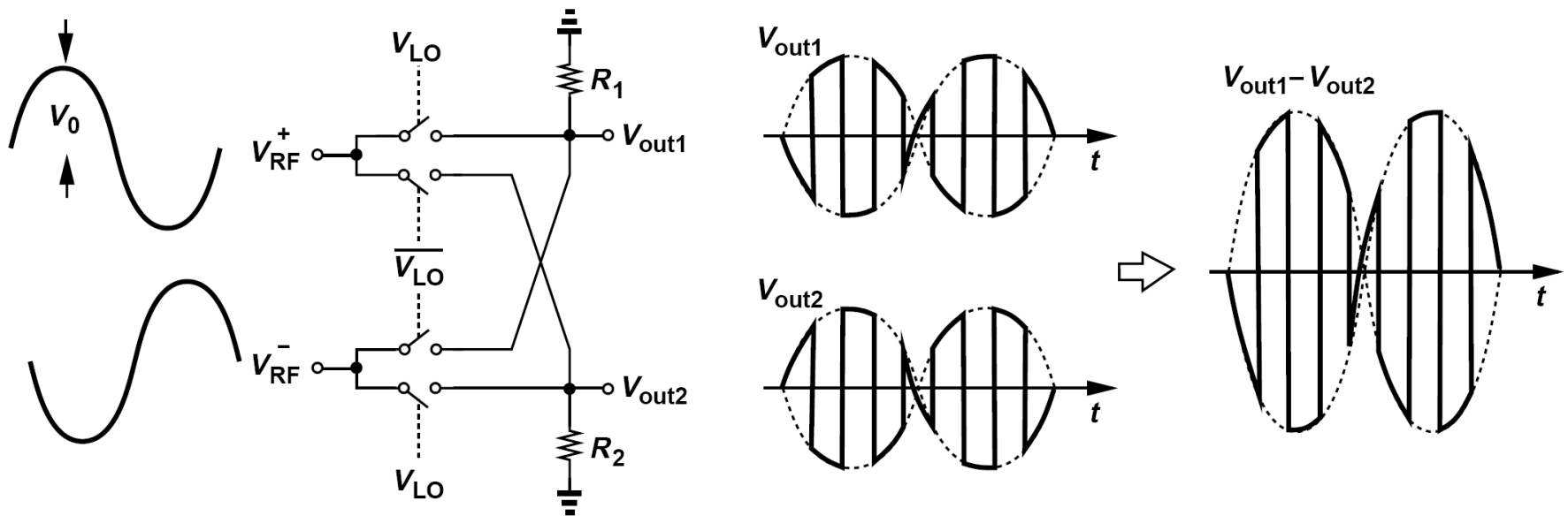
# Example 6.6

- The second output is similar to the first but shifted by  $180^\circ$ . The differential output contains twice the amplitude of each single-ended output. The conversion gain is therefore equal to  $2/\pi$  ( $\approx -4$  dB). Providing differential outputs and twice the gain, this circuit is superior to the single-ended topology above.

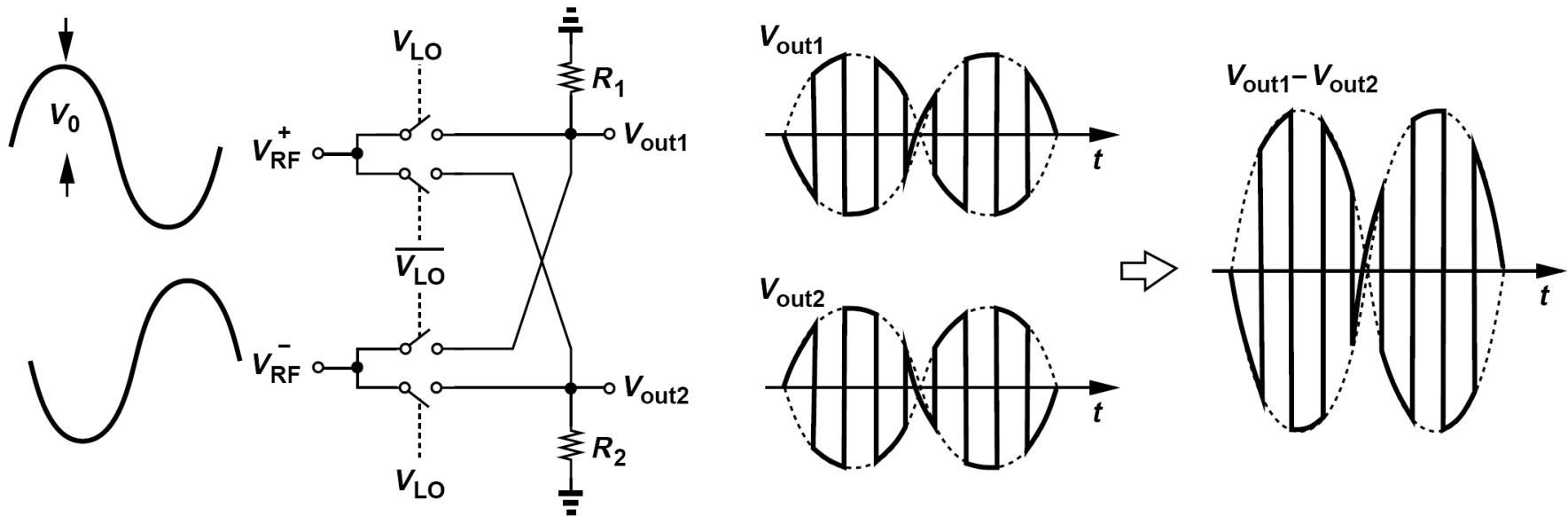


# Example 6.7

- Determine the voltage conversion gain of a double-balanced version of the above topology. (Decompose the differential output to return-to-zero waveforms.)



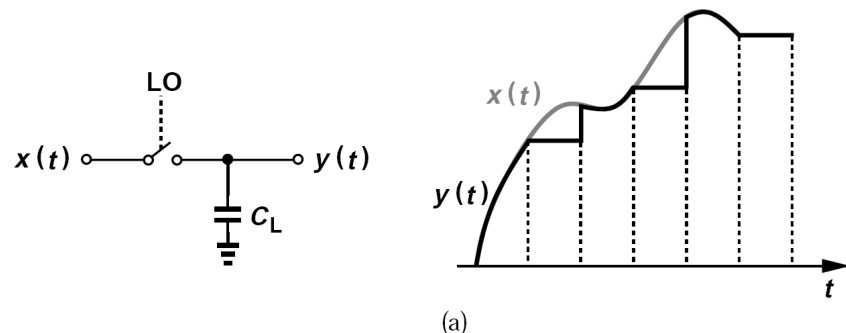
# Example 6.7



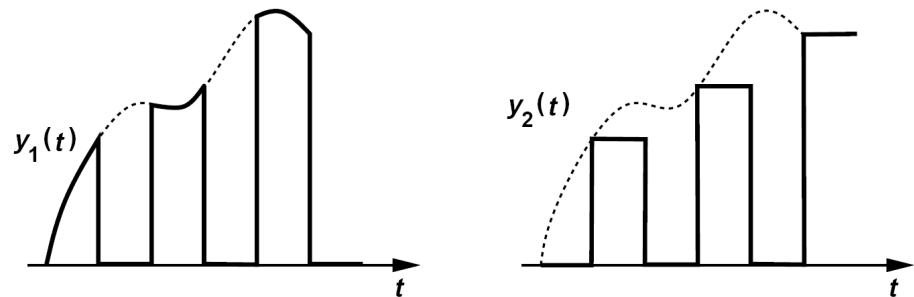
In this case,  $V_{out1}$  is equal to  $V_{RF}^+$  for one half of the LO cycle and equal to  $V_{RF}^-$  for the other half, i.e.,  $R_1$  and  $R_2$  can be omitted because the outputs do not “float.” From the waveforms shown in Fig. 6.20(b), we observe that  $V_{out1} - V_{out2}$  can be decomposed into two return-to-zero waveforms, each having a peak amplitude of  $2V_0$  (why?). Since each of these waveforms generates an IF amplitude of  $(1/\pi)2V_0$  and since the outputs are  $180^\circ$  out of phase, we conclude that  $V_{out1} - V_{out2}$  contains an IF amplitude of  $(1/\pi)(4V_0)$ . Noting that the peak differential input is equal to  $2V_0$ , we conclude that the circuit provides a voltage conversion gain of  $2/\pi$ , equal to that of the single-balanced counterpart.

# Sampling Mixer

- If the resistor  $R_L$  is replaced with a capacitor, such an arrangement operates as a sample-and-hold circuit and exhibits a higher gain because the output is held - rather than reset - when the switch turns off.



- The output waveform can be decomposed into two waveforms

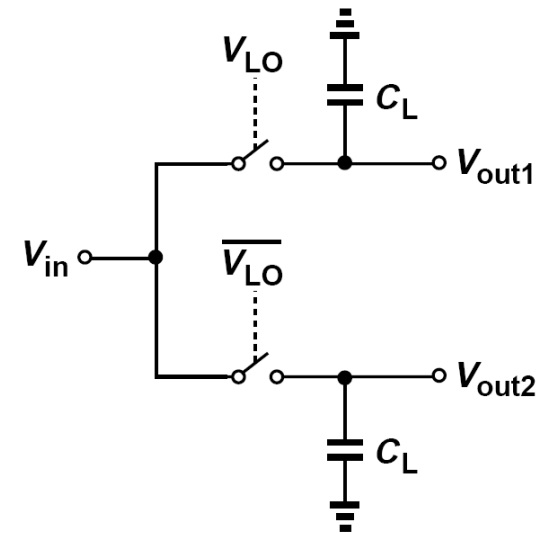


# Sampling Mixer

- We can prove (p. 358-361) that the total IF output is:

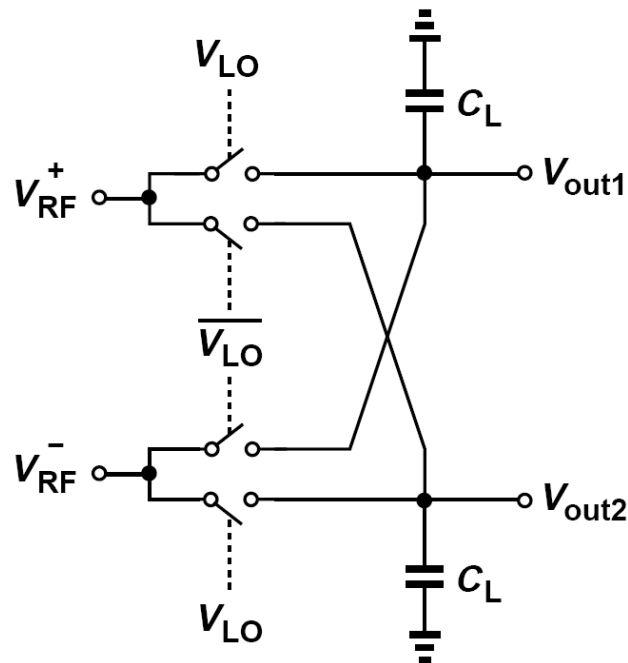
$$\begin{aligned}
 |Y_1(f) + Y_2(f)|_{IF} &= \sqrt{\frac{1}{\pi^2} + \frac{1}{4}} [|X(f - f_{LO})| + |X(f + f_{LO})|] \\
 &= 0.593 [|X(f - f_{LO})| + |X(f + f_{LO})|].
 \end{aligned}$$

- If realized as a single-balanced topology, the circuit provides a gain twice this value (1.19=1.48 dB)
- A passive topology with higher than unity gain!



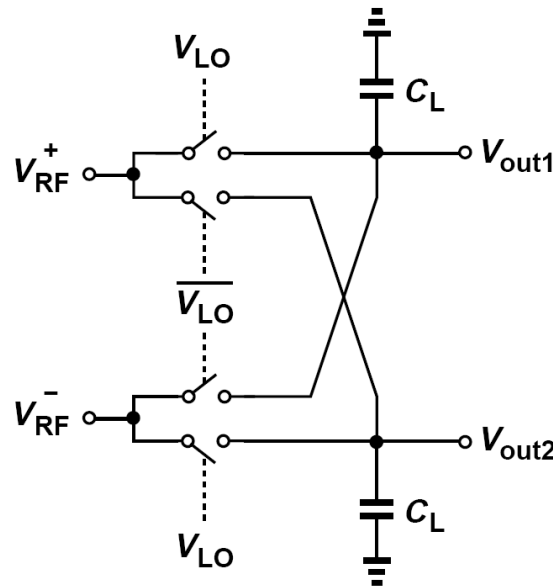
## Example 6.8

- Determine the voltage conversion gain of a double-balanced sampling mixer



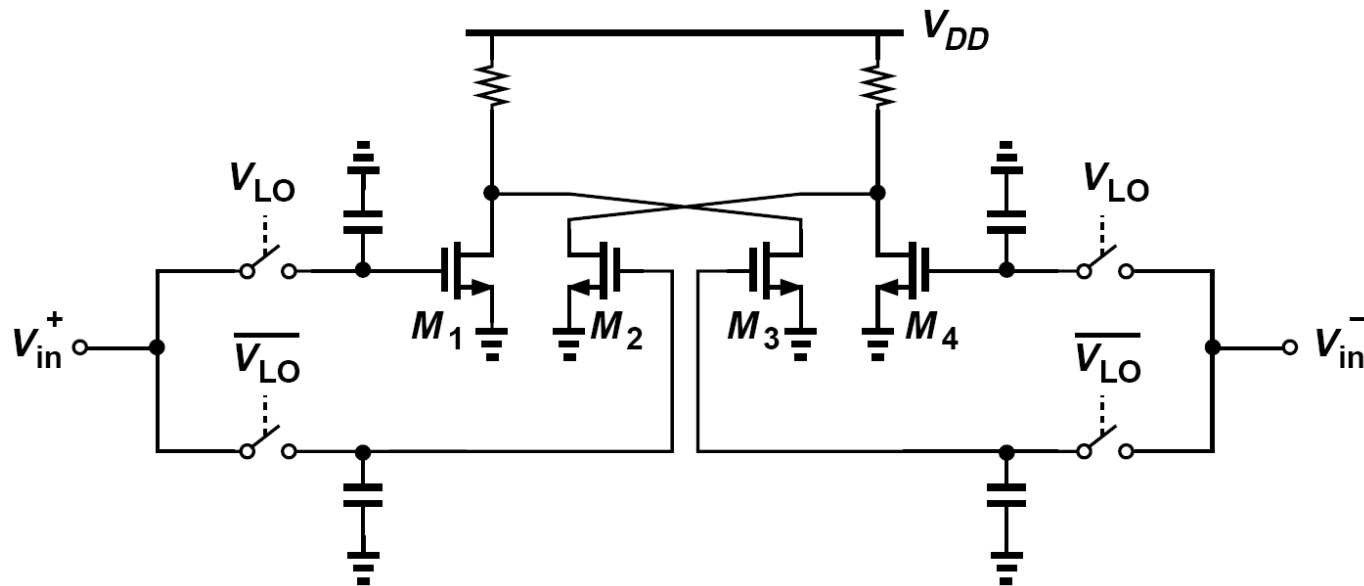
## Example 6.8

- The capacitors play no role here because each output is equal to one of the inputs at any given point in time. The conversion gain is therefore equal to  $2/\pi$ , about 5.5 dB lower than that of the single-balanced topology discussed above.



# Sampling Mixer

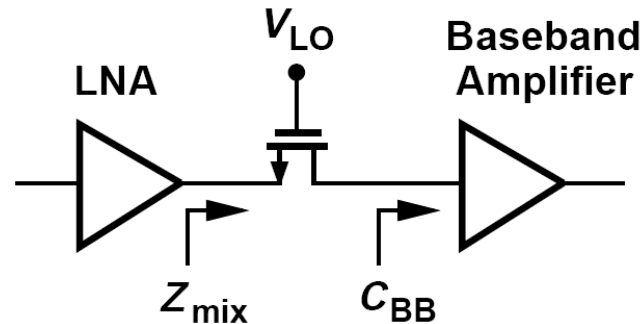
- Double-balanced operation can be realized through the use of two single-balanced mixers whose outputs are summed in the current domain.
- Mixer conversion gain is equal to 1.48 dB.





# Mixer Noise (summary)

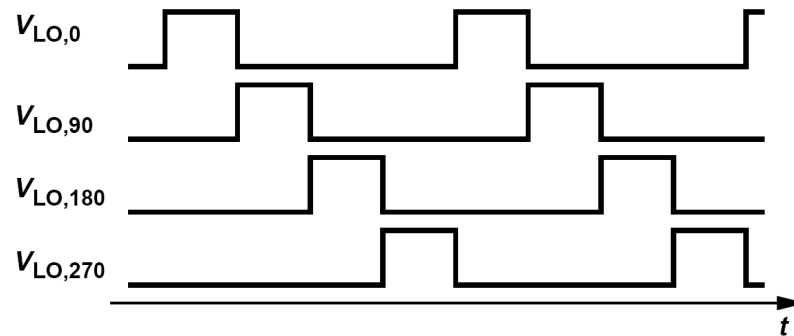
- An important advantage of passive mixers over their active counterparts is their much lower output flicker noise.



- MOSFETs produce little flicker noise if they carry a small current, a condition satisfied in a passive sampling mixer if the load capacitance is relatively small.
- However, the low gain of passive mixers makes the  $1/f$  noise contribution of the subsequent stage critical.
- Passive MOS mixers require large (rail-to-rail) LO swings, a disadvantage compared to active mixers.

# Duty cycle

- Passive mixers need not employ a 50 % LO duty cycle.
- In fact, passive mixers utilizing a 25 % duty cycle provide a higher gain.



- Voltage-driven the RF current entering each switch generates an IF current given by

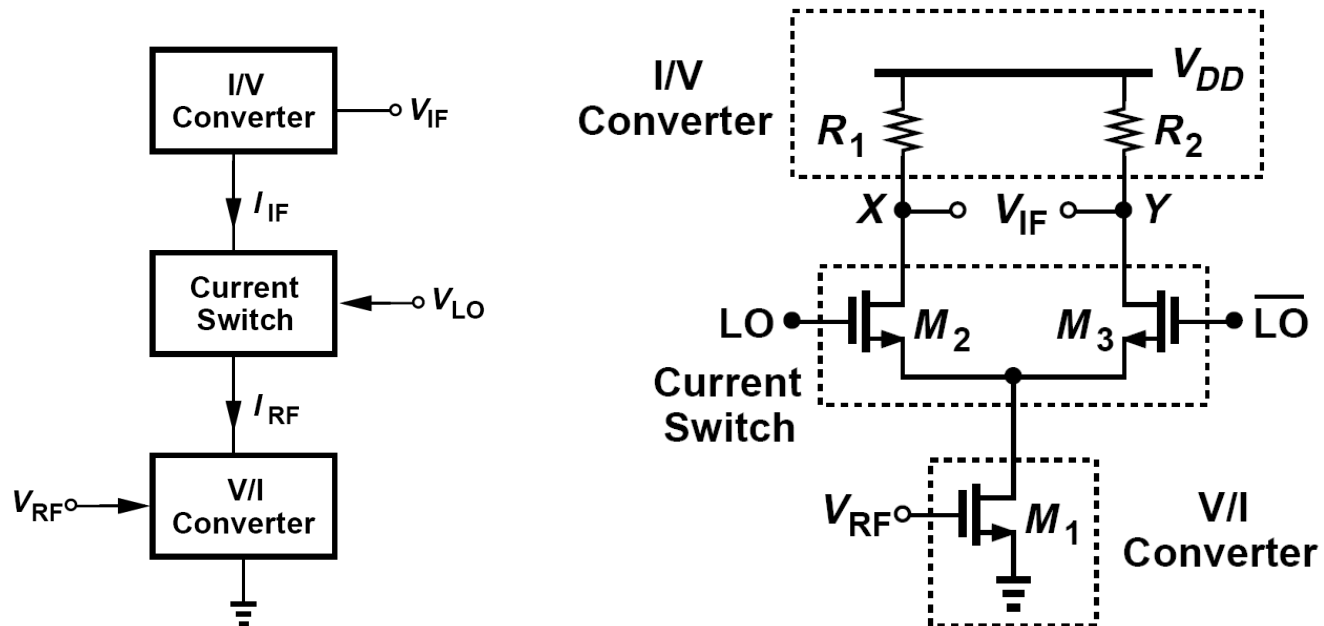
$$I_{IF}(t) = \frac{2}{\pi} \frac{\sin \pi d}{2d} I_{RF0} \cos \omega_{IF} t$$

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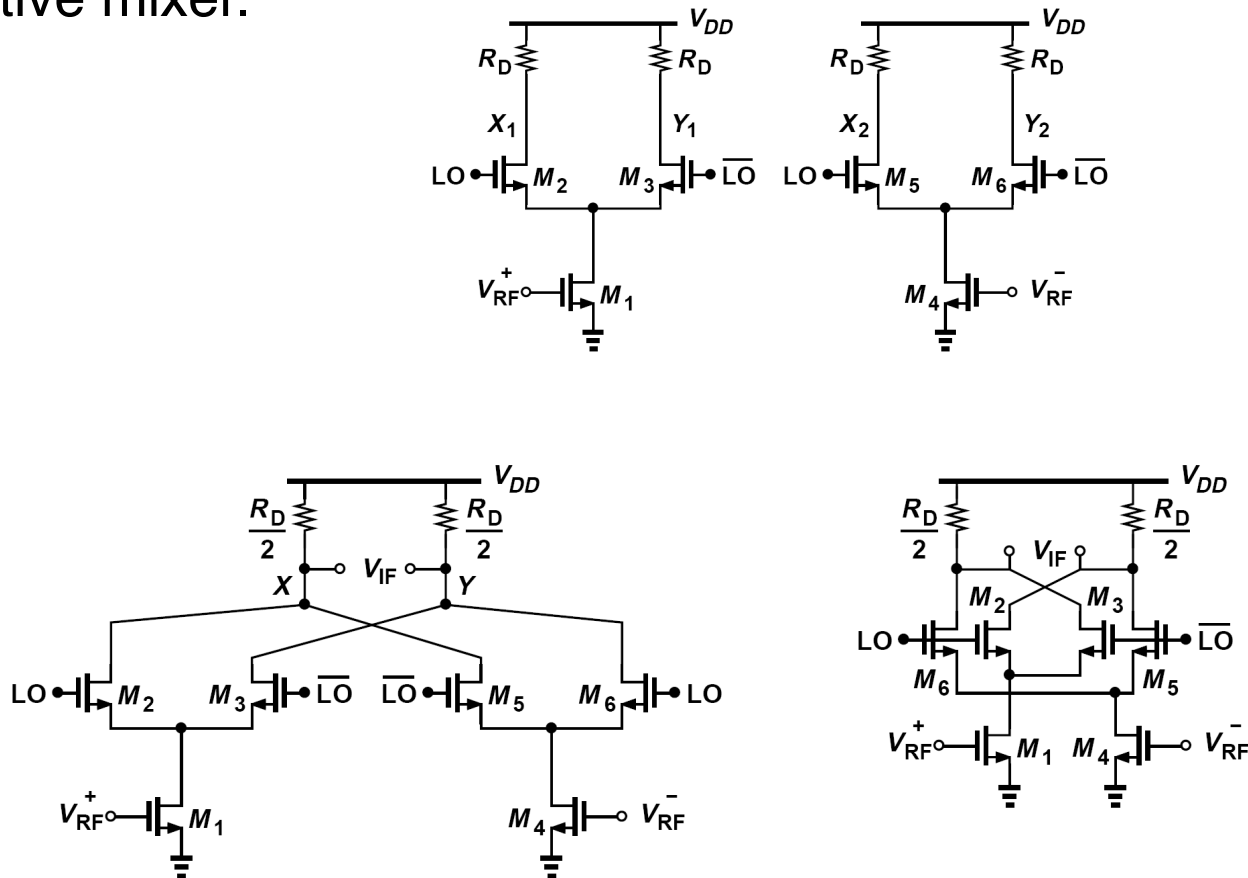
## 6.3 Active Mixers

- Active mixers perform three functions:
  - (1) convert the RF voltage to a current,
  - (2) “commutate” (steer) the RF current by the LO,
  - (3) convert the IF current to voltage.



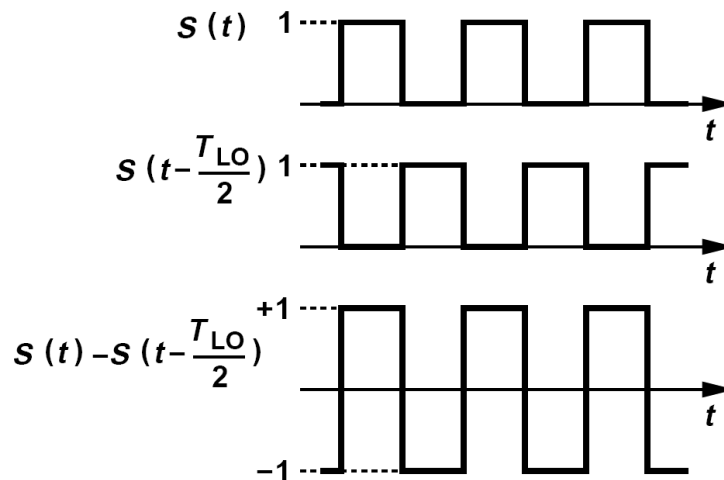
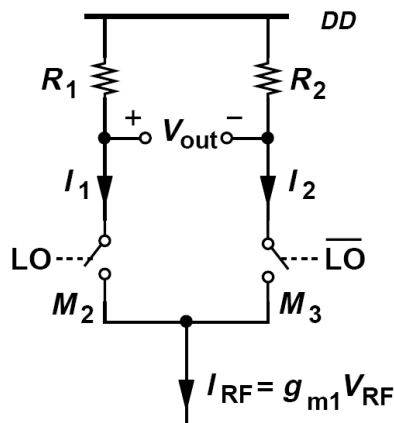
# Double-Balanced Active Mixers

- For differential inputs, we can create a double-balanced active mixer.



## 6.3.1 Conversion Gain

- With abrupt LO switching, the circuit reduces to that shown in figure below (left)



$$I_1 = I_{RF} \cdot S(t) \quad (6.55)$$

$$I_2 = I_{RF} \cdot S\left(t - \frac{T_{LO}}{2}\right) \quad (6.56)$$

# Conversion Gain

- We have for  $R_1 = R_2 = R_D$

$$V_{out}(t) = I_{RF} R_D \left[ S \left( t - \frac{T_{LO}}{2} \right) - S(t) \right] \quad (6.57)$$

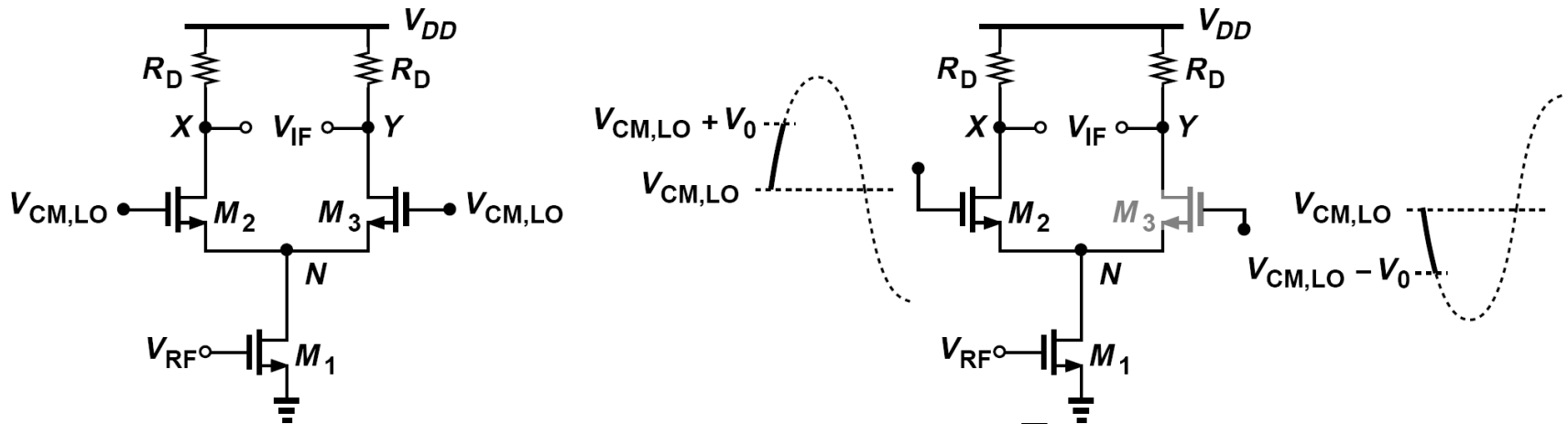
- Square-wave toggle (-1 to 1) with fundamental amplitude equal to  $4/\pi$

$$V_{out}(t) = I_{RF}(t) R_D \cdot \frac{4}{\pi} \cos \omega_{LO} t + \dots \quad (6.58)$$

If  $I_{RF}(t) = g_{m1} V_{RF} \cos \omega_{RF} t$ , then the IF component at  $\omega_{RF} - \omega_{LO}$  is equal to

$$V_{IF}(t) = \frac{2}{\pi} g_{m1} R_D V_{RF} \cos(\omega_{RF} - \omega_{LO})t \quad (6.59) \quad \Rightarrow \quad \frac{V_{IF,p}}{V_{RF,p}} = \frac{2}{\pi} g_{m1} R_D \quad (6.60)$$

# Active Mixer with LO at CM Level



$$V_{R,max} = V_{DD} - \left[ V_{GS1} - V_{TH1} + \left( 1 + \frac{\sqrt{2}}{2} \right) (V_{GS2,3} - V_{TH2}) \right] \quad (6.64)$$

$$R_{D,max} = \frac{2V_{R,max}}{I_{D1}} \quad A_{V,max} = \frac{2}{\pi} g_{m1} R_{D,max} = \frac{8}{\pi} \frac{V_{R,max}}{V_{GS1} - V_{TH1}} \quad (6.67)$$

- Low supply voltage limits the gain of active mixers



## Example 6.12

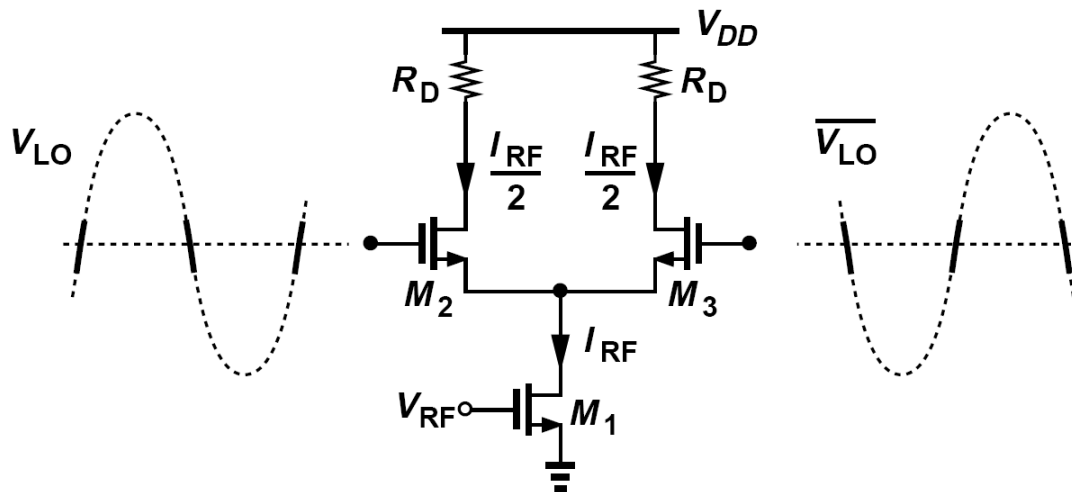
- A single-balanced active mixer requires an overdrive voltage of 300 mV for the input V/I converter transistor.  
If each switching transistor has an equilibrium overdrive of 150 mV and the peak LO swing is 300 mV, how much conversion gain can be obtained with a 1 V supply?
- Eq (6.64):  $V_{R,max} = 444$  mV and hence

$$\begin{aligned} A_{V,max} &= 3.77 \\ &\approx 11.5 \text{ dB} \end{aligned}$$

- Relatively low conversion gain => the noise contributed by the load resistors and following stages may become significant.

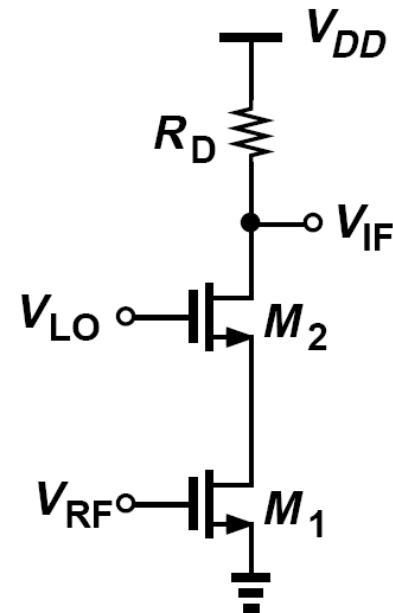
# Conversion Gain

- The conversion gain may also fall if the LO swing is lowered.
- While  $M_2$  and  $M_3$  are near equilibrium, the RF current produced by  $M_1$  is split approximately equally between them, thus appearing as a common-mode current and yielding little conversion gain for that period of time. Reduction of the LO swing tends to increase this time and lower the gain.



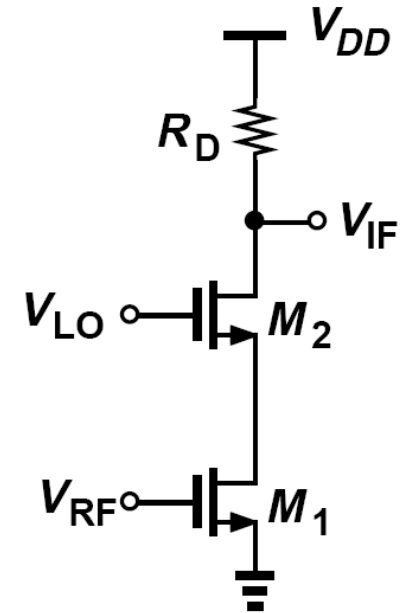
## Example 6.13

- The figure shows a “dual-gate mixer,” where  $M_1$  and  $M_2$  can be viewed as one transistor with two gates. Identify the drawbacks of this circuit.



## Example 6.13

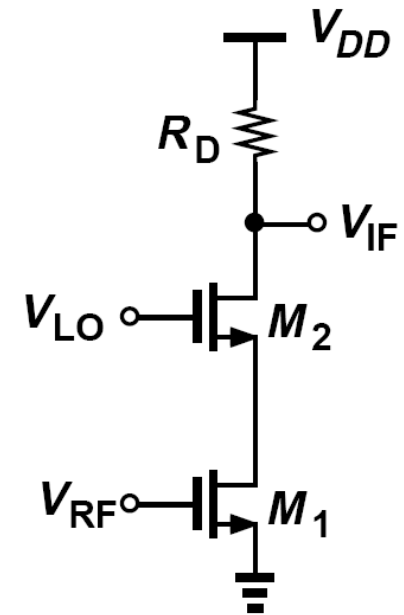
- For  $M_2$  to operate as a switch, its gate voltage must fall to  $V_{TH2}$  above zero regardless of the overdrive voltages of the two transistors.
- For this reason, the dual-gate mixer typically calls for larger LO swings than the single-balanced active topology does. Furthermore, since the RF current of  $M_1$  is now multiplied by a square wave toggling between 0 and 1, the conversion gain is half:



$$A_V = \frac{1}{\pi} g_{m1} R_D$$

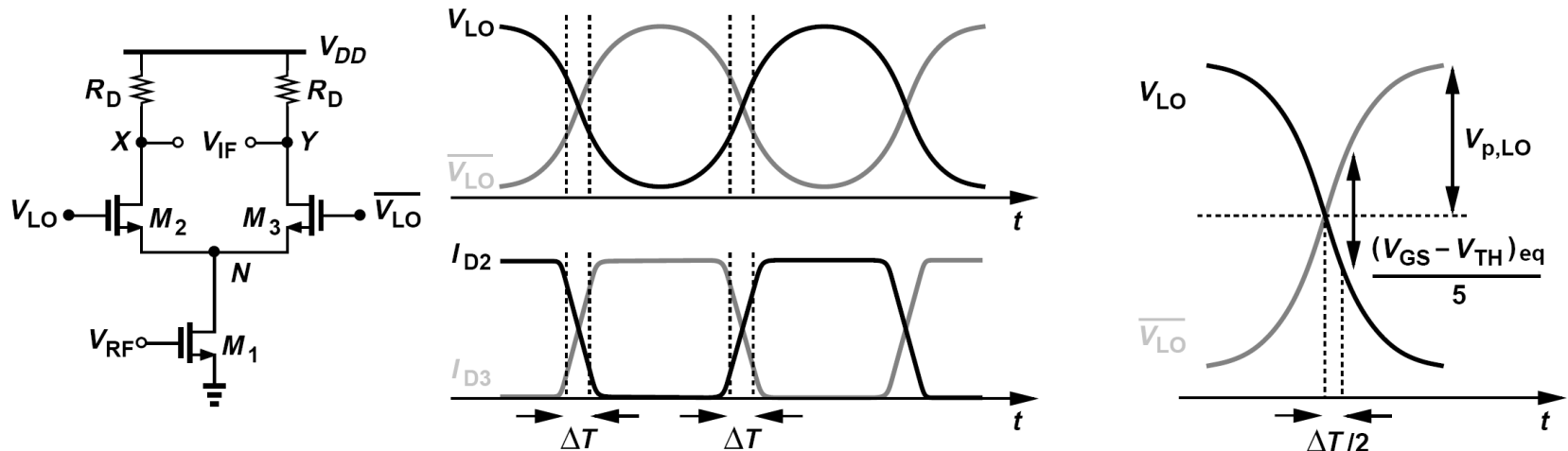
# Example 6.13

- Additionally, all of the frequency components produced by  $M_1$  appear at the output without translation because they are multiplied by the average value of the square wave =  $1/2$ .
- Thus, half of the flicker noise of  $M_1$  — a high-frequency device and hence small — emerges at IF.
- Also, low-frequency beat components resulting from even-order distortion in  $M_1$  directly corrupt the output, leading to a low  $IP_2$ . The dual-gate mixer does not require differential LO waveforms, a minor advantage. For these reasons, this topology is rarely used in modern RF design.



# Effect of Gradual LO Transitions

- With a sinusoidal LO, the drain currents remain approximately equal for a fraction of each half cycle,  $\Delta T$ . The circuit exhibits little conversion gain during these periods.



$$A_V = \frac{2}{\pi} g_{m1} R_D \left( 1 - \frac{2\Delta T}{T_{LO}} \right) = \frac{2}{\pi} g_{m1} R_D \left[ 1 - \frac{(V_{GS} - V_{TH})_{eq}}{5\pi V_{p,LO}} \right]$$

## Example 6.14

- Repeat Example 6.12 but take the gradual LO edges into account.

*A single-balanced active mixer requires an overdrive voltage of 300 mV for the input V/I converter transistor.*

*If each switching transistor has an equilibrium overdrive of 150 mV and the peak LO swing is 300 mV, how much conversion gain can be obtained with a 1 V supply?*

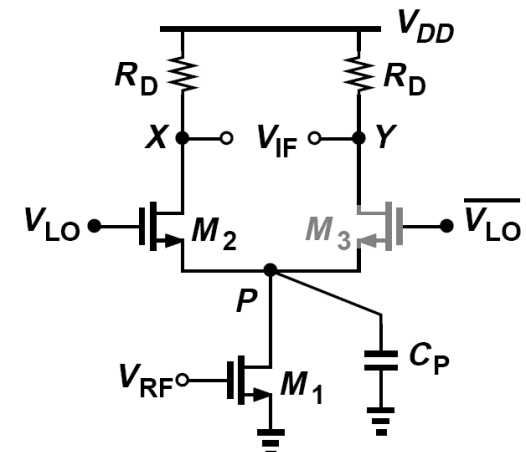
- The gain in the Example 6.12 must be multiplied by  $(1 - 0.0318) \sim 0.97 \Rightarrow$  gain is now 11.3 dB, 0.2 dB lower.
- The gradual LO transitions lower the gain by about 0.2 dB.

# Effect of capacitance

- With abrupt LO edges,  $M_2$  is on and  $M_3$  is off, yielding a total capacitance at node P equal to:

$$C_P = C_{DB1} + C_{GS2} + C_{GS3} + C_{SB2} + C_{SB3}.$$

- The RF current produced by  $M_1$  is split between  $C_P$  and the resistance seen at the source of  $M_2$ ,  $1/g_{m2}$ . The voltage conversion gain is modified as:

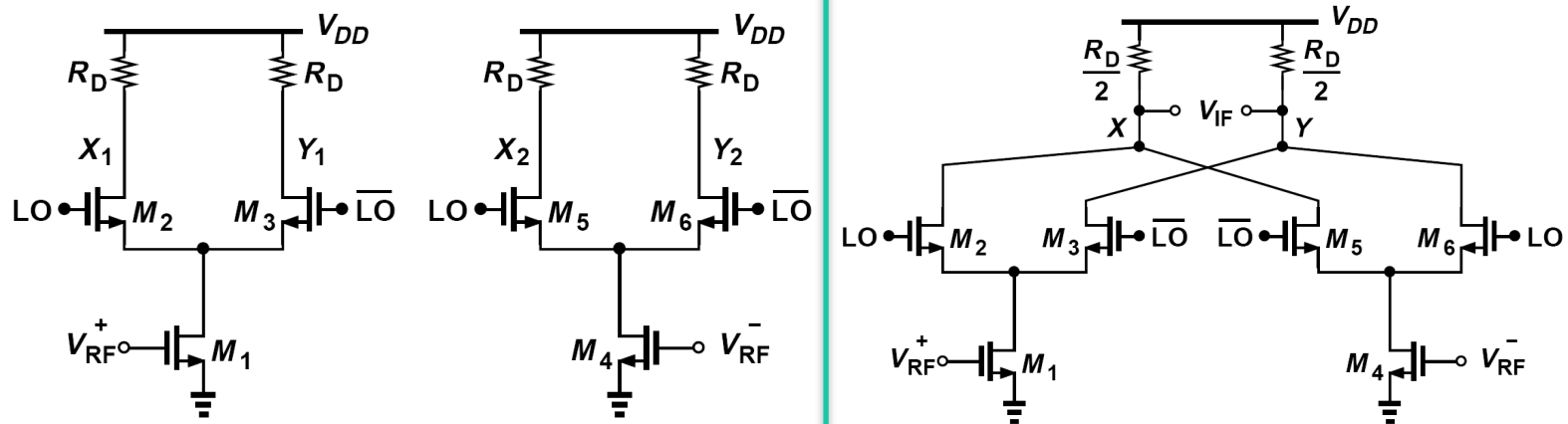


$$A_{V,max} = \frac{2}{\pi} g_{m1} R_D \left[ 1 - \frac{2(V_{GS} - V_{TH})_{eq}}{5\pi V_{P,LO}} \right] \frac{g_{m2}}{\sqrt{C_P^2 \omega^2 + g_{m2}^2}}.$$



# Example 6.16

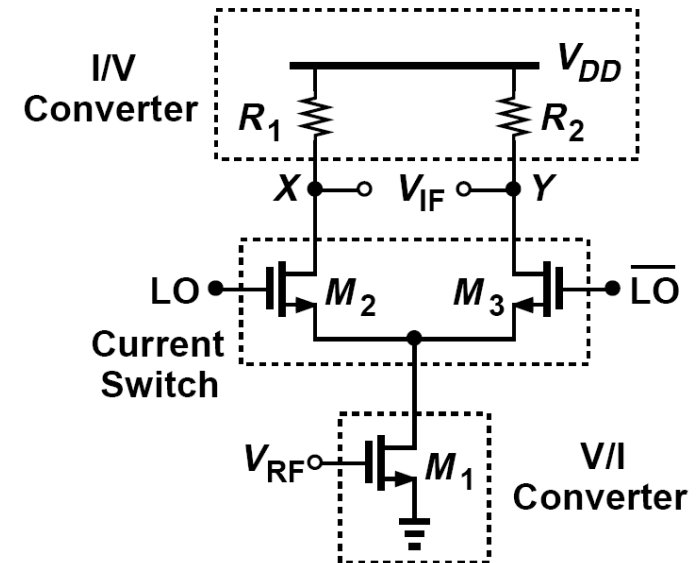
- Compare the voltage conversion gains of single-balanced and double-balanced active mixers.



# Example 6.16

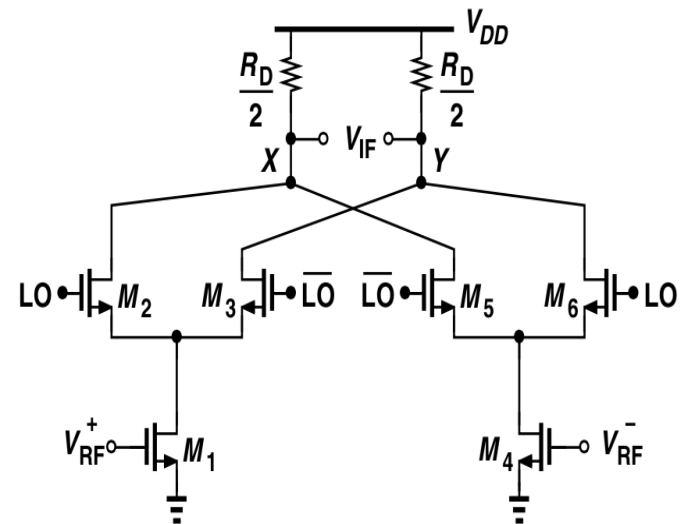
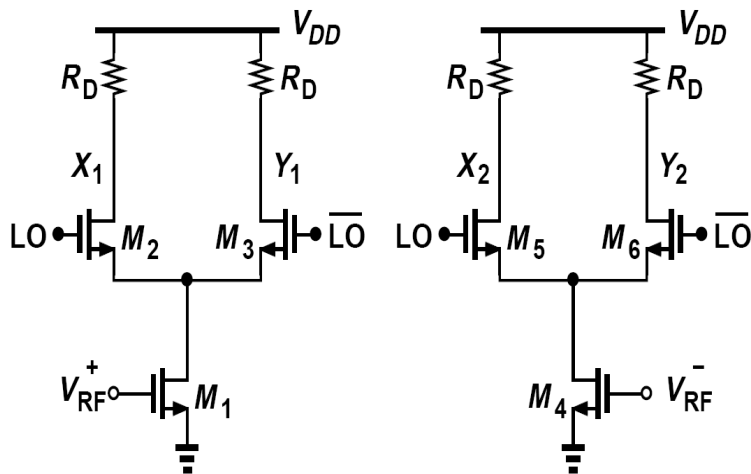
- From previous discussion, we know that  $(V_{X1} - V_{Y1})/V_{RF+}$  is equal to the voltage conversion gain of a single-balanced mixer.
- Also,  $V_{X1} = V_{Y2}$  and  $V_{Y1} = V_{X2}$  if  $V_{RF-} = -V_{RF+}$ . Thus, if Y2 is shorted to X1, and X2 to Y1, these node voltages remain unchanged. The differential voltage conversion gain of the double-balanced topology is therefore given by:

$$\frac{V_X - V_Y}{V_{RF+} - V_{RF-}} = \frac{V_{X1} - V_{Y1}}{2V_{RF+}}$$



# Example 6.16

- This gain is half of that of the single-balanced counterpart. This reduction arises because the limited voltage headroom disallows a load resistance of  $R_D$ .



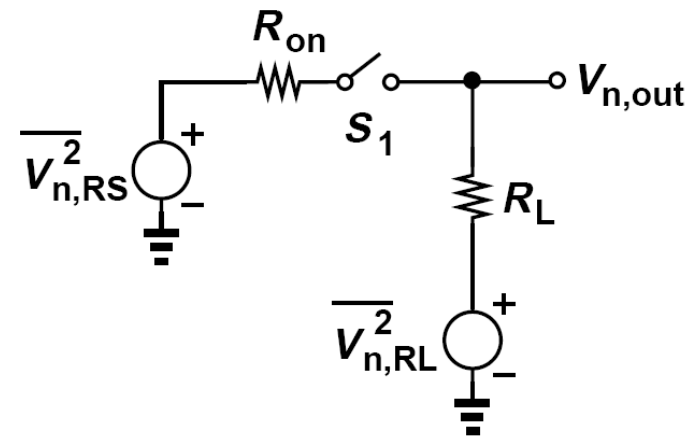
## 6.2.3 Noise in Passive Mixers

- Consider the simple mixer shown below.  
Assuming  $R_L \gg R_{on}$  and the LO has a 50% duty cycle, determine the output noise spectrum due to  $R_S$ , i.e., assume  $R_L$  is noiseless.
- The output noise is given by  $4kT(R_{on} || R_L)$  when  $S_1$  is on and by  $4kTR_L$  when it is off. On the average,

$$\overline{V_{n,out}^2} = 2kT[(R_{on} || R_L) + R_L]$$

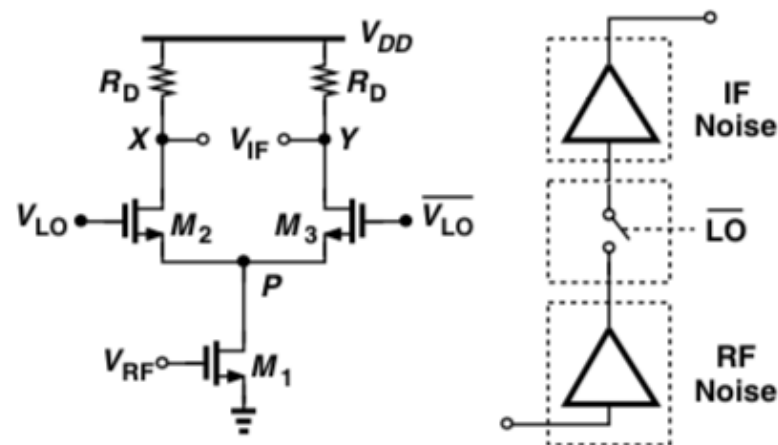
- If we select  $R_{on} \ll R_L$  to minimize the conversion loss and divide by  $1/\pi^2$ ,

$$\begin{aligned} \overline{V_{n,in}^2} &\approx 2\pi^2 kT R_L \\ &\approx 20kT R_L. \end{aligned}$$



## 6.3.2 Noise in Active Mixers

- The noise components of interest lie in the RF range before downconversion and in the IF range after downconversion.



- The frequency translation of RF noise by the switching devices prohibits the direct use of small-signal ac and noise analysis in circuit simulators, necessitating simulations in the time domain.
- Moreover, the noise contributed by the switching devices exhibits time-varying statistics

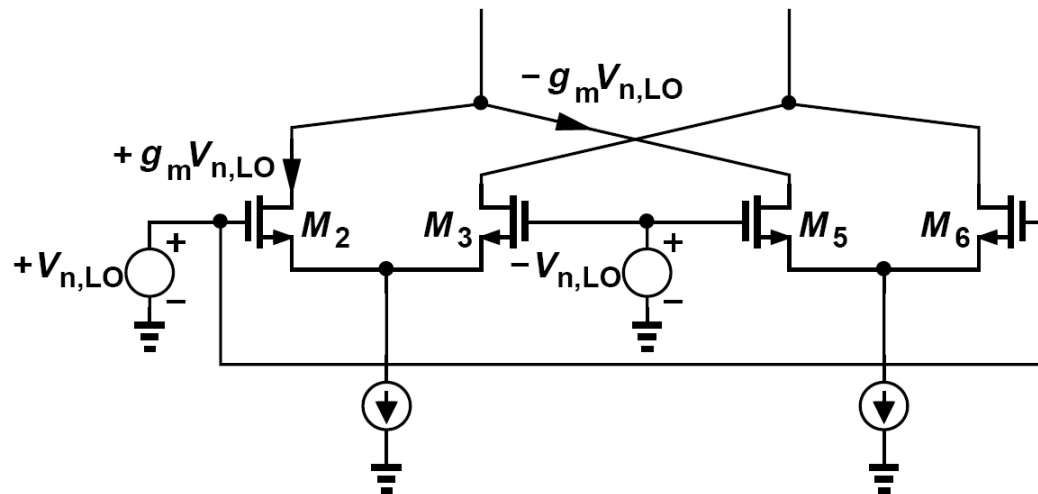
# Quantitative Analysis

- To estimate the input-referred noise voltage, we apply the following procedure:
  1. for each source of noise, determine a “conversion gain” to the IF output.
  2. multiply the magnitude of each noise by the corresponding gain and add up all of the resulting powers, thus obtaining the total noise at the IF output.
  3. divide the output noise by the overall conversion gain of the mixer to refer it to the input.



# Example 6.18

- Drawing the circuit as shown below, we note that the LO noise voltage is converted to current by each switching pair and summed with opposite polarities. Thus, the double-balanced topology is much more immune to LO noise—a useful property obtained at the cost of the 3-dB noise and the higher power dissipation.



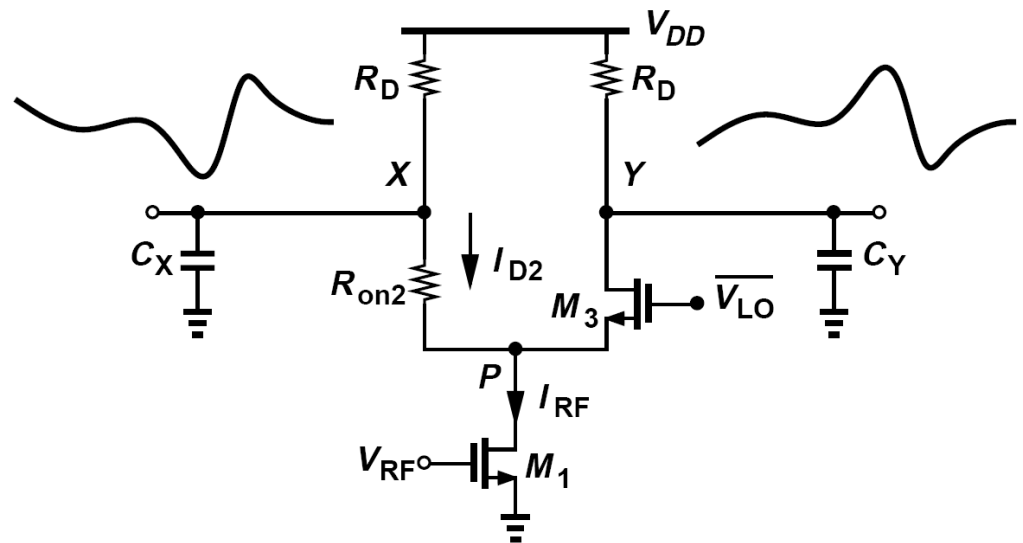


## 6.3.3 Linearity

- The input transistor (RF port) imposes a direct trade-off between nonlinearity and noise:

$$\frac{IP_3}{\overline{V_{n,in}^2}} \propto \frac{V_{GS} - V_{TH}}{\frac{4kT\gamma}{g_m}} = \frac{4kT\gamma}{2I_D}(V_{GS} - V_{TH})$$

The linearity of active mixers degrades if the switching transistors enter the triode region. Thus, the LO swings cannot be arbitrarily large.



## Example 6.22

- An active mixer exhibits a voltage conversion gain of 10 dB and an input 1-dB compression point of 355 mV<sub>pp</sub> (= -5 dBm). Is it possible that the switching devices contribute compression?
- At an input level of -5 dBm, the mixer gain drops to 9 dB, leading to an output differential swing of  $355 \text{ mV}_{pp} \times 2.82 \approx 1 \text{ V}_{pp}$ . Thus, each output node experiences a peak swing of 250 mV (node *X* falls 250 mV below its bias point). If the LO drive is large enough, the switching devices enter the triode region and compress the gain.

# Using Nonlinearity for Mixing

- A nonlinear system can be used as mixer. We can describe the input-output relationship of a nonlinear system by polynomial expansion as:

$$v_{out} = \sum_{n=0}^N c_n v_{in}^n$$

- Considering  $v_{in}$  as sum of the RF input and LO signal, the output of this system includes a DC term and harmonics of the inputs and IM products as  $p\omega_{RF} \pm q\omega_{LO}$
- Normally,  $p = q = 1$  is desired. To avoid higher-order nonlinearities, we can use a square-law mixer.

# Square-Law Mixer

- For a square-law mixer only  $c_1$  and  $c_2$  are non-zero ( $c_0$  is a DC term which can be removed by filtering).

- Input signal: the sum of RF and LO signals

$$v_{in} = v_{RF} \cos(\omega_{RF}t) + v_{LO} \cos(\omega_{LO}t)$$

- Output signal has three components:

$$v_{out} = v_{fund} + v_{sq} + v_{cross}$$

$$v_{fund} = c_1 [v_{RF} \cos(\omega_{RF}t) + v_{LO} \cos(\omega_{LO}t)]$$

$$v_{sq} = c_2 \{ [v_{RF} \cos(\omega_{RF}t)]^2 + [v_{LO} \cos(\omega_{LO}t)]^2 \}$$

$$v_{cross} = 2c_2 v_{RF} v_{LO} [\cos(\omega_{RF}t)] [\cos(\omega_{LO}t)]$$

# Square-Law Mixer

- The useful term is  $v_{cross}$  and can be rewritten as:

$$v_{cross} = c_2 v_{RF} v_{LO} [\cos(\omega_{RF} - \omega_{LO})t + \cos(\omega_{RF} + \omega_{LO})t]$$

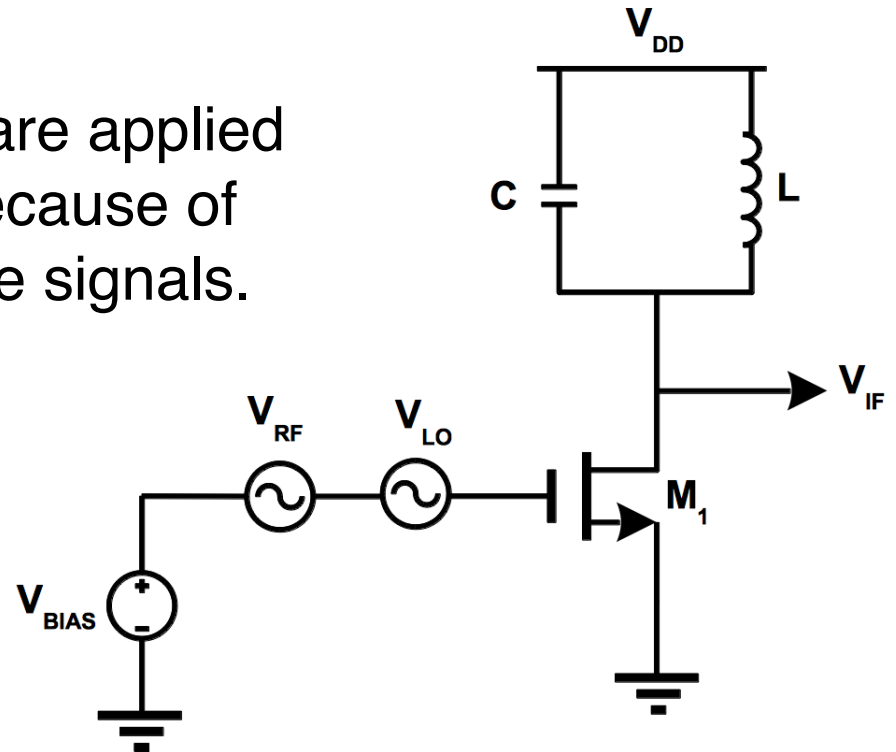
- If LO amplitude is fixed, IF output is proportional to the RF input amplitude.
- We have our mixer!
- Since the current in long-channel MOSFET devices has a quadratic form, they are good candidates for square-law mixers.

$$i_D = \frac{\mu C_{ox} W}{2L} (v_{gs} - V_T)^2$$

# Square-Law Mixer

- Since the current in long-channel MOSFET devices has a quadratic form, they are good candidates for square-law mixers.
- Sum of RF and LO signals are applied to the gate. It is not good because of poor isolation between these signals.
- Conversion gain:

$$G_c = \frac{c_2 v_{RF} v_{LO}}{v_{RF}} = c_2 v_{LO}$$



# Square-Law Mixer

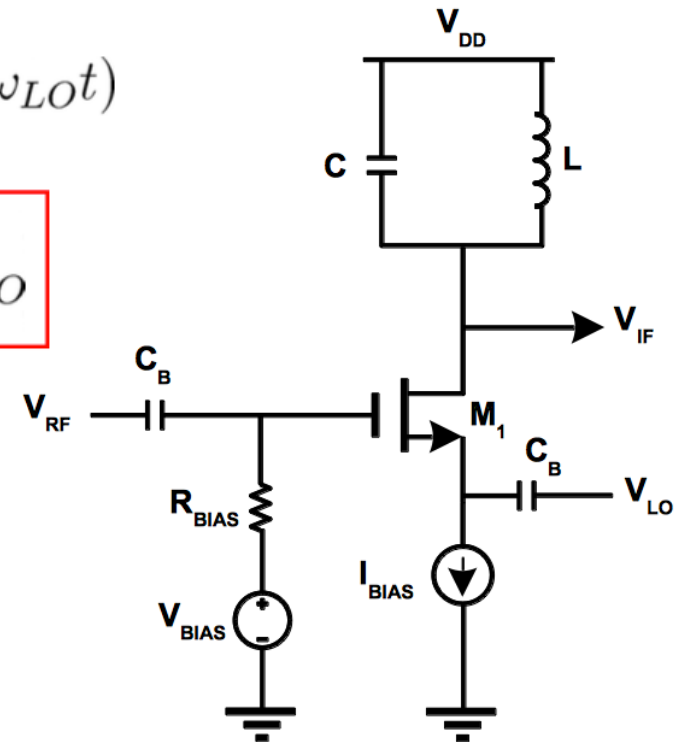
- To reduce the interaction between RF and LO ports, we can apply LO signal to the source

$$v_{gs} = V_{BIAS} + v_{RF} \cos(\omega_{RF}t) + v_{LO} \cos(\omega_{LO}t)$$

- Conversion gain:

$$G_c = \frac{\mu C_{ox} W}{2L} v_{LO}$$

- Independent of bias, but LO will vary with amplitude, temperature.



# Summary

- Mixers are used for frequency conversion.
- Popular radio architectures that need mixing are heterodyne and direct-conversion TRX.
- Any non-linear device can be used for mixing.
- Commonly, mixers are of multiplier type and have three ports: RF, IF, and LO.
- Mixers can be passive or active.
- Square-law mixers use the non-linearity behavior of (often) a single device, a transistor or diode.



# When to use an active or passive mixer?

- According to Li (\*), passive mixers are useful at microwave frequencies, active at lower frequencies.

**TABLE 2.1 Comparison between active and passive mixer**

Item	Active mixer	Passive mixer	Unit
Current drain	~2 to 5	~0	<i>mA</i>
<i>LO</i> Injection	~-10 to 0	~-5 to 10	<i>dB<sub>m</sub></i>
Conversion gain	~-5 to 10	~-5 to -3	<i>dB</i>
Noise figure	~10 to 15	~3 to 5	<i>dB</i>
Bandwidth	Narrower	Wider	
Part count	~13	~6	
Reliability	Lower	Higher	
Cost	Lower	Higher	

(\*) R. C.-H. Li, "RF Circuit Design", p. 78, Wiley 2009.

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