

TSTE05 Elektronik & mätteknik

Föreläsning 5

Växelströmsteori

– $j\omega$ -metoden – effektbegrepp

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Ämnesområdet Elektroniska kretsar och system

$j\omega$ -metoden

1. Ersätt strömmar, spänningar och källor med deras komplexa motsvarigheter:

$$a(t) = \hat{A} \sin(\omega t + \varphi) \Rightarrow$$

$$A = \hat{A} e^{j\varphi} = b + jc$$

$$b = \hat{A} \cos \varphi \quad c = \hat{A} \sin \varphi$$

2. Ersätt R , L , C med deras impedanser:

$$Z_L = j\omega L \quad Z_C = \frac{1}{j\omega C} \quad Z_R = R$$

3. Lös problemet med likströmsteori.

4. Gör omvändningen till punkt 1:

$$A = \hat{A} e^{j\varphi} = b + jc \Rightarrow$$

$$a(t) = \hat{A} \sin(\omega t + \varphi)$$

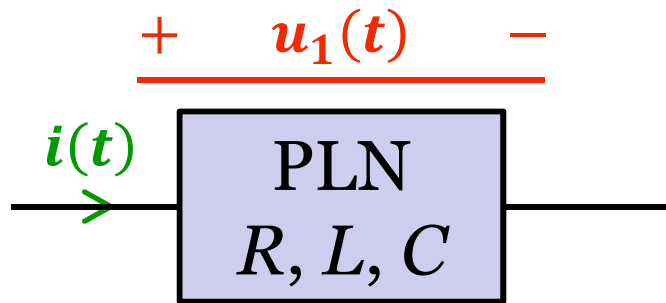
$$\hat{A} = \sqrt{b^2 + c^2}$$

$$\varphi = \arg(b + jc) = \operatorname{atan} \frac{c}{b} \quad (\pm\pi)$$

Om $b < 0$

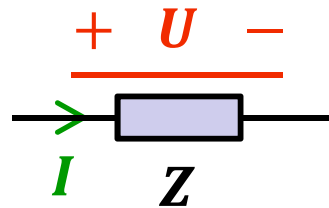
Effektbegrepp 1(4)

Passivt Linjärt Nät – PLN



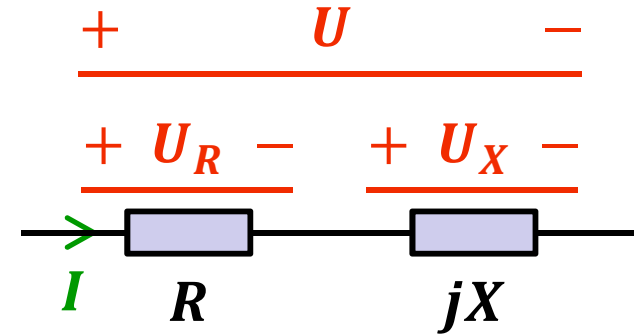
$$i(t) = \hat{I} \sin(\omega t)$$

$$u(t) = \hat{U} \sin(\omega t + \varphi)$$



$$I = \hat{I}$$

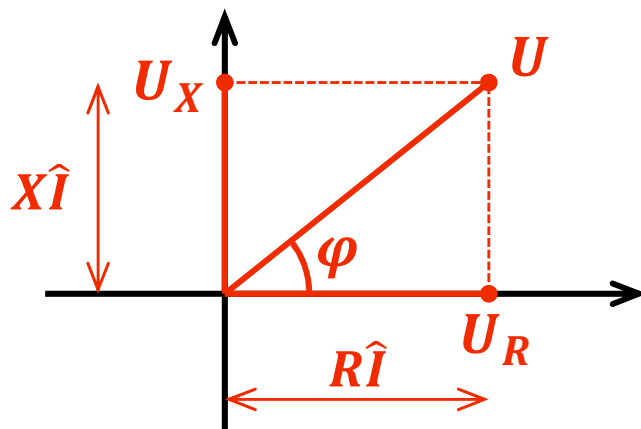
$$U = \hat{U} e^{j\varphi}$$



$$U_R = RI$$

$$U_X = jXI$$

$$U = U_R + U_X = (R + jX)I = ZI$$



Toppvärden

$$\left. \begin{aligned} R\hat{I} &= \hat{U} \cos(\varphi) \\ X\hat{I} &= \hat{U} \sin(\varphi) \end{aligned} \right\} *$$

Effektivvärden

$$\left. \begin{aligned} U_e &= \hat{U} / \sqrt{2} \\ I_e &= \hat{I} / \sqrt{2} \end{aligned} \right\} **$$

$$u(t) = R\hat{I} \sin(\omega t) + X\hat{I} \sin(\omega t + \pi/2)$$

Effektbegrepp 2(4)

$$\left. \begin{aligned} R\hat{I} &= \hat{U} \cos(\varphi) \\ X\hat{I} &= \hat{U} \sin(\varphi) \end{aligned} \right\} * \quad \left. \begin{aligned} U_e &= \hat{U}/\sqrt{2} \\ I_e &= \hat{I}/\sqrt{2} \end{aligned} \right\} **$$
$$i(t) = \hat{I} \sin(\omega t)$$
$$u(t) = R\hat{I} \sin(\omega t) + X\hat{I} \sin(\omega t + \pi/2)$$

Momentan effekt:

$$\begin{aligned} p(t) = u(t)i(t) &= R\hat{I}^2 \sin^2(\omega t) + X\hat{I}^2 \sin(\omega t) \sin(\omega t + \pi/2) \\ &= \underbrace{\frac{R\hat{I}^2}{2} (1 - \cos(2\omega t))}_{p_R(t)} + \underbrace{\frac{X\hat{I}^2}{2} \sin(2\omega t)}_{p_X(t)} \end{aligned}$$

Utnyttja *, ** och $\omega = 2\pi/T$:

$$p_R(t) = U_e I_e \cos(\varphi) (1 - \cos(2\omega t)) = U_e I_e \cos(\varphi) (1 - \cos(4\pi t/T))$$

$$p_X(t) = U_e I_e \sin(\varphi) \sin(2\omega t) = U_e I_e \sin(\varphi) \sin(4\pi t/T)$$

Effektbegrepp 3(4)

$$\left. \begin{array}{l} R\hat{I} = \hat{U} \cos(\varphi) \\ X\hat{I} = \hat{U} \sin(\varphi) \end{array} \right\} * \quad \left. \begin{array}{l} U_e = \hat{U}/\sqrt{2} \\ I_e = \hat{I}/\sqrt{2} \end{array} \right\} **$$

$$p_R(t) = U_e I_e \cos(\varphi) (1 - \cos(4\pi t/T))$$

$$p_X(t) = U_e I_e \sin(\varphi) \sin(4\pi t/T)$$

Aktiv effekt:

$$\begin{aligned} P &= \frac{1}{T} \int_0^T p(t) dt = \frac{1}{T} \int_0^T p_R(t) dt + \frac{1}{T} \int_0^T p_X(t) dt \\ &= \frac{1}{T} (TU_e I_e \cos(\varphi) + 0) + \frac{1}{T} 0 = U_e I_e \cos(\varphi) = \underset{\substack{\uparrow \\ *, **}}{R} I_e^2 \quad \text{Enhet: W} \end{aligned}$$

Reaktiv effekt: $*, **$

$$Q = U_e I_e \sin(\varphi) = \underset{\substack{\downarrow \\ *, **}}{X} I_e^2$$

Enhet: VAr

Effektbegrepp 4(4)

$$P = RI_e^2$$

$$Z = R + jX$$

$$Q = XI_e^2$$

$$\varphi = \arg(Z)$$

Komplex effekt:

$$S = P + jQ = (R + jX)I_e^2 = ZI_e^2 = \frac{UI^*}{2} = \frac{U_e^2}{Z^*}$$

Enhet: VA

Skenbar effekt:

$$P_S = U_e I_e = |S| = \sqrt{P^2 + Q^2} = |Z|I_e^2 = \frac{U_e^2}{|Z|}$$

Enhet: VA

Effektfaktor:

$$\cos(\varphi) = \frac{P}{P_S} = \frac{R}{|Z|}$$

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