Department of Science and Technology
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Exam for TNE041, **Modern Physics**, 8 June 2023, time 14.00 – 18.00.

Allowed examination material: Physics handbook (Studentlitteratur)
calculator (with no wifi)
additional formulae (attached)
one hand-written sheet (A4, not copied, with notes on one side)

Define all quantities you use and give a clear answer, including unit if a numerical value is given. No points are given if only the answer is submitted, with the exception of true/false questions. The maximum score is 24 points (6x4). The limits for different grades given below is with bonus included. The solutions may be given in English or in Swedish.

The following limits for grades apply:

Grade 3	$\geq$ 10 points
Grade 4	$\geq$ 15 points
Grade 5	$\geq$ 19 points

Questions are answered by Michael Hörnquist who will visit the exam room around 3 pm and 4.30 pm. Answers and short solutions will be available at Studieinfo at 8 pm at the latest. Results will be reported not later than 15 working days after the exam.

Good luck!

1. Are the following statements true or false?

   (a) i. In special relativity, energy is *not* a conserved quantity.

      ii. If electrons had spin $s = 1$ (instead of $s = 1/2$), the maximum number of electrons in the $1s$ subshell would increase from 2 to 3.

      iii. The position of the Fermi energy with respect to the conduction band and the valence band is critical for a material being a conductor or an insulator.

      iv. The main reason why one is doping semiconductors is to increase the resistance for charge carriers by inserting more obstacles.

      Only the answers (true/false) are required. (2p)

   (b) What potential difference is nessary to accelerate an electron from $0,6c$ to $0,9c$? Here c denotes the speed of light in vacuum. (2p)

2. Determine the wavelength of a photon that can impart at most 85 keV of kinetic energy to a free electron.
   Hint: Compton effect. (4p)

3. Start from the *time-dependent* Schrödinger equation,

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + U(x)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}.$$

   (a) Derive the *time-independent* Schrödinger equation and its corresponding temporal part. (2p)

   (b) Solve the temporal part for a harmonic oscillator $U(x) = \frac{1}{2}kx^2$. (2p)

4. A QM-particle is bound by a potential V in one dimension. The potential has the form

$$V(x) = \begin{cases} \infty & \text{if } x < -a/2 \text{ or } x > a/2 \\ 0 & \text{if } -a/2 \leq x \leq a/2 \end{cases}$$

   where a is a length on the nanoscale, $a > 0$

   Determine the allowed energies and the corresponding normalized wave functions for this potential. (4p)

5. A hydrogen atom is in the $(1, 0, 0, -\frac{1}{2})$ state. Calculate the probability of finding the electron in the region between the nucleus and a distance of a_0 from the nucleus. Here $a_0 = 4\pi\varepsilon_0\hbar^2/(me^2) = 0,0529$ nm is the Bohr radius. Necessary wave functions can be taken directly from Physics handbook, it is not necessary to derive them from the Schrödinger equation. (4p)

6. Determine the average energy (that is, the energy per particle) for a three-dimensional electron gas at low temperature ($T << E_F/k_B$), expressed in terms of the Fermi energy, E_F, of the system. (4p)