

TSDT14 Signal Theory

Lecture 12

Complex Signals

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Reminder – AM

$$x(t) = m(t) \cos(2\pi f_c t),$$

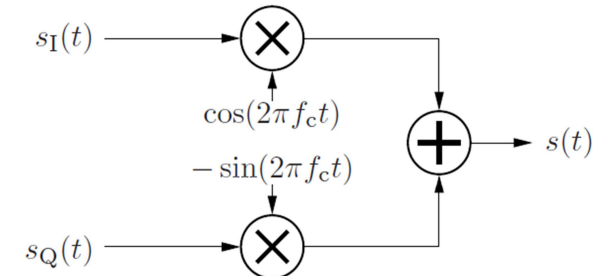
$$y(t) = m(t) \sin(2\pi f_c t).$$

$$X(f) = \mathcal{F}\{x(t)\} = \frac{1}{2} [M(f - f_c) + M(f + f_c)],$$

$$Y(f) = \mathcal{F}\{y(t)\} = \frac{1}{j2} [M(f - f_c) - M(f + f_c)].$$

Modulation

BP signal: $s(t) = s_I(t) \cos(2\pi f_c t) - s_Q(t) \sin(2\pi f_c t)$



Complex envelope: $\tilde{s}(t) = s_I(t) + js_Q(t)$

Figure 12.1

Bandlimited Signals

$$S(f) = \mathcal{F}\{s(t)\}, \quad \tilde{S}(f) = \mathcal{F}\{\tilde{s}(t)\}$$

$$S_I(f) = \mathcal{F}\{s_I(t)\}, \quad S_Q(f) = \mathcal{F}\{s_Q(t)\},$$

$$S(f) = 0 \quad \text{for} \quad |f| \leq f_c - B \quad \text{and for} \quad |f| \geq f_c + B,$$

$$\tilde{S}(f) = 0 \quad \text{for} \quad |f| \geq B,$$

$$S_I(f) = 0 \quad \text{for} \quad |f| \geq B,$$

$$S_Q(f) = 0 \quad \text{for} \quad |f| \geq B,$$

Modulation in the frequency domain 1(2)

$$s(t) = s_I(t) \cos(2\pi f_c t) - s_Q(t) \sin(2\pi f_c t)$$

$$s(t) = \operatorname{Re} \left\{ \tilde{s}(t) e^{j2\pi f_c t} \right\} = \frac{1}{2} (\tilde{s}(t) e^{j2\pi f_c t} + \tilde{s}^*(t) e^{-j2\pi f_c t})$$

$$S(f) = \frac{1}{2} (\tilde{S}(f - f_c) + \tilde{S}^*(-f - f_c))$$

$$\tilde{S}(f) = 2S(f + f_c)u(f + f_c)$$

Odd and Even Signals 1(2)

$$X(f) = X_{\text{even}}(f) + X_{\text{odd}}(f), \quad X_{\text{even}}(f) = \frac{1}{2}(X(f) + X(-f)),$$

$$X_{\text{odd}}(f) = \frac{1}{2}(X(f) - X(-f)).$$

$$\tilde{s}(t) = s_I(t) + js_Q(t) \quad \tilde{S}(f) = S_I(f) + jS_Q(f).$$

$$\operatorname{Re}\{\tilde{S}(f)\} = \operatorname{Re}\{S_I(f)\} - \operatorname{Im}\{S_Q(f)\},$$

$$\operatorname{Im}\{\tilde{S}(f)\} = \operatorname{Im}\{S_I(f)\} + \operatorname{Re}\{S_Q(f)\}.$$

Modulation in the frequency domain 2(2)

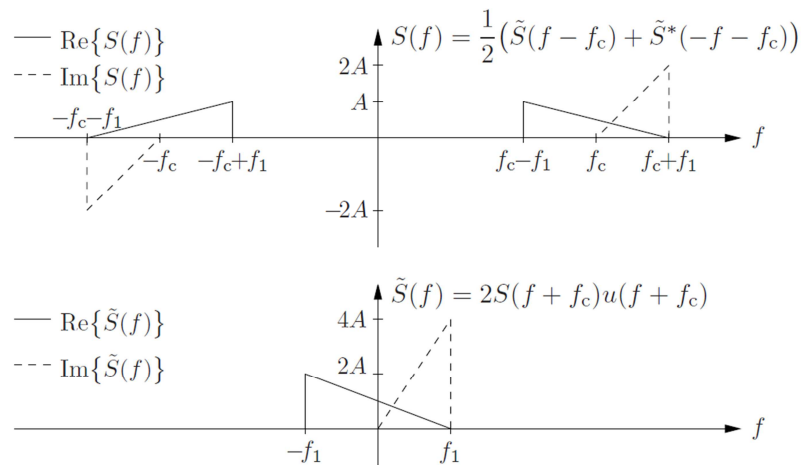


Figure 12.2

Odd and Even Signals 2(2)

$$\operatorname{Re}\{\tilde{S}(f)\} = \operatorname{Re}\{S_I(f)\} - \operatorname{Im}\{S_Q(f)\},$$

$$\operatorname{Im}\{\tilde{S}(f)\} = \operatorname{Im}\{S_I(f)\} + \operatorname{Re}\{S_Q(f)\}.$$

$$\operatorname{Re}\{S_I(f)\} = \frac{1}{2}\operatorname{Re}\{\tilde{S}(f) + \tilde{S}(-f)\},$$

$$\operatorname{Im}\{S_I(f)\} = \frac{1}{2}\operatorname{Im}\{\tilde{S}(f) - \tilde{S}(-f)\},$$

$$\operatorname{Re}\{S_Q(f)\} = \frac{1}{2}\operatorname{Im}\{\tilde{S}(-f) + \tilde{S}(f)\},$$

$$\operatorname{Im}\{S_Q(f)\} = \frac{1}{2}\operatorname{Re}\{\tilde{S}(-f) - \tilde{S}(f)\}.$$

In Phase and Quadrature Phase

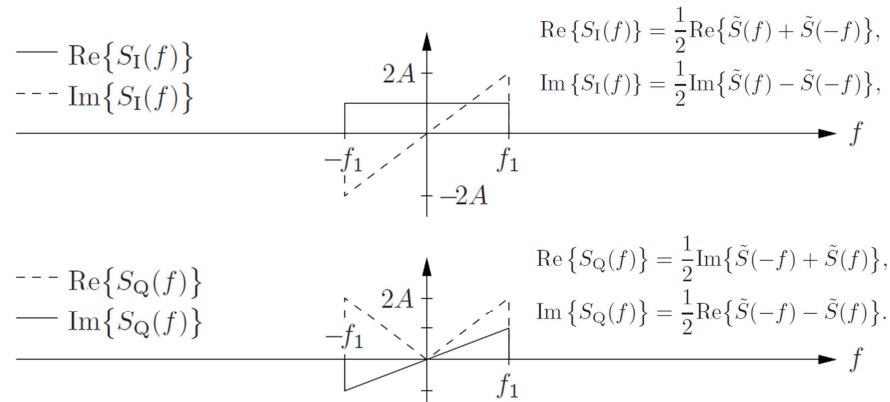


Figure 12.3

Demodulation 2(2)

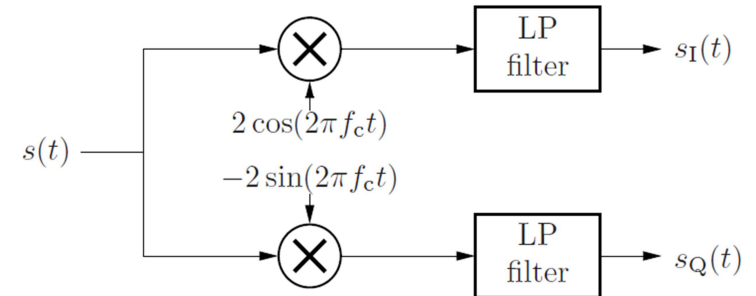


Figure 12.4

Demodulation 1(2)

$$s(t) = s_I(t) \cos(2\pi f_c t) - s_Q(t) \sin(2\pi f_c t),$$

$$\tilde{s}(t) = s_I(t) + js_Q(t).$$

$$2s(t) \cos(2\pi f_c t) = 2s_I(t) \cos^2(2\pi f_c t) - 2s_Q(t) \sin(2\pi f_c t) \cos(2\pi f_c t)$$

$$= s_I(t)(1 + \cos(4\pi f_c t)) - s_Q(t) \sin(4\pi f_c t)$$

$$-2s(t) \sin(2\pi f_c t) = -2s_I(t) \cos(2\pi f_c t) \sin(2\pi f_c t) + 2s_Q(t) \sin^2(2\pi f_c t)$$

$$= -s_I(t) \sin(4\pi f_c t) + s_Q(t)(1 - \cos(4\pi f_c t))$$

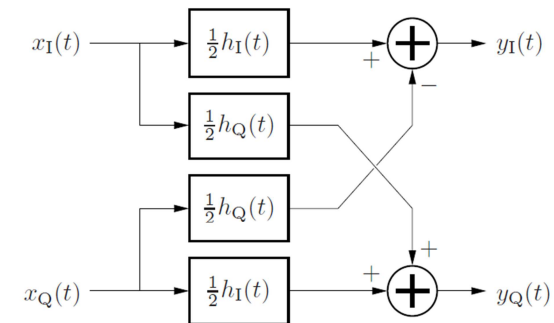
$$\mathcal{F}\{2s(t) \cos(2\pi f_c t)\} =$$

$$= \underline{S_I(f)} + \frac{1}{2}(S_I(f-2f_c) + S_I(f+2f_c) + jS_Q(f-2f_c) - jS_Q(f+2f_c))$$

$$\mathcal{F}\{-2s(t) \sin(2\pi f_c t)\} =$$

$$= \underline{S_Q(f)} + \frac{1}{2}(S_I(f-2f_c) - S_I(f+2f_c) - jS_Q(f-2f_c) - jS_Q(f+2f_c)).$$

Filtering in the Baseband



$$y_I(t) = \frac{1}{2}((x_I * h_I)(t) - (x_Q * h_Q)(t)),$$

$$y_Q(t) = \frac{1}{2}((x_I * h_Q)(t) + (x_Q * h_I)(t)).$$

Figure 12.5

Alternative Filtering in the Baseband

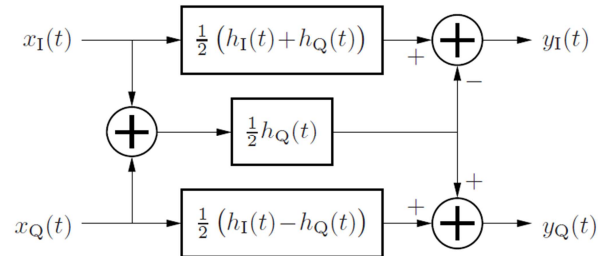


Figure 12.6

ACF and PSD of WSS Complex Process

$$\begin{aligned} r_X(\tau) &= E\{X(t+\tau)X^*(t)\} = E\{(X_1(t+\tau) + jX_2(t+\tau))(X_1(t) - jX_2(t))\} \\ &= E\{X_1(t+\tau)X_1(t)\} + E\{X_2(t+\tau)X_2(t)\} \\ &\quad + j(E\{X_2(t+\tau)X_1(t)\} - E\{X_1(t+\tau)X_2(t)\}) \\ &= r_{X_1}(\tau) + r_{X_2}(\tau) + j(r_{X_2, X_1}(\tau) - r_{X_1, X_2}(\tau)) \\ &= r_{X_1}(\tau) + r_{X_2}(\tau) + j(r_{X_1, X_2}(-\tau) - r_{X_1, X_2}(\tau)) \end{aligned}$$

$$\begin{aligned} R_X(f) &= \mathcal{F}\{r_X(\tau)\} \\ &= \mathcal{F}\{r_{X_1}(\tau) + r_{X_2}(\tau)\} + j\mathcal{F}\{r_{X_1, X_2}(-\tau) - r_{X_1, X_2}(\tau)\} \\ &= R_{X_1}(f) + R_{X_2}(f) + j(R_{X_1, X_2}(f) - R_{X_1, X_2}(f)) \end{aligned}$$

Complex Process

Complex process: $X(t) = X_1(t) + jX_2(t)$

ACF: $r_X(t_1, t_2) = E\{X(t_1)X^*(t_2)\}$

Pseudo-ACF: $\tilde{r}_X(t_1, t_2) = E\{X(t_1)X(t_2)\}$

Circular process: $\tilde{r}_X(t_1, t_2) = 0$

WSS Complex process:

$X_1(t)$ and $X_2(t)$ jointly WSS.

Modulation of a Stochastic Process 1(3)

$$S(t) = S_I(t) \cos(2\pi f_c t) - S_Q(t) \sin(2\pi f_c t)$$

$$\tilde{S}(t) = S_I(t) + jS_Q(t)$$

$$S(t) = \text{Re}\{\tilde{S}(t)e^{j2\pi f_c t}\} = \frac{1}{2}(\tilde{S}(t)e^{j2\pi f_c t} + \tilde{S}^*(t)e^{-j2\pi f_c t})$$

$S(t)$ WSS iff $\tilde{S}(t)$ circular and WSS with mean zero.

Modulation of a Stochastic Process 2(3)

$$S(t) = S_I(t) \cos(2\pi f_c t + \Psi) - S_Q(t) \sin(2\pi f_c t + \Psi)$$

Ψ uniform on $[0, 2\pi)$, independent of $\tilde{S}(t)$

$$r_S(\tau) = \frac{r_{S_I}(\tau) + r_{S_Q}(\tau)}{2} \cos(2\pi f_c \tau) - \frac{r_{S_I, S_Q}(-\tau) - r_{S_I, S_Q}(\tau)}{2} \sin(2\pi f_c \tau)$$

$$r_{\tilde{S}}(\tau) = r_{S_I}(\tau) + r_{S_Q}(\tau) + j(r_{S_I, S_Q}(-\tau) - r_{S_I, S_Q}(\tau))$$

Rounding Up the Course

Stochastic processes: Stationarity, ergodicity, mean, ACF, PSD...

LTI filtering: Mean, ACF, PSD.

Cross-correlation and cross-spectrum. Joint stationarity.

Poisson processes.

Prediction.

Non-linearities: Squaring and such, saturation, quantization.

Modulation: AM, FM, PM, noise.

Estimation (only on laborations).

Linear mappings: Sampling, PAM, reconstruction.

Two-dimensional: Signals, systems,...

Complex processes

Modulation of a Stochastic Process 3(3)

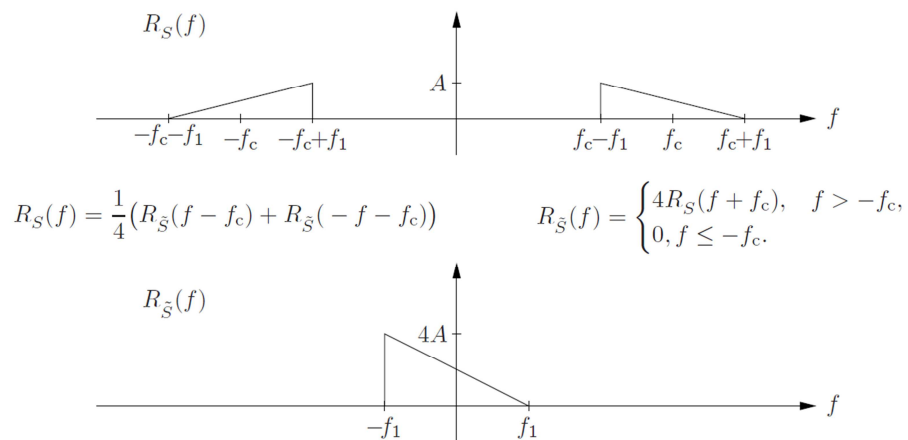


Figure 13.1

Written Examination

When: Wednesday 2019-10-23, 14.00-18.00.

Allowed aids:

Olofsson: Tables and Formulas for Signal Theory

Pocket calculator with empty memory

A German 10 mark note of the fourth series (1991-2001)

What:

A three-part introductory task (simple, 2/3 must be OK).

Five problems – 5 points each, pass is 10 points.

Written Examination – cont'd

A German 10 mark note of the fourth series (1991-2001)



Good Practices at Exams

Rules according to the exam cover:

- Only one task on the same piece of paper.
- Use only one side of the paper.
- Number the pages.
(see common sense →)
- **Do not use a red pen(cil).**
(that's my color)

Let me add:

- Hand in readable solutions.
- Do not hand in scribbles!

Common sense:

1. Solve the exam problems.
2. Sort the papers according to task numbering.
3. Number the pages last!
4. Now hand in your exam.

Do not do it in any other order!

Finally:

- Always provide solid arguments for steps taken in your solutions.

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