TSTE05 Elektronik & mätteknik

Föreläsning 5
Labförberedelser
Växelströmsteori
– jω-metoden – effektbegrepp

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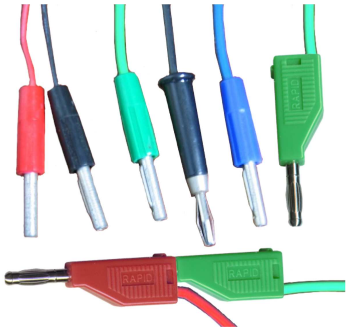
Labutrustningen – Kontakter

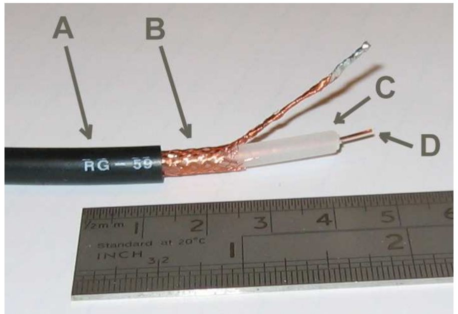
Banankontakter



BNC-kontakter

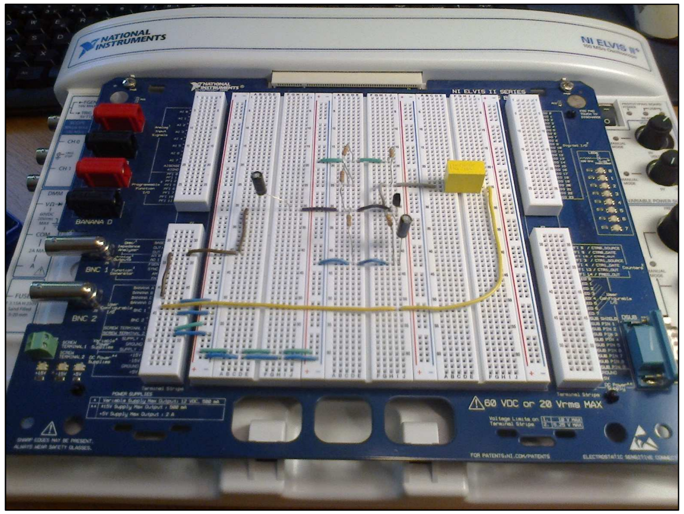
Källa: Wikipedia

Labutrustningen – Koaxialkablar

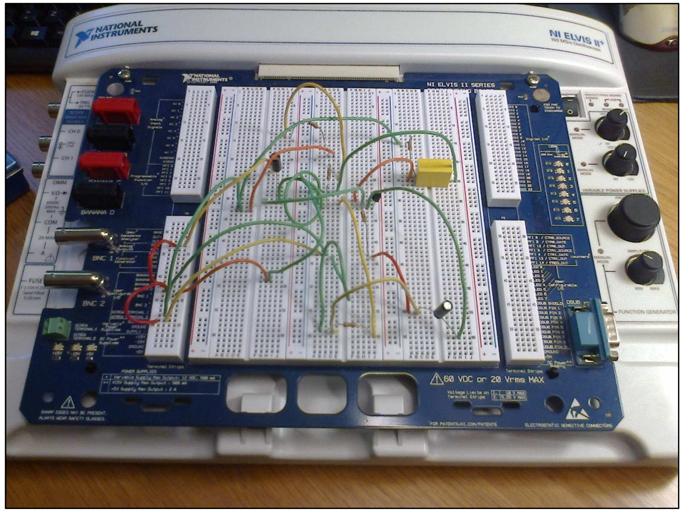


Källa: Wikipedia

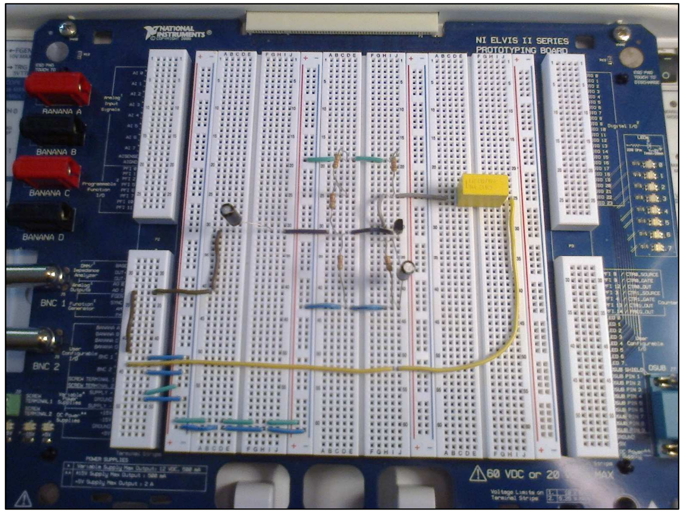
Labutrustningen – Elvis II – Prydlig uppkoppling



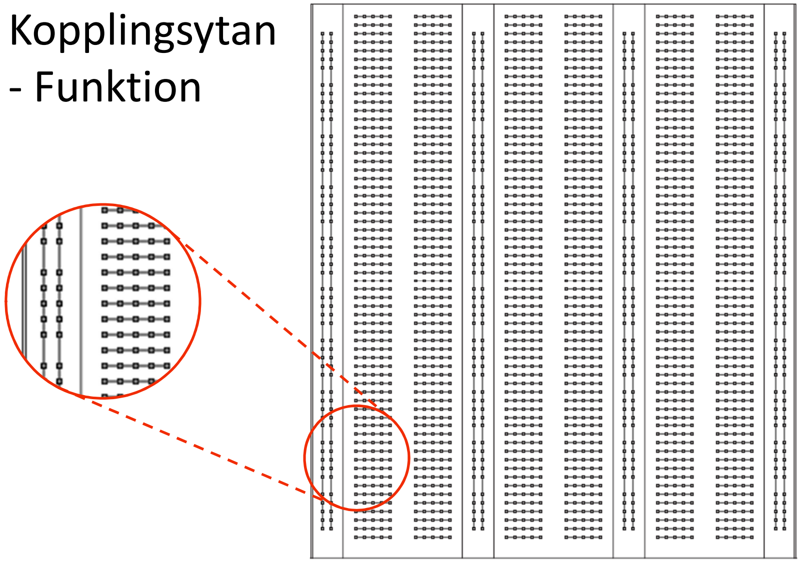
Elvis II – Mindre prydlig uppkoppling



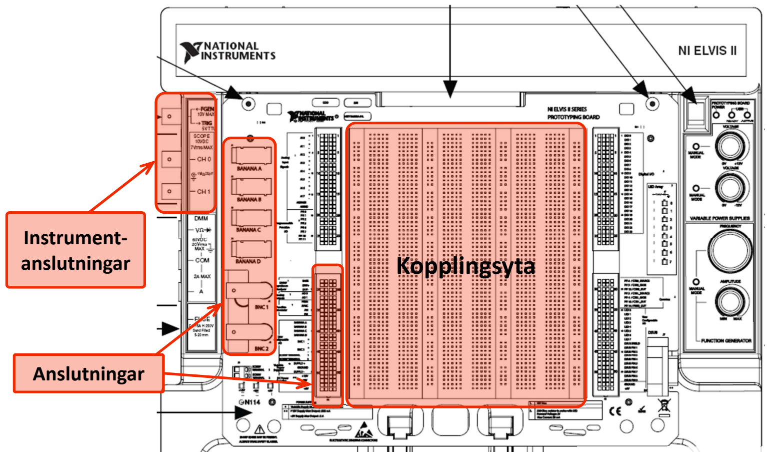
Elvis II – Kopplingsytan, breadboard



Kopplingsytan
- Funktion

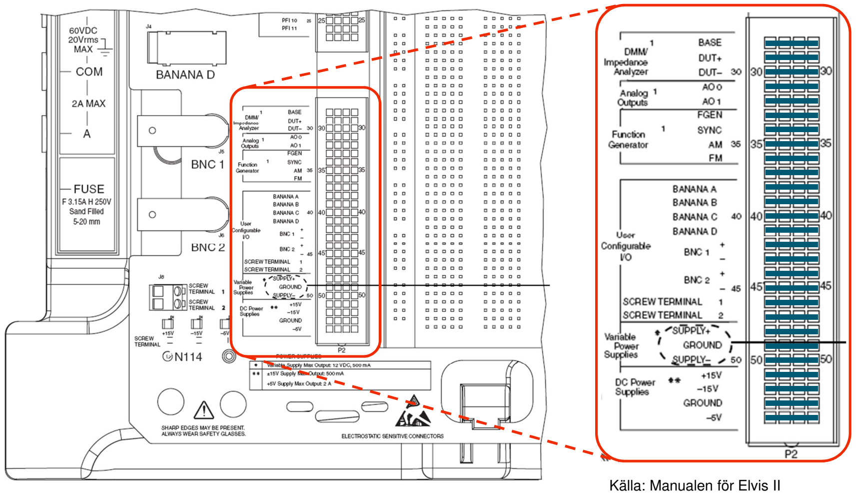


Elvis II – Översikt



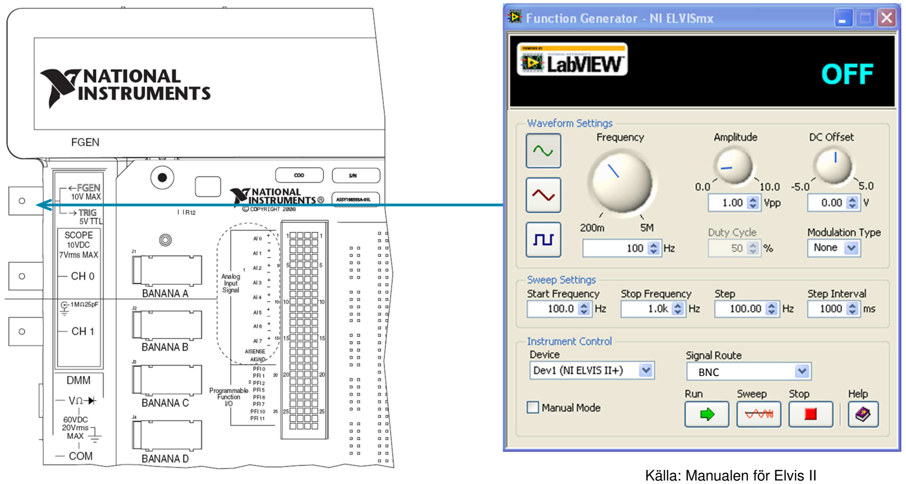
Källa: Manualen för Elvis II

Elvis II – Nedre vänstra hörnet



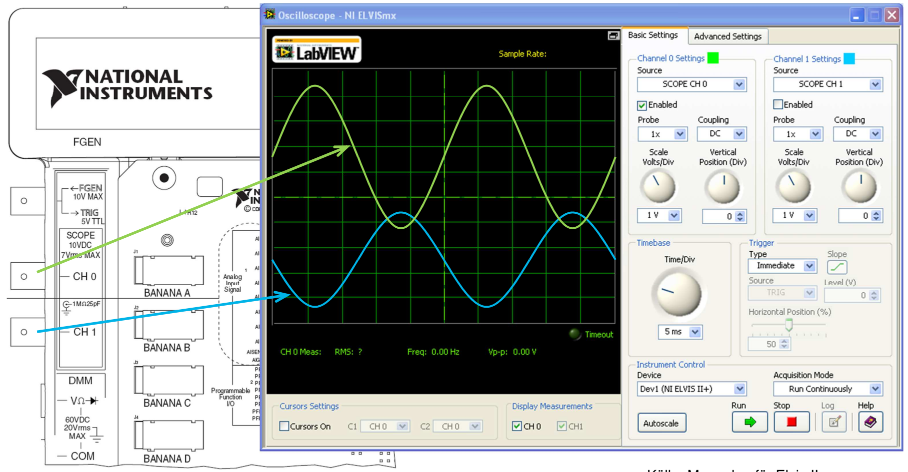
Källa: Manualen för Elvis II

Elvis II – Funktionsgeneratorn



Källa: Manualen för Elvis II

Elvis II – Oscilloskopet



Källa: Manualen för Elvis II

jω-metoden

1. Ersätt strömmar, spänningar och källor med deras komplexa motsvarigheter:

$$a(t) = \hat{A}\sin(\omega t + \varphi) \Rightarrow$$

$$A = \hat{A}e^{j\varphi} = b + jc$$

$$b = \hat{A}\cos\varphi \qquad c = \hat{A}\sin\varphi$$

2. Ersätt R, L, C med deras impedanser:

$$Z_L = j\omega L \qquad Z_C = \frac{1}{j\omega C} \qquad Z_R = R$$

3. Lös problemet med likströmsteori.

4. Gör omvändningen till punkt 1:

$$A = \hat{A}e^{j\varphi} = b + jc \Rightarrow$$

$$a(t) = \hat{A}\sin(\omega t + \varphi)$$

$$\hat{A} = \sqrt{b^2 + c^2}$$

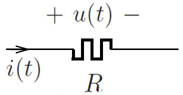
$$\varphi = arg(b + jc) = \text{atan}\frac{c}{b} \qquad (\pm\pi)$$

Om $b < 0$

Härledning jω-metoden 1(2)

$$u(t) = \hat{U}\sin(\omega t + \phi_u)$$
$$= \mathrm{Im}\left\{\hat{U}e^{j(\omega t+\phi_u)}\right\} = \mathrm{Im}\left\{\overbrace{\hat{U}e^{j\phi_u}}^{U}e^{j\omega t}\right\} = \mathrm{Im}\left\{Ue^{j\omega t}\right\}$$
$$i(t) = \hat{I}\sin(\omega t + \phi_i)$$
$$= \mathrm{Im}\left\{\hat{I}e^{j(\omega t+\phi_i)}\right\} = \mathrm{Im}\left\{\overbrace{\hat{I}e^{j\phi_i}}^{I}e^{j\omega t}\right\} = \mathrm{Im}\left\{Ie^{j\omega t}\right\}$$

Resistans



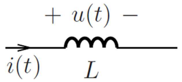
$$u(t) = Ri(t)$$

$$\mathrm{Im}\left\{Ue^{j\omega t}\right\} = R\,\mathrm{Im}\left\{Ie^{j\omega t}\right\} = \mathrm{Im}\left\{RIe^{j\omega t}\right\}$$

Lösning: $U = RI$
Impedans: $Z_R = R$

Härledning jω-metoden 2(2)

Induktans

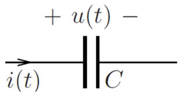


$$u(t) = L\frac{d}{dt}i(t)$$

$$\mathrm{Im}\left\{Ue^{j\omega t}\right\} = L\frac{d}{dt}\mathrm{Im}\left\{Ie^{j\omega t}\right\} = \mathrm{Im}\left\{LI\frac{d}{dt}e^{j\omega t}\right\} = \mathrm{Im}\left\{LIj\omega e^{j\omega t}\right\}$$

Lösning: $U = j\omega LI$
Impedans: $Z_L = j\omega L$

Kapacitans



$$i(t) = C\frac{d}{dt}u(t)$$

$$\mathrm{Im}\left\{Ie^{j\omega t}\right\} = C\frac{d}{dt}\mathrm{Im}\left\{Ue^{j\omega t}\right\} = \mathrm{Im}\left\{CU\frac{d}{dt}e^{j\omega t}\right\} = \mathrm{Im}\left\{CUj\omega e^{j\omega t}\right\}$$

Lösning: $I = j\omega CU \Rightarrow U = \frac{1}{j\omega C}I$

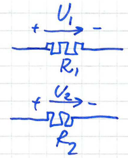
Impedans: $Z_C = \frac{1}{j\omega C}$

dB-begreppet

P_1 & P_2 effekter.

P_1 är $\log_{10}\frac{P_1}{P_2}$ Bel större än P_2.

P_1 är $10\log_{10}\frac{P_1}{P_2}$ dB större än P_2.



$P_1 = \frac{U_1^2}{R_1}$

$P_2 = \frac{U_2^2}{R_2}$

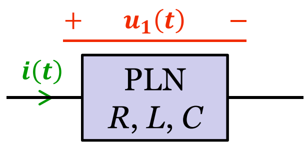
$$10\log_{10}\frac{P_1}{P_2} = 10\log_{10}\frac{U_1^2/R_1}{U_2^2/R_2} = 10\log_{10}\left(\frac{U_1}{U_2}\right)^2 + 10\log_{10}\frac{R_2}{R_1}$$
$$= 20\log_{10}\frac{U_1}{U_2} + 10\log_{10}\frac{R_2}{R_1} \underset{R_1=R_2}{=} 20\log_{10}\frac{U_1}{U_2}$$

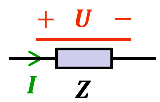
U_1 är $20\log_{10}\frac{U_1}{U_2}$ större än U_2.

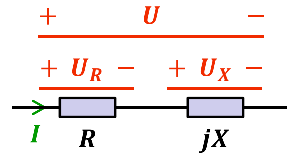
$20\log_{10}\frac{1}{\sqrt{2}} \approx -3.0103 \approx -3$ dB. Gränsfrekv. när $|H(\omega)|$ sjunkit 3dB.

Effektbegrepp 1(4)

Passivt Linjärt Nät – PLN

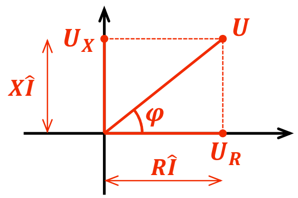






$i(t) = \hat{I}\sin(\omega t)$ $\quad I = \hat{I}$ $\quad U_R = RI$ $\quad U_X = jXI$

$u(t) = \hat{U}\sin(\omega t + \varphi)$ $\quad U = \hat{U}e^{j\varphi}$ $\quad U = U_R + U_X = (R + jX)I = ZI$



Toppvärden

$$\left.\begin{aligned} R\hat{I} &= \hat{U}\cos(\varphi) \\ X\hat{I} &= \hat{U}\sin(\varphi) \end{aligned}\right\} *$$

Effektivvärden

$$\left.\begin{aligned} U_e &= \hat{U}/\sqrt{2} \\ I_e &= \hat{I}/\sqrt{2} \end{aligned}\right\} **$$

$$u(t) = R\hat{I}\sin(\omega t) + X\hat{I}\sin(\omega t + \pi/2)$$

Effektbegrepp 2(4)

$$\left.\begin{aligned} R\hat{I} &= \hat{U}\cos(\varphi) \\ X\hat{I} &= \hat{U}\sin(\varphi) \end{aligned}\right\} * \qquad \left.\begin{aligned} U_e &= \hat{U}/\sqrt{2} \\ I_e &= \hat{I}/\sqrt{2} \end{aligned}\right\} **$$

$$i(t) = \hat{I}\sin(\omega t)$$

$$u(t) = R\hat{I}\sin(\omega t) + X\hat{I}\sin(\omega t + \pi/2)$$

Momentan effekt:

$$p(t) = u(t)i(t) = R\hat{I}^2\sin^2(\omega t) + X\hat{I}^2\sin(\omega t)\sin(\omega t + \pi/2)$$

$$= \underbrace{\frac{R\hat{I}^2}{2}(1-\cos(2\omega t))}_{p_R(t)} + \underbrace{\frac{X\hat{I}^2}{2}\sin(2\omega t)}_{p_X(t)}$$

Utnyttja $*$, $**$ och $\omega = 2\pi/T$:

$$p_R(t) = U_e I_e\cos(\varphi)(1-\cos(2\omega t)) = U_e I_e\cos(\varphi)(1-\cos(4\pi t/T))$$

$$p_X(t) = U_e I_e\sin(\varphi)\sin(2\omega t) = U_e I_e\sin(\varphi)\sin(4\pi t/T)$$

Effektbegrepp 3(4)

$$\left.\begin{aligned} R\hat{I} &= \hat{U}\cos(\varphi) \\ X\hat{I} &= \hat{U}\sin(\varphi) \end{aligned}\right\} * \qquad \left.\begin{aligned} U_e &= \hat{U}/\sqrt{2} \\ I_e &= \hat{I}/\sqrt{2} \end{aligned}\right\} **$$

$$p_R(t) = U_e I_e\cos(\varphi)(1-\cos(4\pi t/T))$$

$$p_X(t) = U_e I_e\sin(\varphi)\sin(4\pi t/T)$$

Aktiv effekt:

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{T}\int_0^T p_R(t)\,dt + \frac{1}{T}\int_0^T p_X(t)\,dt$$

$$= \frac{1}{T}(TU_e I_e\cos(\varphi) + 0) + \frac{1}{T}0 = U_e I_e\cos(\varphi) = R{I_e}^2$$ Enhet: W

$*, **$

Reaktiv effekt: $*, **$

$$Q = U_e I_e\sin(\varphi) = X{I_e}^2$$ Enhet: VAr

Effektbegrepp 4(4)

$$P = R{I_e}^2 \qquad Z = R + jX$$

$$Q = X{I_e}^2 \qquad \varphi = \arg(Z)$$

Komplex effekt:

$$S = P + jQ = (R + jX){I_e}^2 = Z{I_e}^2 = \frac{UI^*}{2} = \frac{{U_e}^2}{Z^*}$$ Enhet: VA

Skenbar effekt:

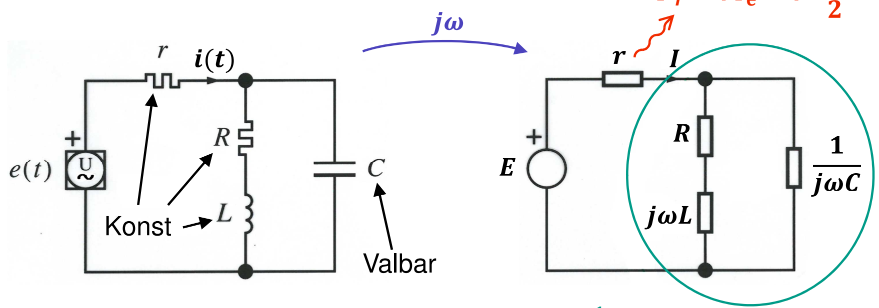
$$P_S = U_e I_e = |S| = \sqrt{P^2 + Q^2} = |Z|{I_e}^2 = \frac{{U_e}^2}{|Z|}$$ Enhet: VA

Effektfaktor:

$$\cos(\varphi) = \frac{P}{P_S} = \frac{R}{|Z|}$$

Faskompensering

$$P_r = rI_e^2 = r\frac{|I|^2}{2}$$



Mål: Minimera P_r

Antagande: r litet.

$$Z = \frac{(R + j\omega L)\frac{1}{j\omega C}}{\frac{1}{j\omega C} + j\omega L + R} = \frac{R + j\omega L}{1 - \omega^2 LC + j\omega RC}$$

$$\min P_r \Rightarrow \min I_e \Rightarrow \max|Z| \Rightarrow \min|1 - \omega^2 LC + j\omega RC|$$

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