LINKÖPINGS UNIVERSITET

Department of Science and Technology
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Exam TEN1 for TNE041, **Modern Physics**, 5 June 2024, 14.00 – 18.00.

Allowed examination material: Physics handbook (Studentlitteratur)
calculator (with no wifi)
additional formulae (attached)
one hand-written sheet (A4, not copied, with notes on one side)

Define all quantities you use and give a clear answer, including unit if a numerical value is given. No points are given if only the answer is submitted, with the exception of true/false questions. The maximum score is 24 points (6x4). The limits for different grades given below are with bonus included. The solutions may be given in English or in Swedish.

The following limits for grades apply:

Grade 3	$\geq$ 10 points
Grade 4	$\geq$ 15 points
Grade 5	$\geq$ 19 points

Questions are answered by Michael Hörnquist who will visit the exam room around 3 pm and 4.30 pm. Answers and short solutions will be available at Studieinfo at 9 pm at the latest. Results will be reported not later than 15 working days after the exam.

Good luck!

1. (a) Are the following statements true or false?
   i. It is possible to solve the Schrödinger equation for an H-atom analytically.
   ii. It is possible to solve the Schrödinger equation for a Li-atom analytically.
   iii. It is possible to solve the Schrödinger equation for a Li^{+}-ion analytically.
   iv. It is possible to solve the Schrödinger equation for a Li^{2+}-ion analytically.

   Only the answers (true/false) are required. (2p)

   (b) The maximum wavelength for which photo electric effect is possible in Wolfram is 230 nm. Determine the highest kinetic energy possible for electrons emitted from a surface of Wolfram radiated by UV-light of wavelength 180 nm. The value of the work function *must not* be taken from Physics Handbook. (2p)

2. Consider the case of a QM-particle in a 1D-box, i.e., a QM-particle confined by the potential

$$V(x) = \begin{cases} \infty & \text{if } x < 0 \text{ or } x > a \\ 0 & \text{if } 0 \leq x \leq a \end{cases}$$

where a is a length on the nanoscale, $a > 0$ Let b and Δx be two lengths such that the interval $[b,\, b + \Delta x]$ is inside the box.

   (a) If the particle is in the state n, where $n = 1$ is the ground state, what is the probability to find the particle in the interval $[b,\, b + \Delta x]$? (2p)

   (b) Let $n \to \infty$ in the probability expression from (a). Compare with the non-QM scenario where the particle is a little ball with no wave-properties. (2p)

3. (a) Determine the coefficient of reflection for electrons with kinetic energy 0,1 eV incident on a Sodium surface. The metal surface can be approximated with an instant potential *drop* of 5,0 eV. (3p)

   (b) What would the value be if the electrons could be considered as classical particles? (1p)

4. Consider a particle with spin $s = 3/2$.

   (a) List the possible values of its quantum number m_s. (1p)

   (b) Determine the smallest angle between the spin vector $\mathbf{S}$ and the z-axis. (3p)

5. (a) Give a physical interpretation of the expression

$$\int_{E_1}^{E_2} N(E)D(E)dE,$$

   where $N(E)$ and $D(E)$ are defined according to "Additional formulae". (2p)

   (b) Determine, using the expression i (a), the probability that an electron is found to have the energy $E > 0,9E_F$, where E_F is the Fermi energy of the system. Assume room temperature and the free-electron-model. (2p)

6. When the semiconductor gallium arsenide (GaAs) is doped with selenium (Se) the result is an excess of electrons (n-doping) since the selenium atoms replace arsenide atoms and contain an "extra" electron compared to these. Using a crude model, the selenium atoms can be regarded as a kind of hydrogen-like atoms with the extra donor electron being bound to the rest of the selenium atom, which has a net charge equal to $+e$.

   The energy levels of this "hydrogen atom" can be calculated by changing the expression for the energy levels of the ordinary hydrogen atom, so that the permittivity ε_0 is replaced by $\varepsilon_r \varepsilon_0$ where the relative permittivity ε_r is equal to 13,5 for GaAs.

   Use this model to estimate how much energy is needed to free a donor electron in a selenium atom in its ground state, and compare the result to the band gap in GaAs that is equal to 1,4 eV. Does it seem reasonable to consider the donor electrons as free?

ADDITIONAL FORMULAE TNE041 MODERN PHYSICS

Special relativity:

Momentum $\mathbf{p} = \gamma_u m\mathbf{u}$ where $\gamma_u = \dfrac{1}{\sqrt{1-\dfrac{u^2}{c^2}}}$

Energy $E = \gamma_u mc^2$ $E^2 = p^2c^2 + m^2c^4$

Internal (rest) energy $E_{int} = mc^2$ Kinetic energy $E_{kin} = (\gamma_u - 1)mc^2$

Quantum mechanics:

Penetration depth $\delta = \dfrac{\hbar}{\sqrt{2m(U_0 - E)}}$ Gaussian wave packet $\psi(x) = Ae^{-(x/2\varepsilon)^2} e^{ik_0 x}$

If $\delta << L$ (barrier width) then the transmission coefficient can be approximated as

$$T \approx 16\frac{E}{U_0}(1-\frac{E}{U_0})e^{-2(\sqrt{2m(U_0-E)}/\hbar)L}$$

Solutions of the time independent Schrödinger equation for a particle with mass m in an infinite well, side lengths L_x, L_y, L_z:

$$\psi_{n_x,n_y,n_z}(x,y,z) = A\sin\frac{n_x\pi x}{L_x}\sin\frac{n_y\pi y}{L_y}\sin\frac{n_z\pi z}{L_z} \text{ and } E_{n_x,n_y,n_z} = \left(\frac{n_x^2}{L_x^2}+\frac{n_y^2}{L_y^2}+\frac{n_z^2}{L_z^2}\right)\frac{\pi^2\hbar^2}{2m}$$

where $n_x, n_y, n_z = 1,2,\ldots$

Statistical mechanics:

Distribution functions (the probability that a state with energy E (E_i) is occupied)

Maxwell-Boltzmann: $N(E) = Ae^{-E/k_BT}$ (continuous) or $N(E_i) = \dfrac{g_i}{Z}e^{-E_i/k_BT}$ (discrete) ,

g_i: degree of degeneracy for energy level E_i ,partition function $Z = \sum_i g_i e^{-E_i/k_BT}$

Fermi-Dirac: N(E) = $\frac{1}{e^{(E-E_F)/k_BT}+1}$ Bose-Einstein: N(E) = $\frac{1}{e^{\alpha+E/k_BT}-1}$

Average values

Discrete $\overline{Q} = \frac{\sum_n Q_n N(E_n)}{\sum_n N(E_n)}$ Continuous $\overline{Q} = \frac{\int Q(E)N(E)D(E)dE}{\int N(E)D(E)dE}$ where *D(E)* is the density of states.

Solid state physics, some crystal lattices: