LINKÖPINGS UNIVERSITET

Department of Science and Technology
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Exam for TNE041, **Modern Physics**, 18 March 2024, 14.00 – 18.00.

Allowed examination material: Physics handbook (Studentlitteratur)
calculator (with no wifi)
additional formulae (attached)
one hand-written sheet (A4, not copied, with notes on one side)

Define all quantities you use and give a clear answer, including unit if a numerical value is given. No points are given if only the answer is submitted, with the exception of true/false questions. The maximum score is 24 points (6x4). The limits for different grades given below is with bonus included. The solutions may be given in English or in Swedish.

The following limits for grades apply:

Grade 3	$\geq$ 10 points
Grade 4	$\geq$ 15 points
Grade 5	$\geq$ 19 points

Questions are answered by Michael Hörnquist who will visit the exam room around 3 pm and 4.30 pm. Answers and short solutions will be available at Studieinfo at 9 pm at the latest. Results will be reported not later than 15 working days after the exam.

Good luck!

1. (a) Are the following statements true or false?

    i. The photoelectric effect shows that electrons have wave properties.

    ii. The function $\Psi(x,t) = A\sin(kx - \omega t)$ is a solution to the time-dependent Schrödinger equation for a free particle.

    iii. The transmissison coefficient T for a free electron with energy 20 eV incident on a potential barrier of height 15 eV and width 2 nm is $T = 1$.

    iv. Undoped semiconductors are insulutors at $T = 0\text{K}$.

    Only the answers (true/false) are required. (2p)

    (b) Determine the voltage it takes to accelerate an electron

    i. from rest to $0,9c$. (1p)

    ii. from $0,9c$ to $0,99c$. (1p)

2. Starting from the time-independent Schrödinger equation for a symmetric harmonic potential U in three dimensions,

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r}) + U(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

with the potential given as

$$U(\mathbf{r}) = \frac{1}{2}kr^2.$$

(a) Show how this equation can be turned into three one-dimensional, time-independent Schrödinger equations. (2p)

(b) Given the energy levels for the one-dimensional, time-independent Schrödinger equation, determine the energy levels for the three-dimensional equation above. (1p)

(c) What is the degeneracy for the first excited state? (1p)

3. Consider a QM-particle (1D) described by the wave function ψ given as

$$\psi(x) = \begin{cases} 0 & \text{if } x < 0 \\ C\sqrt{x}e^{-ax^2} & \text{if } x \geq 0 \end{cases}$$

with $a = 1,0\ (\text{nm})^{-2}$ and C is a normalization constant.

(a) Determine the value of C. (2p)

(b) Determine the probability to detect the particle in the interval $0 < x < a^{-1/2}$. (2p)

4. An electron is in a $n = 2$ state of a hydrogen atom.

(a) What is its energy? (1p)

(b) What other quantum numbers are there to fully determine the state of the system? List the possible values of these quantum numbers when $n = 2$. (1p)

(c) One of these quantum numbers does *not* originate from the Schrödinger equation? Which? What is the origin of this quantum number? (2p)

5. Consider a dilute gas of singly ionized He-atoms, i.e., He^+-ions, at room temperature. Determine the ratio of ions in the first excited state ($n = 2$) to those in the ground state ($n = 1$). The mean distance between ions can be assumed to be so large that MB-statistics is applicable. (4p)

6. Consider a two-dimensional electron gas consisting of N non-interacting electrons at room temperature. Within FEM, the free electron model, one can show the density of states for electrons (with spin taken into account) is given by

$$D(E) = \frac{m_e A}{\pi \hbar^2},$$

where A is the area of the system.

   (a) Determine an expression for the Fermi energy E_F, assuming room temperature can be considerad as cool. (2p)

   (b) Calculate its numerical value if $N = 10^8$ electrons and $A = 10^{-10}\text{m}^2$. (1p)

   (c) Determine the Fermi temperature $T_F = E_F/k_B$. Is room temperature still to be considered "cold" also in 2D? (1p)

ADDITIONAL FORMULAE TNE041 MODERN PHYSICS

Special relativity:

Momentum $\mathbf{p} = \gamma_u m\mathbf{u}$ where $\gamma_u = \dfrac{1}{\sqrt{1-\dfrac{u^2}{c^2}}}$

Energy $E = \gamma_u mc^2$ $E^2 = p^2c^2 + m^2c^4$

Internal (rest) energy $E_{int} = mc^2$ Kinetic energy $E_{kin} = (\gamma_u - 1)mc^2$

Quantum mechanics:

Penetration depth $\delta = \dfrac{\hbar}{\sqrt{2m(U_0 - E)}}$ Gaussian wave packet $\psi(x) = Ae^{-(x/2\varepsilon)^2}e^{ik_0x}$

If $\delta << L$ (barrier width) then the transmission coefficient can be approximated as

$$T \approx 16\frac{E}{U_0}(1-\frac{E}{U_0})e^{-2(\sqrt{2m(U_0-E)}/\hbar)L}$$

Solutions of the time independent Schrödinger equation for a particle with mass m in an infinite well, side lengths L_x, L_y, L_z:

$$\psi_{n_x,n_y,n_z}(x,y,z) = A\sin\frac{n_x\pi x}{L_x}\sin\frac{n_y\pi y}{L_y}\sin\frac{n_z\pi z}{L_z} \text{ and } E_{n_x,n_y,n_z} = \left(\frac{n_x^2}{L_x^2}+\frac{n_y^2}{L_y^2}+\frac{n_z^2}{L_z^2}\right)\frac{\pi^2\hbar^2}{2m}$$

where $n_x, n_y, n_z = 1,2,\ldots$

Statistical mechanics:

Distribution functions (the probability that a state with energy E (E_i) is occupied)

Maxwell-Boltzmann: $N(E) = Ae^{-E/k_BT}$ (continuous) or $N(E_i) = \dfrac{g_i}{Z}e^{-E_i/k_BT}$ (discrete) ,

g_i: degree of degeneracy for energy level E_i ,partition function $Z = \sum_i g_i e^{-E_i/k_BT}$

Fermi-Dirac: $N(E) = \frac{1}{e^{(E-E_F)/k_BT}+1}$ Bose-Einstein: $N(E) = \frac{1}{e^{\alpha+E/k_BT}-1}$

Average values

Discrete $\overline{Q} = \frac{\sum_n Q_n N(E_n)}{\sum_n N(E_n)}$ Continuous $\overline{Q} = \frac{\int Q(E)N(E)D(E)dE}{\int N(E)D(E)dE}$ where *D(E)* is the density of states.

Solid state physics, some crystal lattices: