TSKS21 Signaler, information & bilder

Föreläsning 2

Växelströmsteori, introduktion

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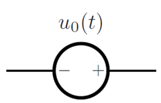
Växelströmsteori

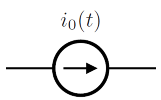
Tidsberoende storheter:

Spänning	$u(t)$
Ström	$i(t)$
Effekt	$p(t)$

Ideal spänningskälla

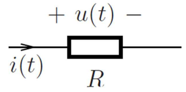


Ideal strömkälla



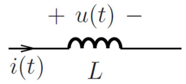
Växelströmsteori – Passiva komponenter

Resistans



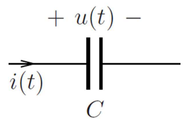
$$u(t) = Ri(t)$$

Induktans



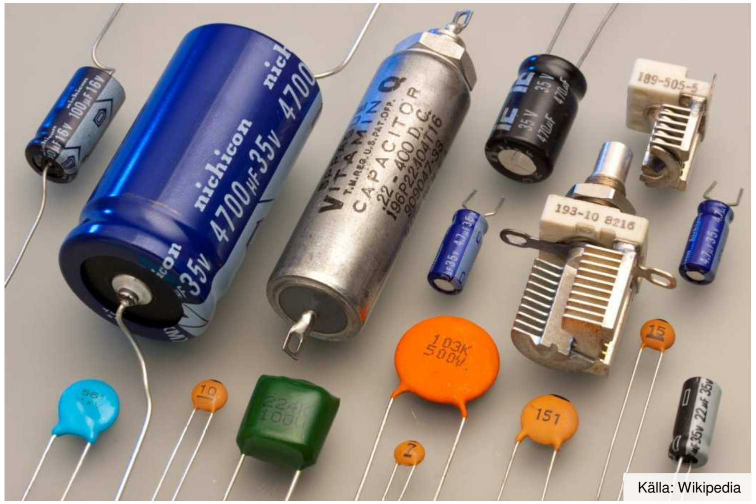
$$u(t) = L\frac{d}{dt}i(t)$$

Kapacitans



$$i(t) = C\frac{d}{dt}u(t)$$

Kapacitans – Kondensatorer



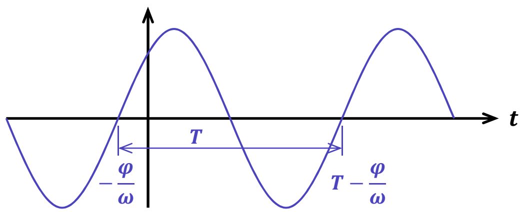
Källa: Wikipedia

Induktans – Spolar

Källa: Wikipedia

Stationär sinussignal

$x(t) = \hat{X}\sin(\omega t + \varphi)$



Symbol	Förklaring
$x(t)$	Momentanvärde
$\hat{X}$	Amplitud (toppvärde)
ω	Vinkelfrekvens [rad/s]
φ	Fasvinkel [rad]
T	Periodtid [s]
f	Frekvens [Hz]

$f = \frac{1}{T}$ $\quad\omega = 2\pi f$

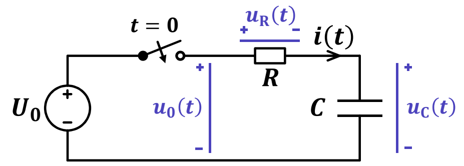
Momentan effekt

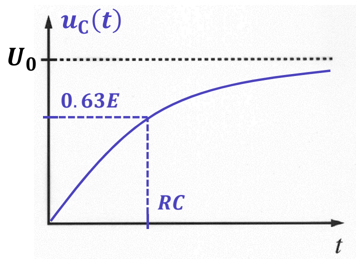
$p(t) = u(t)i(t)$

Aktiv effekt: $P = \frac{1}{T}\int_0^T p(t)\,dt$

Effektivvärde: $X_e = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt} = \frac{\hat{X}}{\sqrt{2}}$ (Sinus)

Uppladdning av en kapacitans





Initialtillstånd: $u_C(0-) = 0$ $\quad u_0(t) = \begin{cases} 0, & t < 0, \\ U_0, & t \geq 0. \end{cases}$ $\quad u_C(t) + u_R(t) = u_0(t)$

$u_R(t) = Ri(t) = RC\frac{d}{dt}u_C(t)$

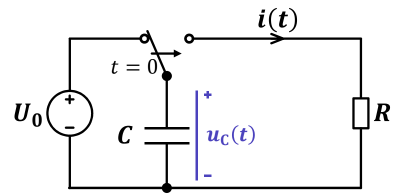
$i(t) = C\frac{d}{dt}u_C(t)$

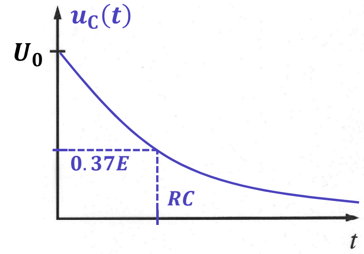
$t \geq 0$: $\quad u_C(t) + RC\frac{d}{dt}u_C(t) = U_0$

Homogen och partikulär lösning ⇒

$u_C(t) = (1 - e^{-t/RC})U_0$

Urladdning av kapacitans





Initialtillstånd: $u_C(0-) = U_0$

$t \geq 0$:

$u_C(t) = Ri(t) = -RC\frac{d}{dt}u_C(t)$

$i(t) = -C\frac{d}{dt}u_C(t)$

$u_C(t) + RC\frac{d}{dt}u_C(t) = 0$

Homogen (och partikulär) lösning ⇒

$u_C(t) = U_0 e^{-t/RC}$

jω-metoden

1. Ersätt strömmar, spänningar och källor med deras komplexa motsvarigheter:

   $a(t) = \hat{A}\sin(\omega t + \varphi) \Rightarrow$

   $A = \hat{A}e^{j\varphi} = b + jc$

   $b = \hat{A}\cos\varphi \quad c = \hat{A}\sin\varphi$

2. Ersätt R, L, C med deras impedanser:

   $Z_L = j\omega L \quad Z_C = \dfrac{1}{j\omega C} \quad Z_R = R$

3. Lös problemet med likströmsteori.

4. Gör omvändningen till punkt 1:

   $A = \hat{A}e^{j\varphi} = b + jc \Rightarrow$

   $a(t) = \hat{A}\sin(\omega t + \varphi)$

   $\hat{A} = \sqrt{b^2 + c^2}$

   $\varphi = arg(b + jc) = \text{atan}\dfrac{c}{b} \quad (\pm\pi)$

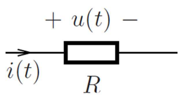
   Om $b < 0$

Härledning jω-metoden 1(2)

$$u(t) = \hat{U}\sin(\omega t + \phi_u) = \text{Im}\left\{\hat{U}e^{j(\omega t + \phi_u)}\right\} = \text{Im}\left\{\overbrace{\hat{U}e^{j\phi_u}}^{U}e^{j\omega t}\right\} = \text{Im}\left\{Ue^{j\omega t}\right\}$$

$$i(t) = \hat{I}\sin(\omega t + \phi_i) = \text{Im}\left\{\hat{I}e^{j(\omega t + \phi_i)}\right\} = \text{Im}\left\{\overbrace{\hat{I}e^{j\phi_i}}^{I}e^{j\omega t}\right\} = \text{Im}\left\{Ie^{j\omega t}\right\}$$

Resistans



$$u(t) = Ri(t)$$

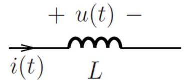
$$\text{Im}\left\{Ue^{j\omega t}\right\} = R\text{Im}\left\{Ie^{j\omega t}\right\} = \text{Im}\left\{RIe^{j\omega t}\right\}$$

Lösning: $U = RI$

Impedans: $Z_R = R$

Härledning jω-metoden 2(2)

Induktans



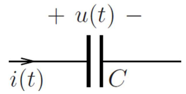
$$u(t) = L\frac{d}{dt}i(t)$$

$$\text{Im}\left\{Ue^{j\omega t}\right\} = L\frac{d}{dt}\text{Im}\left\{Ie^{j\omega t}\right\} = \text{Im}\left\{LI\frac{d}{dt}e^{j\omega t}\right\} = \text{Im}\left\{LIj\omega e^{j\omega t}\right\}$$

Lösning: $U = j\omega LI$

Impedans: $Z_L = j\omega L$

Kapacitans



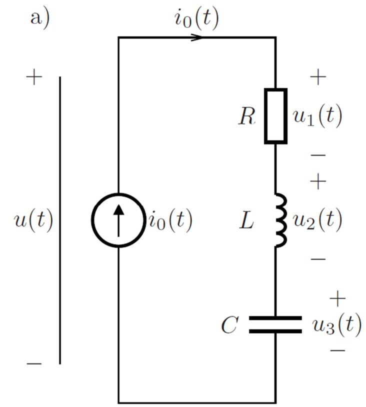
$$i(t) = C\frac{d}{dt}u(t)$$

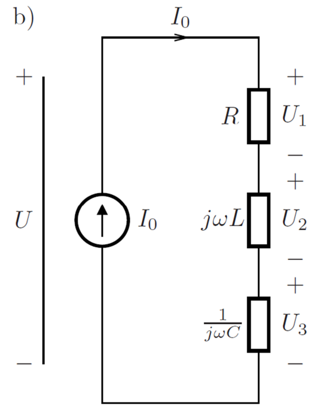
$$\text{Im}\left\{Ie^{j\omega t}\right\} = C\frac{d}{dt}\text{Im}\left\{Ue^{j\omega t}\right\} = \text{Im}\left\{CU\frac{d}{dt}e^{j\omega t}\right\} = \text{Im}\left\{CUj\omega e^{j\omega t}\right\}$$

Lösning: $I = j\omega CU \Rightarrow U = \dfrac{1}{j\omega C}I$

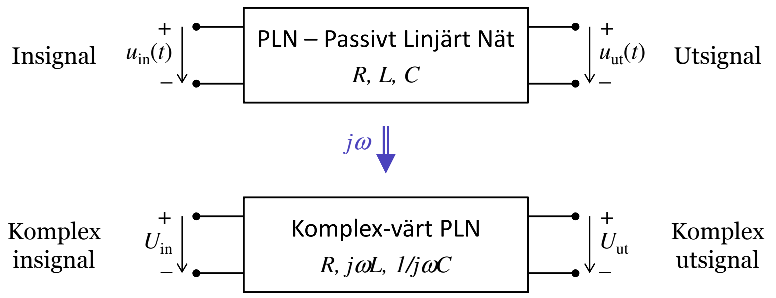
Impedans: $Z_C = \dfrac{1}{j\omega C}$

Exempel jω-metoden





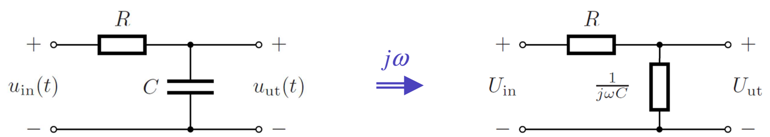
Passiva filter – Introduktion



Samband: $U_{ut} = H(\omega)\, U_{in}$ $\qquad H(\omega) = |H(\omega)| \cdot e^{j\arg\{H(\omega)\}} = U_{ut}/U_{in}$

Frekvensfunktion $\quad$ Amplitudkarakteristik $\quad$ Faskarakteristik

Passiva filter – Exempel 1(2)



Spänningsdelning ger

$$U_{ut} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} U_{in} = \frac{1}{1 + j\omega RC} U_{in}$$

Frekvensfunktion

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

Amplitudkarakteristik

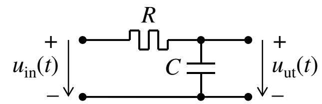
$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

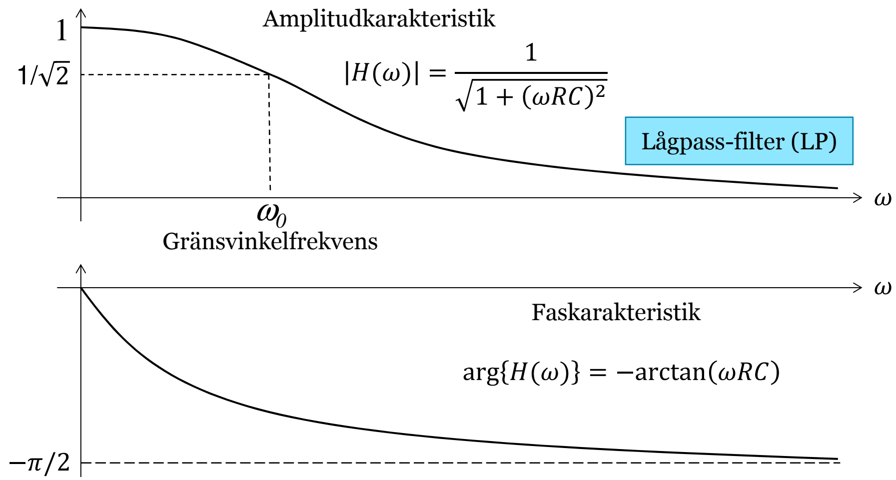
Faskarakteristik

$$\arg\{H(\omega)\} = -\arctan(\omega RC)$$

$$u_{in}(t) = \hat{U}_{in} \sin(\omega t + \varphi) \quad \Rightarrow \quad u_{ut}(t) = \hat{U}_{in} |H(\omega)| \sin(\omega t + \varphi + \arg\{H(\omega)\})$$

Passiva filter – Exempel 2(2)





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