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Modern Physics, TNE041

Exam, 5 June 2024, Answers and short solutions

1. (a) True, False, False, True.

   (b) Work function from $K_{max} = hc/\lambda - \phi =$ and $K_{max} = 0$ eV when $\lambda = 230$nm, yields $\phi = 5,39$ eV. When $\lambda = 180$nm one obtains $K_{max} = 1,5$ eV.

2. (a) The wave function is known $\psi(x) = \sqrt{2/a}\sin(n\pi x/a)$. The probability $\Pr(b \le x \le b + \Delta x) = \int_b^{b+\Delta x} |\psi(x)|^2 dx = (\Delta x)/a - (2n\pi)^{-1}[\sin(2n\pi(b + \Delta x)/a) - \sin(2n\pi b/a)]$. (b) When $n \to \infty$ one obtains $\Pr(b \le x \le b + \Delta x) \to (\Delta x)/a$, which is what to be expected if the electron had been a classical ball. This is an example of the *correspondence principle*, which is described in Harris.

3. (a) Let the wave function in empty space be $e^{ik_1 x} + Ae^{-ik_1 x}$ and in the metal $Be^{ik_2 x}$. From the Schrödinger equation one has $k_1 = \sqrt{2mE}/\hbar$ and $k_1 = \sqrt{2m(E + E_{Na})}/\hbar$, with $E = 0,1$ eV and $E_{Na} = 5,0$ eV. Continuity at $x = 0$ for ψ and its first derivative yields the equations $1 + A = B$ and $k_1 - Ak_1 = Bk_2$ from which we get $R = |A|^2 = [(k_2 - k_1)/(k_2 + k_1)]^2 = [(\sqrt{E + E_{Na}} - \sqrt{E})/(\sqrt{E + E_{Na}} + \sqrt{E})]^2 = 0,57$. (b) Classically, nothing reflects when it hits a drop, so $R = 0$.

4. (a) The m_s takes one of the values $-s, -s + 1, \cdots, s - 1, s$, that is, in this case one of the values $-3/2, -1/2, 1/2, 3/2$. (b) The length of $\mathbf{S}$ is $|\mathbf{S}| = \sqrt{s(s + 1)}\hbar = \sqrt{15}\hbar/2$. Its z-component is $S_z = m_s\hbar$. Least angle corresponds to largest value of m_s which is when $m_s = s = 3/2$). One obtains $\arccos(S_z/|\mathbf{S}|) = 39°$.

5. (a) Here $N(E)$ is the probability that the state with energy E is populated while $D(E)$ is the number of states per energy at the same energy E. The expression $\int_{E_1}^{E_2} N(E)D(E)dE$ is just the number of particles with an energy between E_1 and E_2. (b) The probability searched for is given by the ratio $\int_{0,9E_F}^{\infty} N(E)D(E)dE / \int_0^{\infty} N(E)D(E)dE$, where the denominator is just the total number of particles in the system. Since it is electrons we consider, Fermi-Dirac statistics apply. Room temperature implies the approximation of $N(E)$ with a step function, the Free-electron-model tells us $D(E) = \text{const.} \cdot \sqrt{E}$. The ratio becomes $\int_{0,9E_F}^{E_F} \sqrt{E}dE / \int_0^{E_F} \sqrt{E}dE = 1 - 0,9^{3/2} = 0,146$.

6. The energy levels of a "hydrogen atom" is (PH F6.4) $E_n = -me^4Z^2/(8\varepsilon_0^2 h^2 n^2)$. For the Se-atom here $Z = 1$ but one has to replace the permittivity ε_0 by $\varepsilon_0\varepsilon_r$ where the relative permittivity $\varepsilon_r = 13,5$. The energy to free a donor electron from the Se-atom is then the same to excite it from $n = 1$ to "$n = \infty$", i.e., $\Delta E = E_\infty - E_1 = 0 - (-13,6)/13,5^2$ eV $= 0,075$ eV. This is orders of magnitude less than the band gap 1,4 eV, and the the electrons from the donor atoms can be considered as free.