TSKS21 Signaler, information & bilder

Föreläsning 2
Likströmsteori, effektbegreppet
Växelströmsteori, introduktion

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Effektbegreppet

Grunduttryck: $P = UI$

Källor avger (vanligen) elektrisk effekt

Resistanser konsumerar elektrisk effekt

För ett helt nät gäller

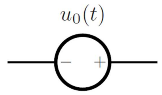
$$\sum_k P_k = 0$$

Växelströmsteori

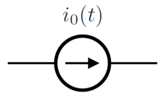
Tidsberoende storheter:

Spänning	$u(t)$
Ström	$i(t)$
Effekt	$p(t)$

Ideal spänningskälla

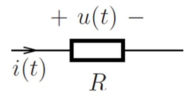


Ideal strömkälla



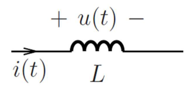
Växelströmsteori – Passiva komponenter

Resistans



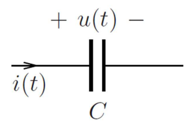
$$u(t) = Ri(t)$$

Induktans



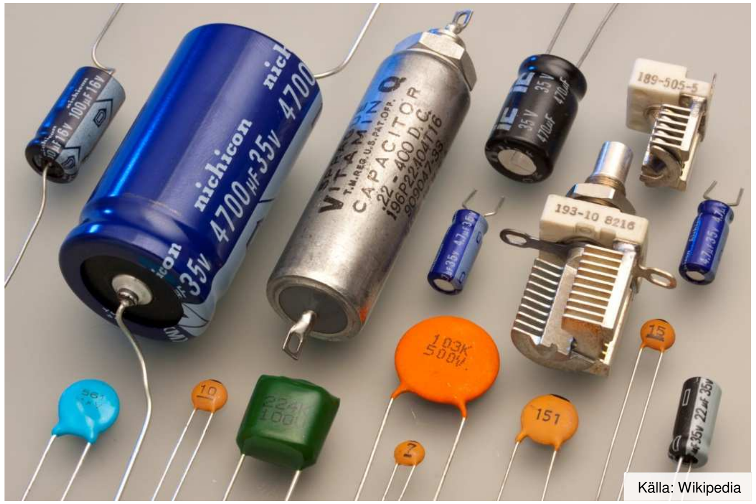
$$u(t) = L\frac{d}{dt}i(t)$$

Kapacitans



$$i(t) = C\frac{d}{dt}u(t)$$

Kapacitans – Kondensatorer



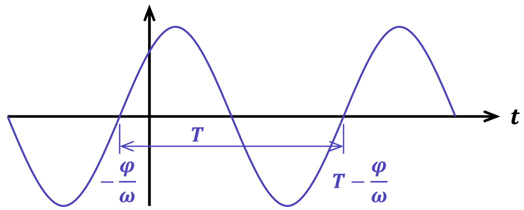
Källa: Wikipedia

Induktans – Spolar

Källa: Wikipedia

Stationär sinussignal

$$x(t) = \hat{X}\sin(\omega t + \varphi)$$



Symbol	Förklaring
$x(t)$	Momentanvärde
$\hat{X}$	Amplitud (toppvärde)
ω	Vinkelfrekvens [rad/s]
φ	Fasvinkel [rad]
T	Periodtid [s]
f	Frekvens [Hz]

$$f = \frac{1}{T} \qquad \omega = 2\pi f$$

Momentan effekt

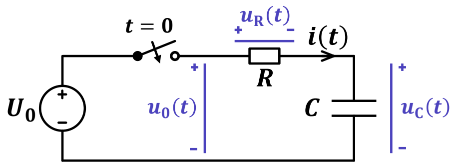
$$p(t) = u(t)i(t)$$

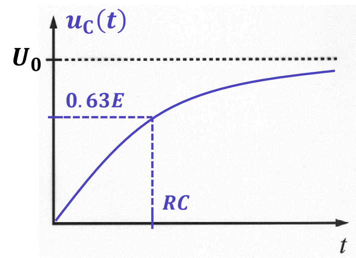
Aktiv effekt: $$P = \frac{1}{T}\int_0^T p(t)\,dt$$

Sinus

Effektivvärde: $$X_\mathrm{e} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt} = \frac{\hat{X}}{\sqrt{2}}$$

Uppladdning av en kapacitans





Initialtillstånd: $u_\mathrm{C}(0-) = 0$ $\quad u_0(t) = \begin{cases} 0, & t < 0, \\ U_0, & t \ge 0. \end{cases}$ $\quad u_\mathrm{C}(t) + u_\mathrm{R}(t) = u_0(t)$

$$u_\mathrm{R}(t) = Ri(t) = RC\frac{d}{dt}u_\mathrm{C}(t)$$

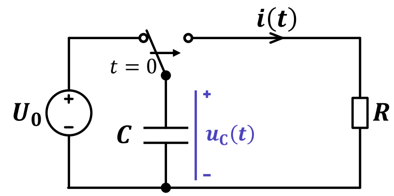
$$i(t) = C\frac{d}{dt}u_\mathrm{C}(t)$$

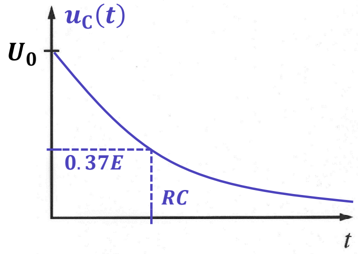
$t \ge 0$: $$u_\mathrm{C}(t) + RC\frac{d}{dt}u_\mathrm{C}(t) = U_0$$

Homogen och partikulär lösning ⇒

$$u_\mathrm{C}(t) = \left(1 - e^{-t/RC}\right)U_0$$

Urladdning av kapacitans





Initialtillstånd: $u_C(0-) = U_0$

$t \geq 0$:

$$u_C(t) = Ri(t) = -RC\frac{d}{dt}u_C(t)$$

$$i(t) = -C\frac{d}{dt}u_C(t)$$

$$u_C(t) + RC\frac{d}{dt}u_C(t) = 0$$

Homogen (och partikulär) lösning ⇒

$$u_C(t) = U_0 e^{-t/RC}$$

jω-metoden

1. Ersätt strömmar, spänningar och källor med deras komplexa motsvarigheter:

$$a(t) = \hat{A}\sin(\omega t + \varphi) \Rightarrow$$

$$A = \hat{A}e^{j\varphi} = b + jc$$

$$b = \hat{A}\cos\varphi \qquad c = \hat{A}\sin\varphi$$

2. Ersätt R, L, C med deras impedanser:

$$Z_L = j\omega L \qquad Z_C = \frac{1}{j\omega C} \qquad Z_R = R$$

3. Lös problemet med likströmsteori.

4. Gör omvändningen till punkt 1:

$$A = \hat{A}e^{j\varphi} = b + jc \Rightarrow$$

$$a(t) = \hat{A}\sin(\omega t + \varphi)$$

$$\hat{A} = \sqrt{b^2 + c^2}$$

$$\varphi = \arg(b + jc) = \operatorname{atan}\frac{c}{b} \quad (\pm\pi)$$

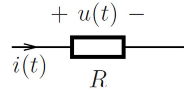
Om $b < 0$

Härledning jω-metoden 1(2)

$$u(t) = \hat{U}\sin(\omega t + \phi_u)$$
$$= \mathrm{Im}\left\{\hat{U}e^{j(\omega t + \phi_u)}\right\} = \mathrm{Im}\left\{\overbrace{\hat{U}e^{j\phi_u}}^{U}e^{j\omega t}\right\} = \mathrm{Im}\left\{Ue^{j\omega t}\right\}$$

$$i(t) = \hat{I}\sin(\omega t + \phi_i)$$
$$= \mathrm{Im}\left\{\hat{I}e^{j(\omega t + \phi_i)}\right\} = \mathrm{Im}\left\{\overbrace{\hat{I}e^{j\phi_i}}^{I}e^{j\omega t}\right\} = \mathrm{Im}\left\{Ie^{j\omega t}\right\}$$

Resistans



$$u(t) = Ri(t)$$

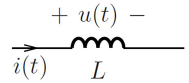
$$\mathrm{Im}\left\{Ue^{j\omega t}\right\} = R\,\mathrm{Im}\left\{Ie^{j\omega t}\right\} = \mathrm{Im}\left\{RIe^{j\omega t}\right\}$$

Lösning: $U = RI$

Impedans: $Z_R = R$

Härledning jω-metoden 2(2)

Induktans



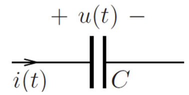
$$u(t) = L\frac{d}{dt}i(t)$$

$$\mathrm{Im}\left\{Ue^{j\omega t}\right\} = L\frac{d}{dt}\mathrm{Im}\left\{Ie^{j\omega t}\right\} = \mathrm{Im}\left\{LI\frac{d}{dt}e^{j\omega t}\right\} = \mathrm{Im}\left\{LIj\omega e^{j\omega t}\right\}$$

Lösning: $U = j\omega LI$

Impedans: $Z_L = j\omega L$

Kapacitans



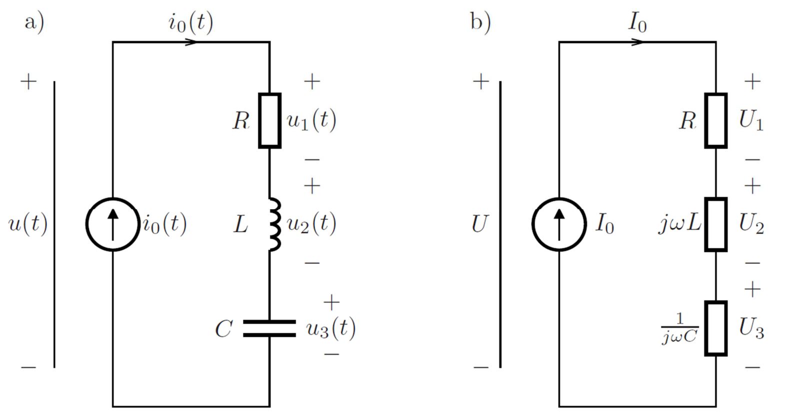
$$i(t) = C\frac{d}{dt}u(t)$$

$$\mathrm{Im}\left\{Ie^{j\omega t}\right\} = C\frac{d}{dt}\mathrm{Im}\left\{Ue^{j\omega t}\right\} = \mathrm{Im}\left\{CU\frac{d}{dt}e^{j\omega t}\right\} = \mathrm{Im}\left\{CUj\omega e^{j\omega t}\right\}$$

Lösning: $I = j\omega CU \Rightarrow U = \frac{1}{j\omega C}I$

Impedans: $Z_C = \frac{1}{j\omega C}$

Exempel jω-metoden



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