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Modern Physics, TNE046

Exam, 8 June 2023, Answers and short solutions

1. (a) False, energy is *always* conserved (but in QM sometimes uncertain for short time-intervals).
False, with $s = 1$ the electron would be a boson, not following the Pauli principle.
True, see Harris.
False, doping is mainly performed to increase the number of charge carriers.

   (b) The kinetic energy is given by $K = \gamma_u mc^2 - mc^2$, which gives the energy neccessary for the requested increase in speed to be $\Delta K = ((\gamma_{0,9c} - \gamma_{0,6c})mc^2 = 8,187 \cdot 10^{-14}$ J, gives us $V = 511$ keV.

2. The maximum amount of energy will be imparted to the electron in the case where the photon loses the maximum amount, that is, when the wavelength difference $\Delta\lambda = \lambda_C(1 - \cos\theta)$ is as greatest. This happens trivially when $\theta = \pi$, i.e., the photon changes direction totally. The energy it loses is 85 keV, so $hc/\lambda - hc/\lambda' = 85$ keV, which gives $\lambda^{-1} - \lambda'^{-1} = 6,855 \cdot 10^{10}\text{m}^{-1}$. This combined with $\Delta\lambda = \lambda' - \lambda = 4,85 \cdot 10^{-12}$ m results in $\lambda = 0,00659$ nm (a quadratic equation has to be solved along the way).

3. Make the ansatz $\Psi(x,t) = \psi(x)\phi(t)$ and insert into the time-dependent equation, see Harris section 5.2. The temporal part becomes $d\phi/dt = (-iE/\hbar)\phi$ (here E as the separation constant) which has the solution $\phi(t) = ae^{-i(E/\hbar)t}$, with a as an arbitrary constant of integration. Notice there is a misprint in Harris Eq. (5-6), a minus-sign is omitted.

4. The easiest solution is to translate the x-values through $x' = x + a/2$ and thereafter just do as "normal" for the infinite well between 0 and L. If one insists on keeping the original values, one has to use the boundary conditions $\psi(-a/2) = \psi(a/2) = 0$. This gives two sets of solutions; $\psi_n(x) = A\sin[(2n)\pi x/a]$, $n = 1, 2, \cdots$, and $\psi_n(x) = B\cos[(2n+1)\pi x/a]$, $n = 0, 1, 2, \cdots$, both with the same value of the normalization constant $A = B = \sqrt{2/a}$ for all n. Merging of the two sets

$$\psi_n(x) = \begin{cases} \sqrt{2/a}\sin(n\pi x/a) & n \geq 2 \text{ even} \\ \sqrt{2/a}\cos(n\pi x/a) & n \geq 1 \text{ odd} \end{cases} \qquad E_n = n^2\frac{\pi^2\hbar^2}{2ma^2}.$$

5. The spin is irrelevant for the spatial distribution, the wave function is $\psi_{1,0,0}(r,\theta,\varphi) = R_{1,0}(r)Y_{0,0}(\theta,\varphi)$. The probability is obtained as

$$\int_0^{a_0}\int_0^{\pi}\int_0^{2\pi} |R(r)|^2|Y_{0,0}(\theta,\varphi)|^2\, r^2\sin\theta\, d\varphi d\theta dr = \int_0^{a_0} |\frac{2}{a_0^{3/2}}e^{-r/a_0}|^2 r^2\, dr = 1 - 5e^{-2} \approx 0,323.$$

6. The average energy is given by

$$\bar{E} = \frac{\int_0^\infty E\, N(E)\, D(E)\, dE}{\int_0^\infty N(E)\, D(E)\, dE},$$

   where $N(E)$ is the mean occupation number, here for FD-statistics for low temperatures, which simplifies to $N(E) = 1$ for $E < E_F$ and $N(E) = 0$ for $E > E_F$. The quantity $D(E)$ is the density of states for an electron gas, in three dimensions proportional to $\sqrt{E}$. Together the expression above simplifies to

$$\bar{E} = \frac{\int_0^{E_F} E\sqrt{E}\, dE}{\int_0^{E_F} \sqrt{E}\, dE} = \frac{3}{5}E_F,$$

   where E_F is the Fermi energy.