TSKS21 Signaler, information & bilder

Föreläsning 9
Kvantisering och brus
Amplitudmodulering

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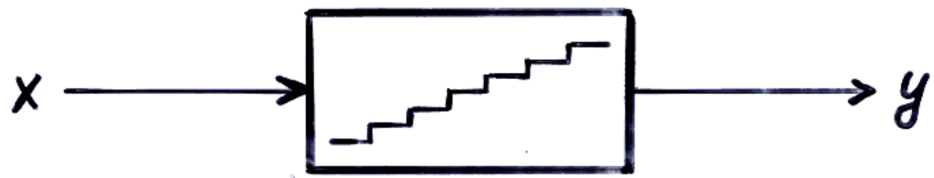
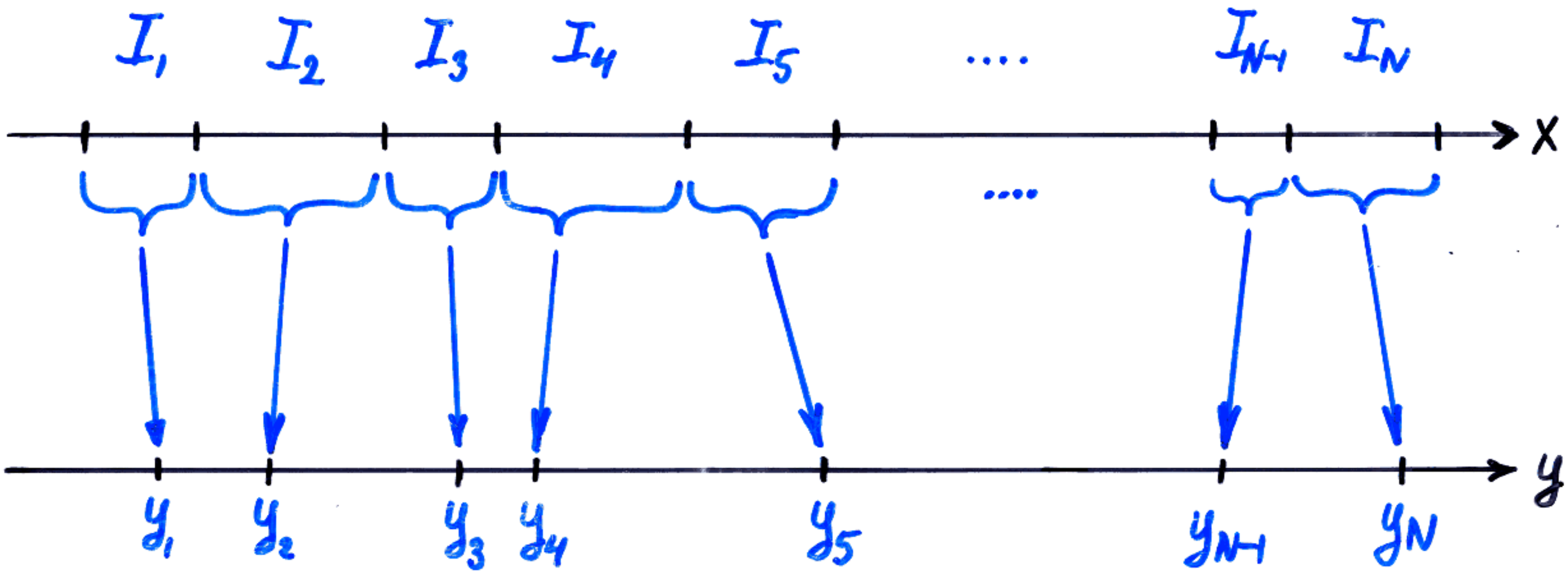
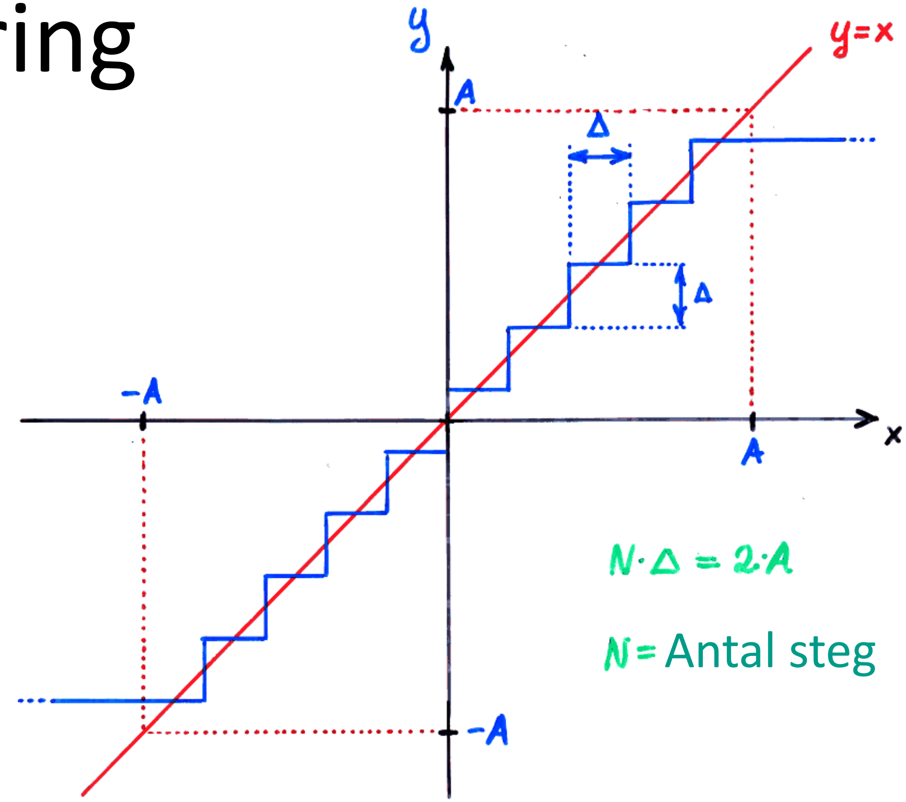
Principen för Kvantisering





Likformig Kvantisering

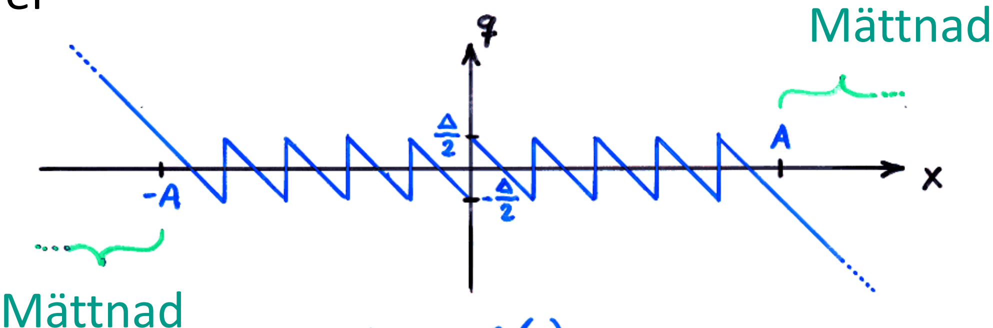
$$y = g(x) = \begin{cases} A - \frac{\Delta}{2}, & x > A \\ \frac{\Delta}{2} + \left\lfloor \frac{x}{\Delta} \right\rfloor \cdot \Delta, & |x| \leq A \\ -A + \frac{\Delta}{2}, & x < -A \end{cases}$$



$N \cdot \Delta = 2 \cdot A$

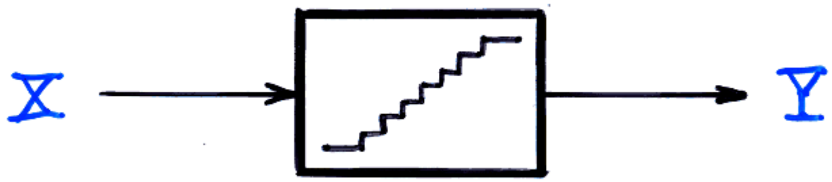
N = Antal steg

Kvantiseringsfel



$$q = y - x = g(x) - x.$$

Kvantiseringsdistorsion 1(2)



The error: $Q = Y - X = g(X) - X$

Quantization distorsion:

$$P_Q = E\{Q^2\} = E\{(g(X)-X)^2\} = \int_{-\infty}^{\infty} (g(x)-x)^2 \cdot f_X(x)\,dx$$

Assumptions:

1. No saturation: $f_X(x) = 0$ for $|x| \geq A$
2. Nice distribution: $f_X(x)$ continuous for $|x| < A$
3. Small Δ: $f_X(x)$ approx. const. in intervals of length Δ.

Kvantiseringsdistorsion 2(2)

$$P_Q = \int_{-A}^{A} (g(x)-x)^2 f_X(x)\,dx = \sum_{k=1}^{N} \int_{y_k-\frac{\Delta}{2}}^{y_k+\frac{\Delta}{2}} (y_k-x)^2 \cdot f_X(x)\,dx = \Big/ \begin{matrix} u = x - y_k \\ du = dx \end{matrix} \Big/$$

$$= \sum_{k=1}^{N} \int_{-\Delta/2}^{\Delta/2} u^2 f_X(y_k+u)\,du \underset{\substack{\uparrow \\ \Delta \text{ small} \\ u \text{ small}}}{\approx} \sum_{k=1}^{N} \int_{-\Delta/2}^{\Delta/2} u^2 \cdot f_X(y_k)\,du$$

$$= \sum_{k=1}^{N} f_X(y_k) \underbrace{\int_{-\Delta/2}^{\Delta/2} u^2\,du}_{=\Delta^3/12} = \frac{\Delta^2}{12} \sum_{k=1}^{N} \Delta \cdot f_X(y_k) \approx \frac{\Delta^2}{12} \underbrace{\sum_{k=1}^{N} Pr\{X \in I_k\}}_{=1} = \frac{\Delta^2}{12}$$

Error distribution: Approx. uniformly distr. over $\left[-\frac{\Delta}{2}, \frac{\Delta}{2}\right)$

Generally without saturation:

$P_Q \leq \frac{\Delta^2}{4}$ since $|Q| \leq \frac{\Delta}{2}$.

SDR för likformig kvantisering

SDR – Signal-till-Distorsions-Förhållande (Ratio)

Fortfarande begränsad till [-*A*,*A*] och med tillräckligt snäll fördelning.

$$P_Q = \mathrm{E}\left\{Q^2\right\} = \int_{-\Delta/2}^{\Delta/2} q^2 \frac{1}{\Delta}\, dq = \frac{\Delta^2}{12} = \frac{A^2}{3N^2} \qquad \Rightarrow \qquad \mathrm{SDR} = \frac{P_X}{P_Q} = \frac{3P_X}{A^2} N^2 = \frac{3P_X}{A^2} 2^{2n}$$

$$\mathrm{SDR_{dB}} = 10\log_{10}(\mathrm{SDR}) = 10\log_{10}\left(\frac{3P_X}{A^2}\right) + n \cdot 20\log_{10}(2) \approx 10\log_{10}\left(\frac{3P_X}{A^2}\right) + 6n.$$

Exempel: Likformig fördelning över [-*A*,*A*].

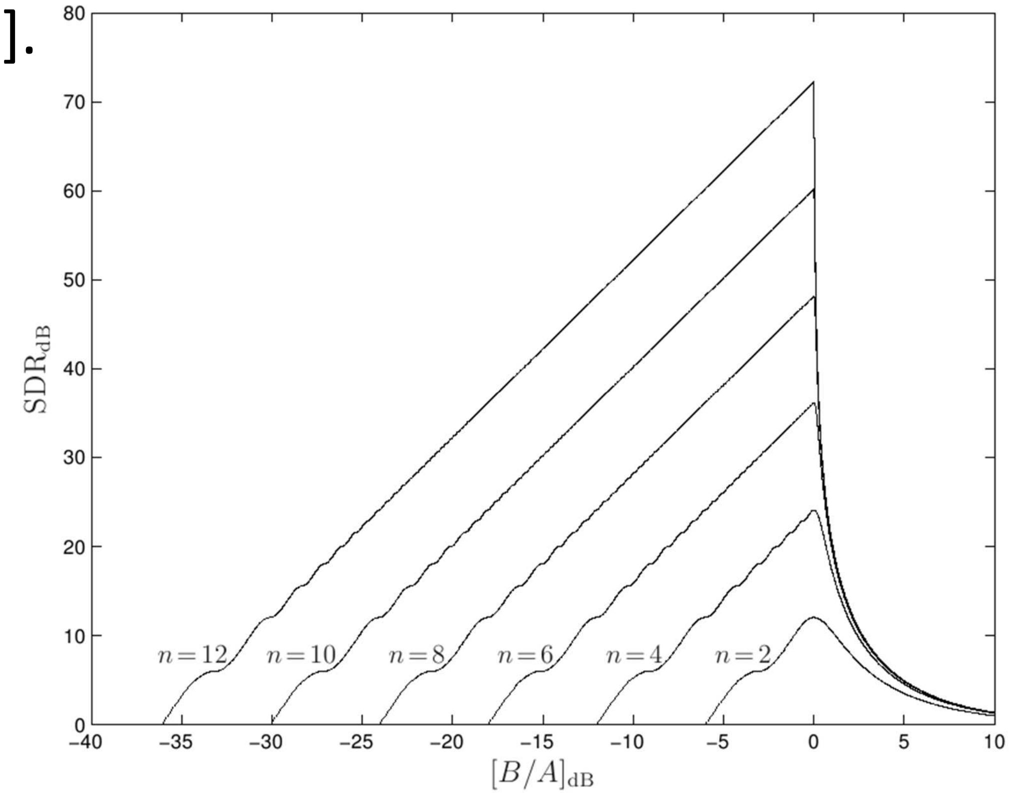
$$P_X = \mathrm{E}\left\{X^2\right\} = \int_{-A}^{A} x^2 \frac{1}{2A}\, dx = \frac{A^2}{3},$$

$$\mathrm{SDR_{dB}} \approx 10\log_{10}\left(\frac{3A^2/3}{A^2}\right) + 6n = 10\log_{10}(1) + 6n = 6n$$

SDR för likformig kvantisering med mättnad

Likformig fördelning över [-*B*,*B*].

$$P_X = \mathrm{E}\left\{X^2\right\} = \int_{-B}^{B} x^2 \frac{1}{2B}\, dx = \frac{B^2}{3}.$$



Q och S okorrelerade:

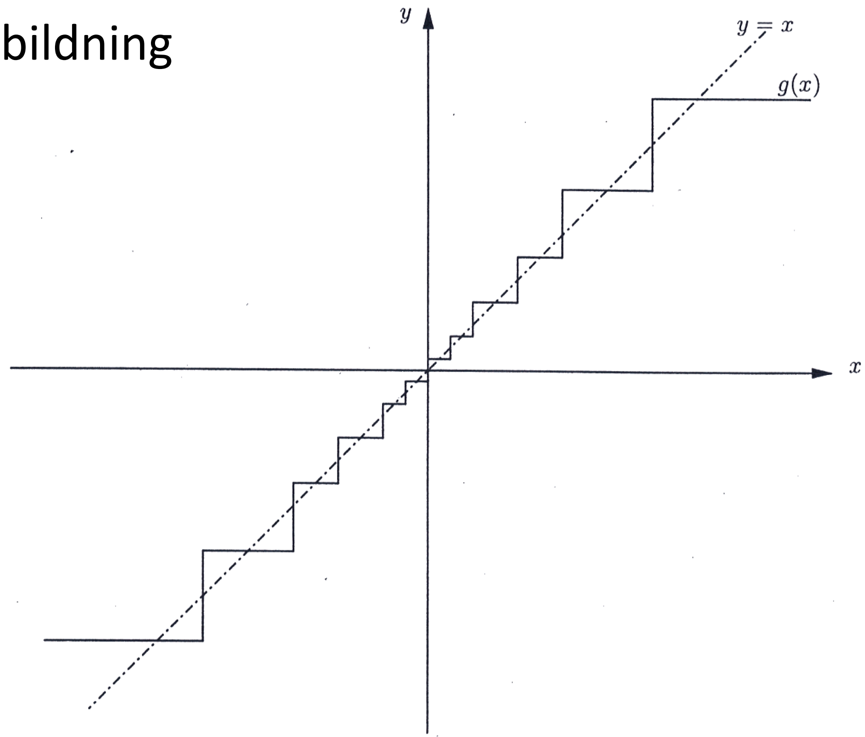
$$P_{Q+S} = P_Q + P_S.$$

$$\mathrm{SDR} = \frac{P_X}{P_Q + P_S},$$

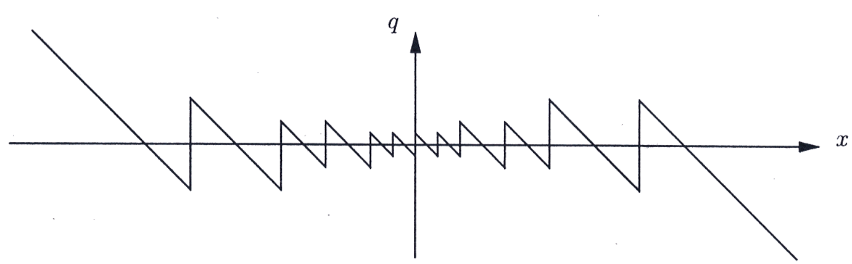
$$P_{Q+S} = \begin{cases} \frac{(2k+1)(\Delta/2)^3 + (B-(2k+1)\Delta/2)^3}{3B}, & k\Delta \le B < (k+1)\Delta,\ k \in \{0, 1, ..., N/2-2\} \\ \frac{A-\Delta/2}{B} \cdot \frac{\Delta^2}{12} + \frac{(B-A+\Delta/2)^3}{3B} & B \ge A - \frac{\Delta}{2} \end{cases}$$

Olikformig kvantisering

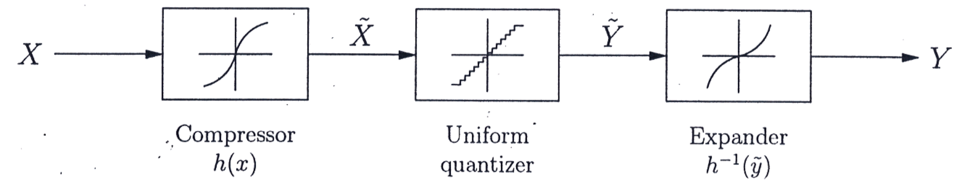
Avbildning

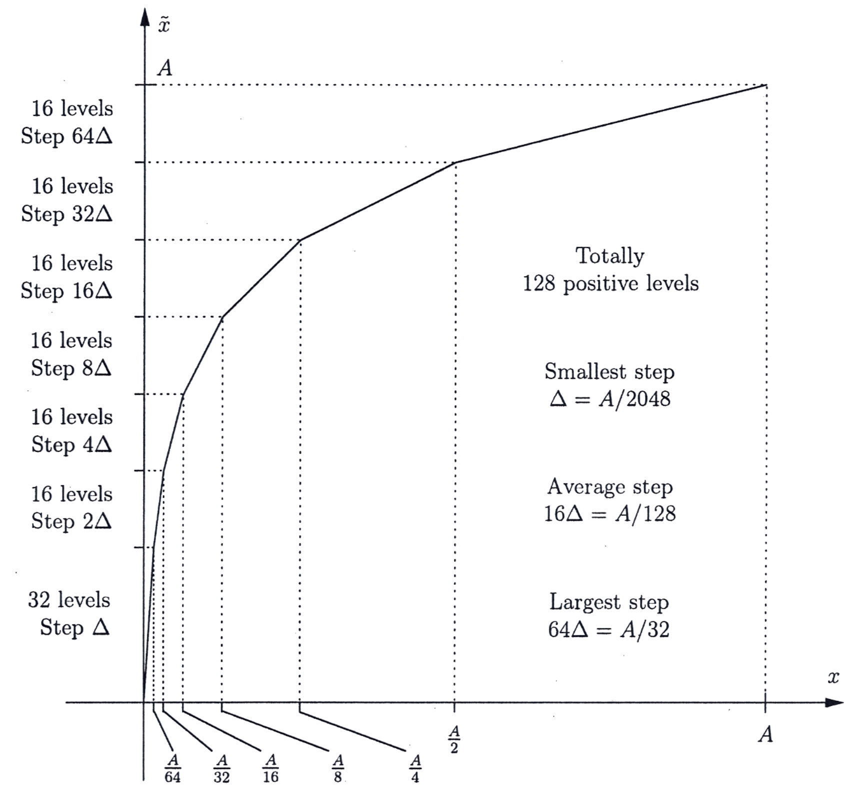


Kvantiseringsfel



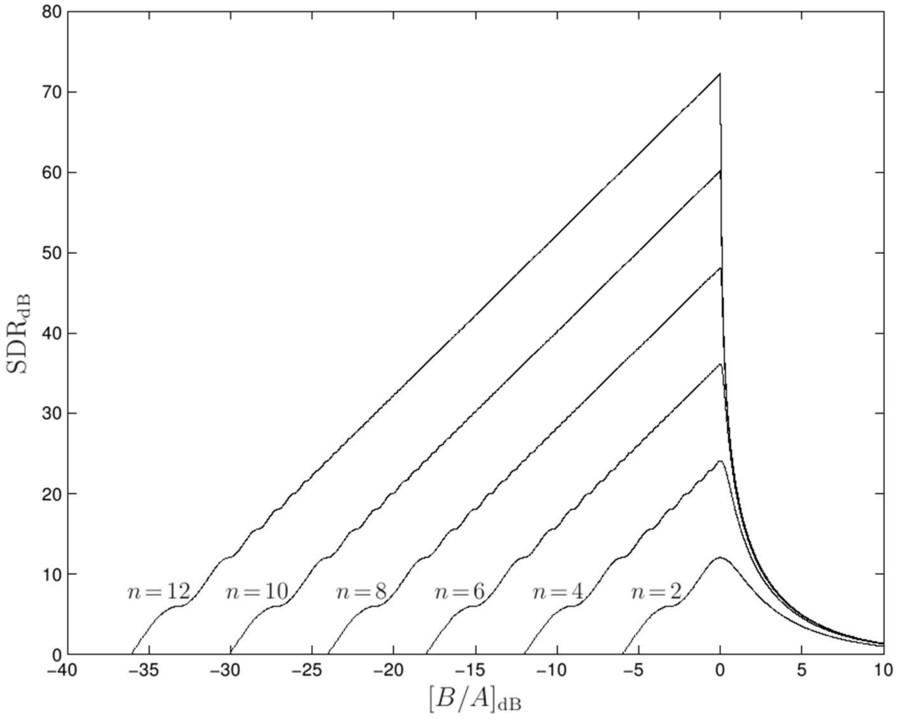
Implementering



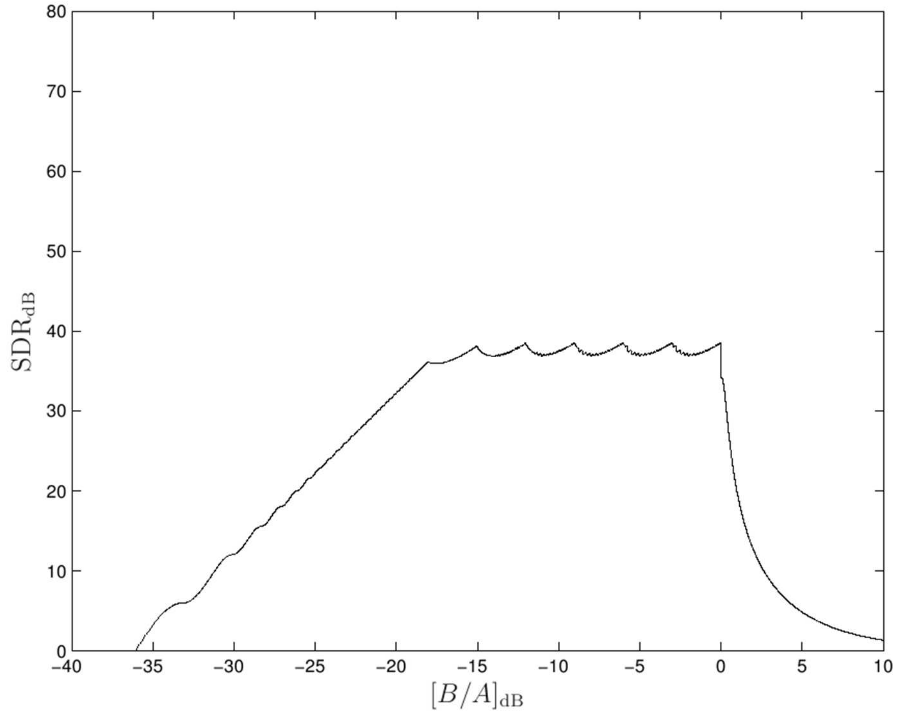


SDR för olikformig kvantisering

Likformig kvantisering

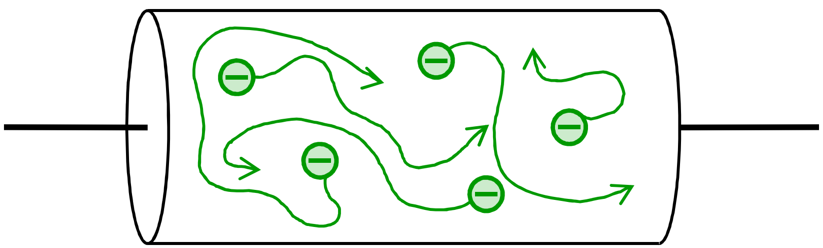


Olikformig kvantisering



Termiskt brus 1(3) – Fysiken bakom

Ett motstånd:



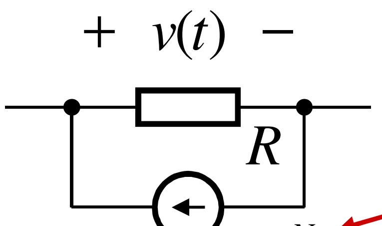
Termiska rörelser hos elektroner

$\Rightarrow$ Slumpmässiga lokala strömmar

$\Rightarrow$ Slumpmässiga lokala spänningar

$\Rightarrow$ Slumpmässig total spänning

Modell:



$$v(t) = R \cdot i(t)$$

Enorm

$$i(t) = \sum_{k=1}^{N} i_k(t)$$

$\Rightarrow$ $i(t)$ gaussisk

$\Rightarrow$ $v(t)$ gaussisk

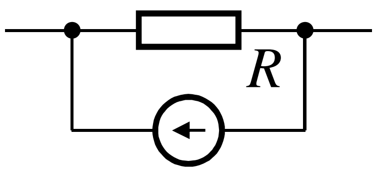
Korta pulser, nästan enhetsimpulser
$\Rightarrow$ $i(t_1)$ & $i(t_2)$ nästan oberoende
för $t_1 \neq t_2$

Vitt gaussiskt brus

Termiskt brus 2(3) – Spektraltäthet

Fortsatt modell:

$+ \ V(t) \ -$



Vitt gaussiskt brus med

$$R_V(f) = \frac{N_0}{2} = 2kTR$$

$k \approx 1.38 \cdot 10^{-23}$ J/K (Boltzmanns konstant)

T = Absolut temperatur i Kelvin.

R = Resistans i Ohm.

$h \approx 6.63 \cdot 10^{-34}$ Js (Plancks konstant)

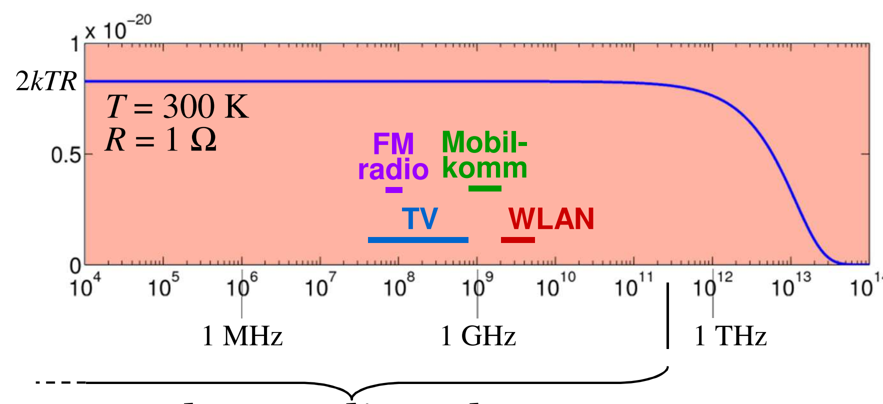
Mer exakt modell:

Gaussiskt brus med

$$R_V(f) = \frac{2Rh|f|}{e^{h|f|/kT} - 1}$$

Notera:

$$R_V(f) \to 2kTR \quad \text{när} \quad f \to 0$$



Reglerat radiospektrum

Termiskt brus 3(3) – Hur stort är stort?

Kol

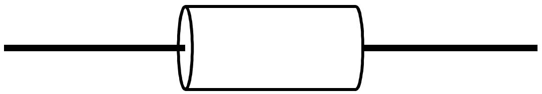
Densitet: 1.5 kg/dm³

#nukleoner: 12

Atommassa: $12 \cdot 1.7 \cdot 10^{-27}$ kg/atom

$$\Rightarrow \frac{1.5}{12 \cdot 1.7 \cdot 10^{-27}} = 7.4 \cdot 10^{25} \text{ atomer/dm}^3$$

Motstånd



Tvärsnittsarea: 0.25 mm²

Längd: 2 mm

$\Rightarrow$ Volym 0.5 mm³ = $0.5 \cdot 10^{-6}$ dm³

$$\Rightarrow 0.5 \cdot 10^{-6} \cdot 7.4 \cdot 10^{25} \approx 3.8 \cdot 10^{19} \text{ kolatomer}$$

Kol har 4 valenselektroner/atom

$$\Rightarrow \text{Totalt} \quad 4 \cdot 3.8 \cdot 10^{19} \approx 1.5 \cdot 10^{20} \text{ valenselektroner.}$$

Klassificering av frekvensselektiva filter

- Ett (frekvensselektivt) filter är ett LTI-system. Vanligen släpper det igenom något frekvensband eller spärrar det.
- Vanligen ett elektriskt nät – antingen passivt eller aktivt.

Notation | Ideal amplitudkarakteristik | Verklig amplitudkarakteristik

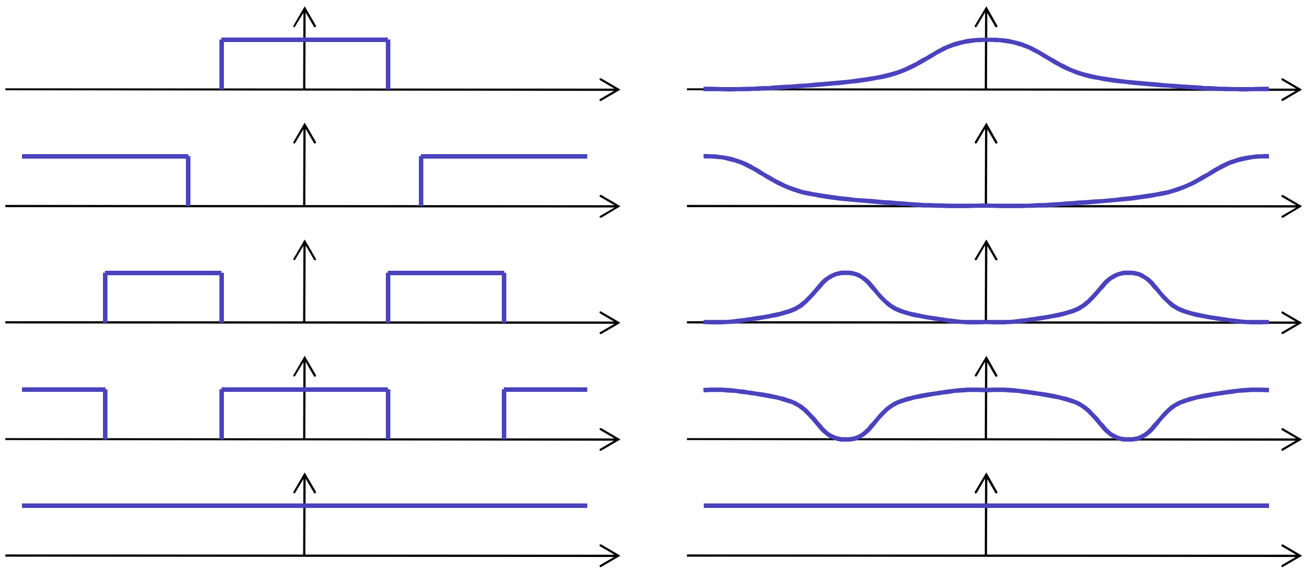
Lågpassfilter (LP-filter):

Högpassfilter (HP-filter):

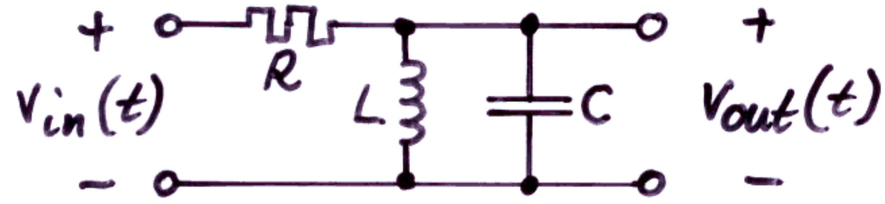
Bandpassfilter (BP-filter):

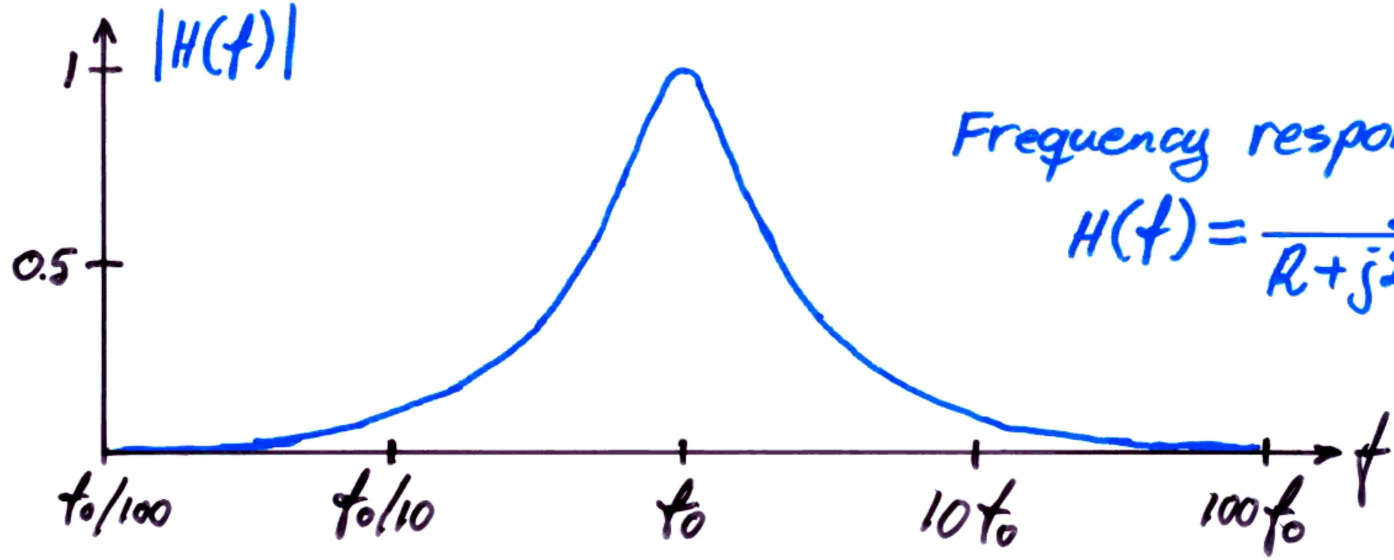
Bandspärrfilter (BS-filter):

Allpassfilter (AP-filter):



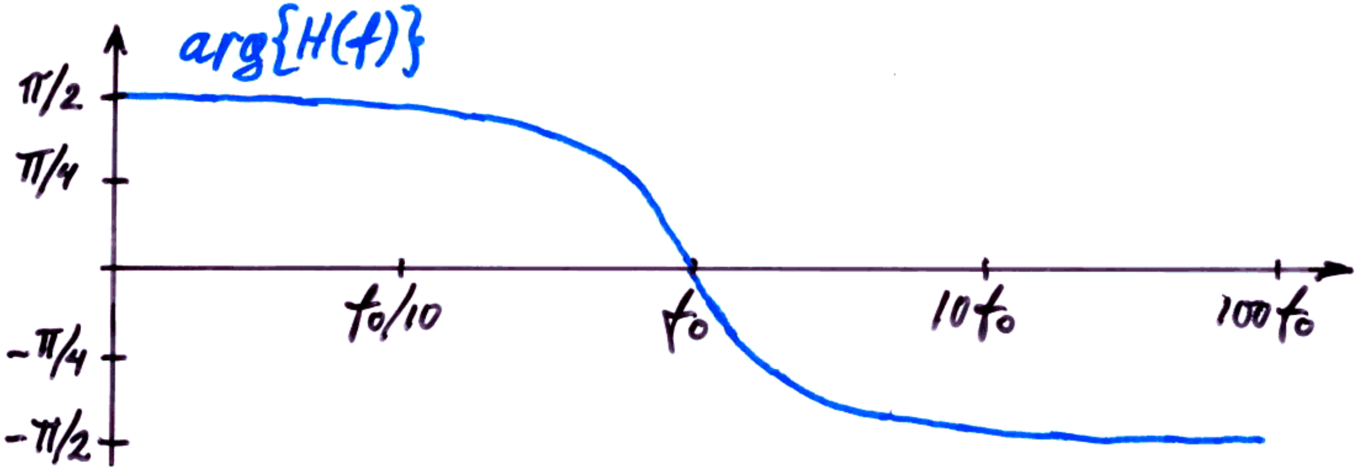
Ett BP-filter





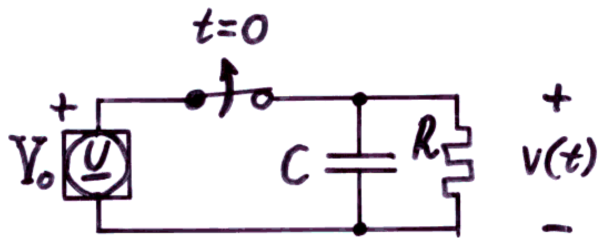
Frequency response:

$$H(f) = \frac{j2\pi fL}{R + j2\pi fL - 4\pi^2 f^2 RLC}$$



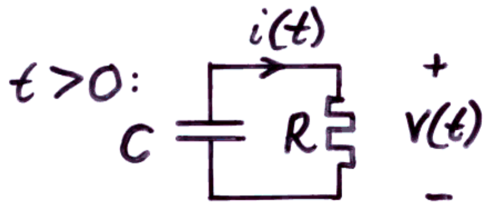
$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Urladdning av en kapacitans



Initial condition

$v(0) = V_0$



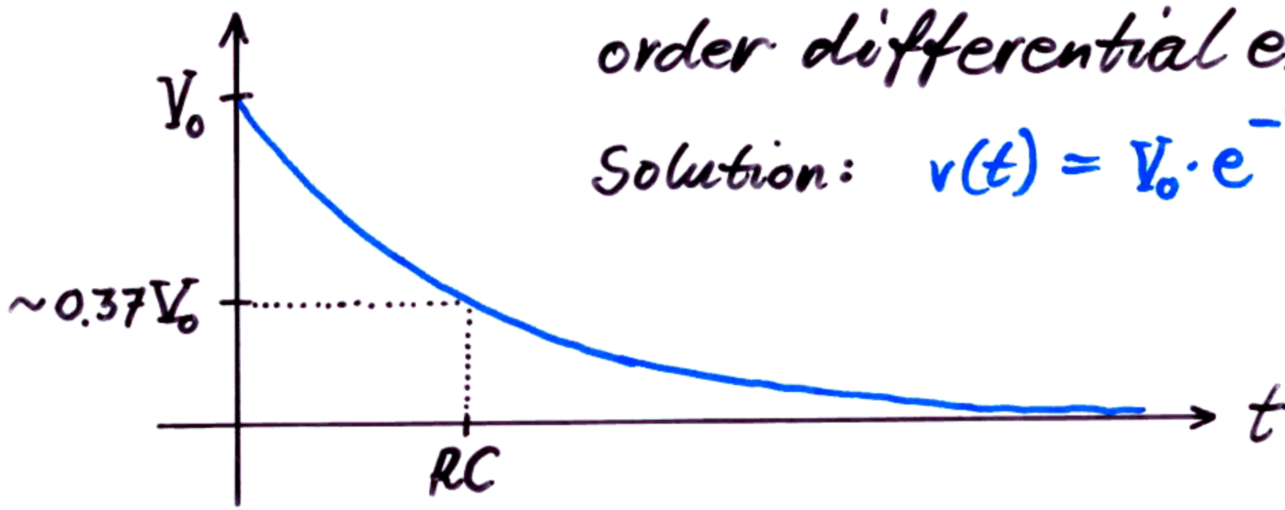
$v(t) = R \cdot i(t) = -RC \frac{d}{dt} v(t)$

$i(t) = -C \frac{d}{dt} v(t)$

$\Rightarrow RC \frac{d}{dt} v(t) + v(t) = 0$

This is the standard example of a first order differential equation.

Solution: $v(t) = V_0 \cdot e^{-t/RC}$ for $t \geq 0$



Amplitudmodulering

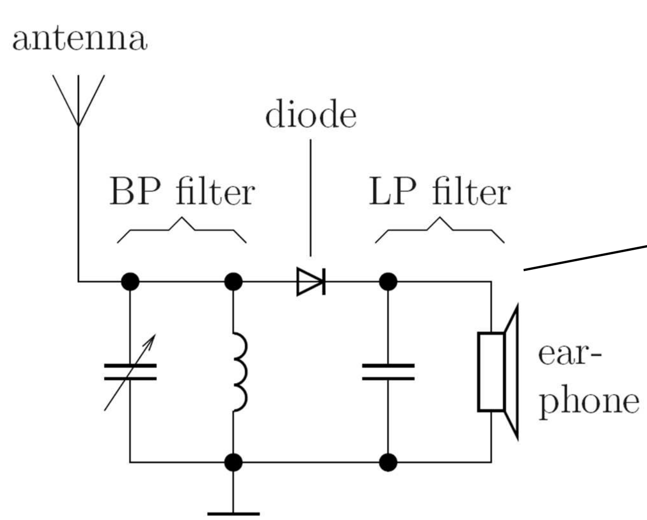
- Den första tekniken för rundradio.
- En linjär modulationsteknik.
- Enkel att analysera.
- Enkel demodulering.
- Bruskänslig.

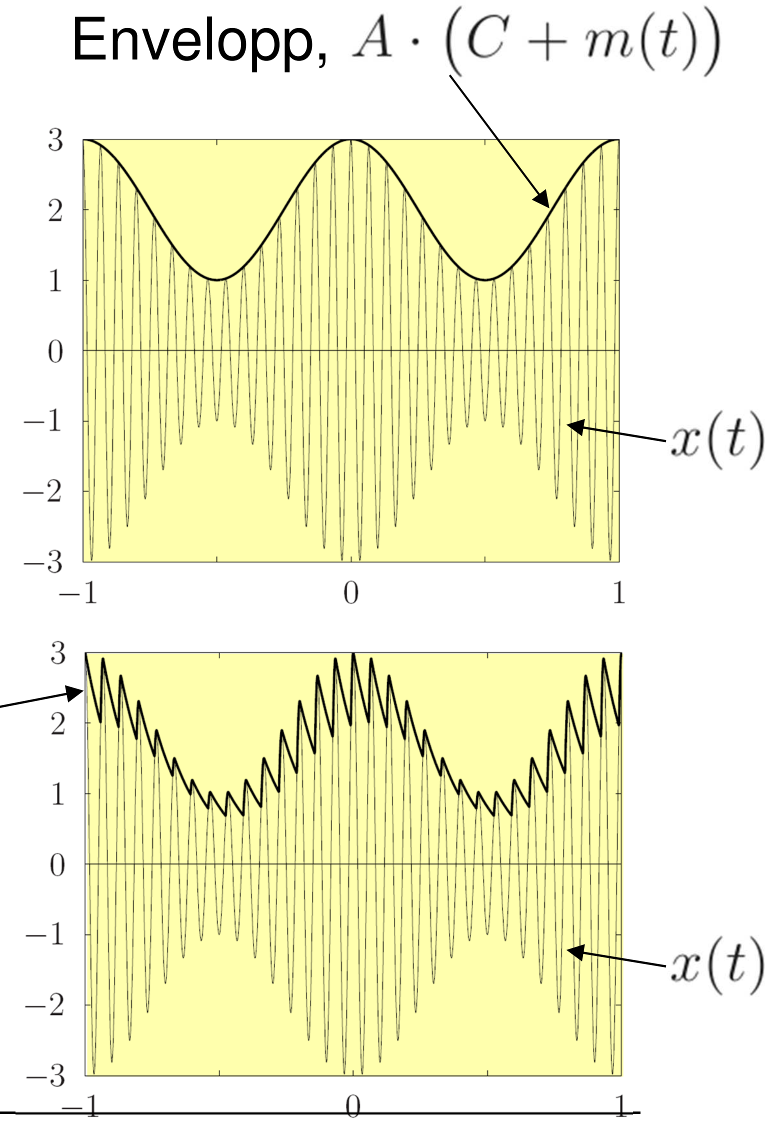
Amplitudmodulering i tidsdomänen

Standard AM:

$$x(t) = A \cdot \left(C + m(t)\right) \cos\left(2\pi f_c t\right)$$

Kristallmottagare, en enveloppdetektor, den första demodulatorn för standard AM:



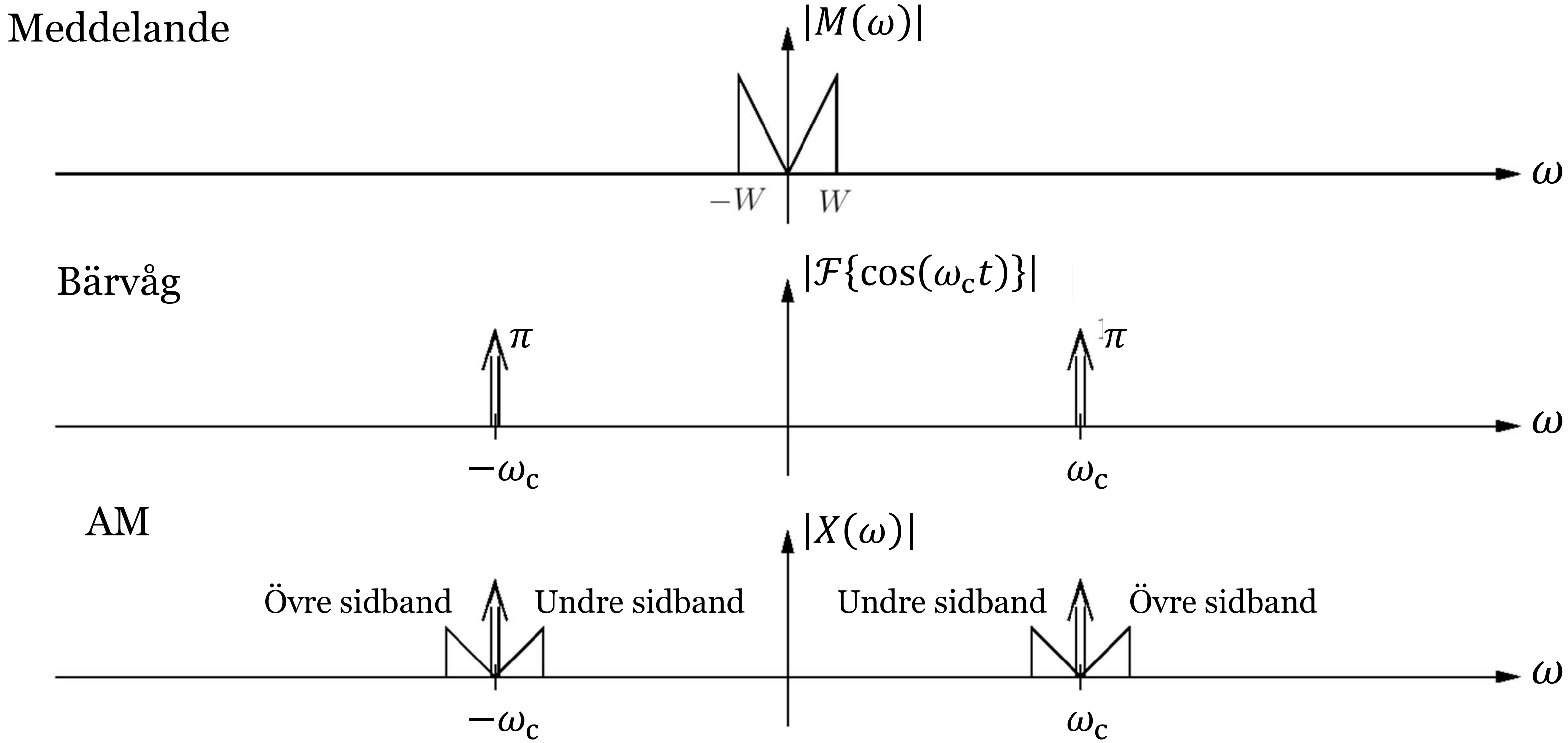


Spektrum för AM

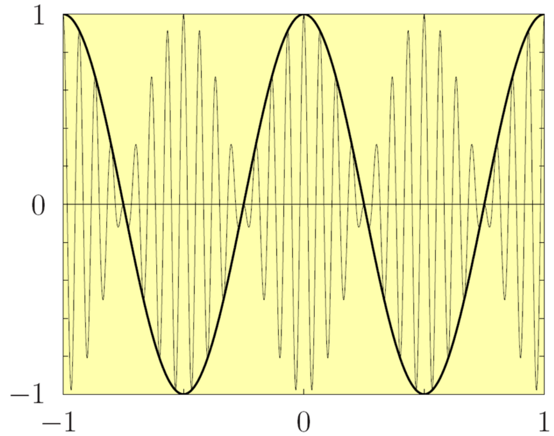
$$x(t) = A \cdot \big(C + m(t)\big)\cos(\omega_\mathrm{c} t)$$

$$X(\omega) = \mathcal{F}\{AC\cos(\omega_\mathrm{c} t)\} + \mathcal{F}\{Am(t)\cos(\omega_\mathrm{c} t)\}$$

$$= AC\pi\big(\delta(\omega + \omega_\mathrm{c}) + \delta(\omega - \omega_\mathrm{c})\big) + \frac{A}{2}\big(M(\omega + \omega_\mathrm{c}) + M(\omega - \omega_\mathrm{c})\big)$$



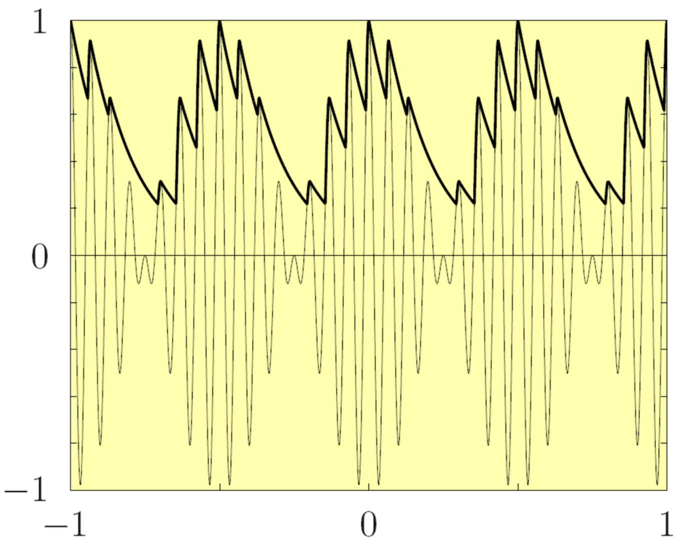
AM-SC – Undertryckt bärvåg



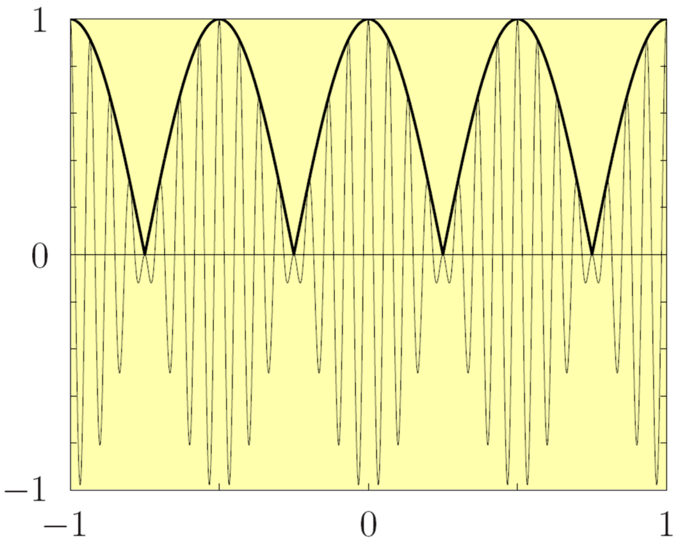
AM-SC: $x(t) = Am(t)\cos(\omega_c t)$

Demodulering av AM-SC med en enveloppdetektor:

Kristalmottagarens utsignal



Envelopp



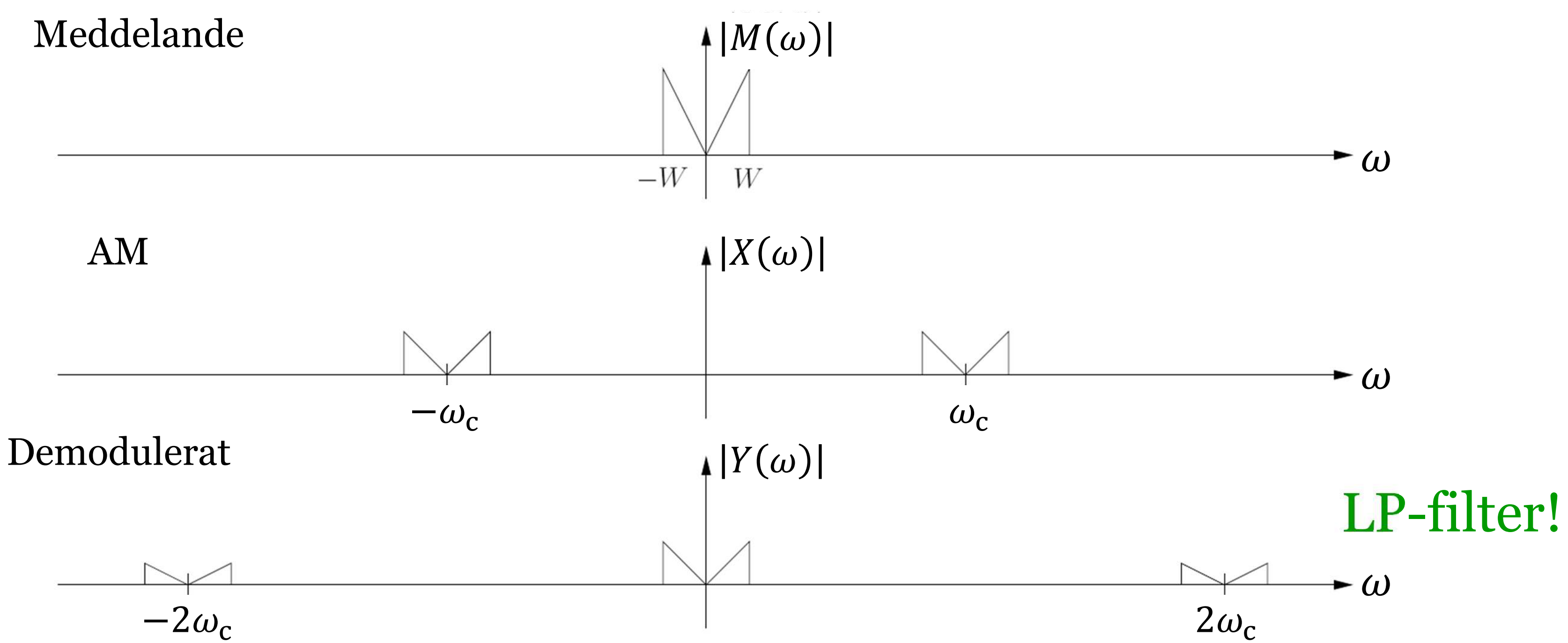
Spektrum för AM-SC – och Demodulering

$$x(t) = Am(t)\cos(\omega_c t) \quad \Rightarrow \quad X(\omega) = \frac{A}{2}\big(M(\omega+\omega_c) + M(\omega-\omega_c)\big)$$

Koherent demodulering:

$$y(t) = 2x(t)\cos(\omega_c t) = 2Am(t)\cos^2(\omega_c t) = Am(t)(1+\cos(2\omega_c t))$$

$$Y(\omega) = X(\omega+\omega_c) + X(\omega-\omega_c) = AM(\omega) + \frac{A}{2}\big(M(\omega+\omega_c) + M(\omega-\omega_c)\big)$$

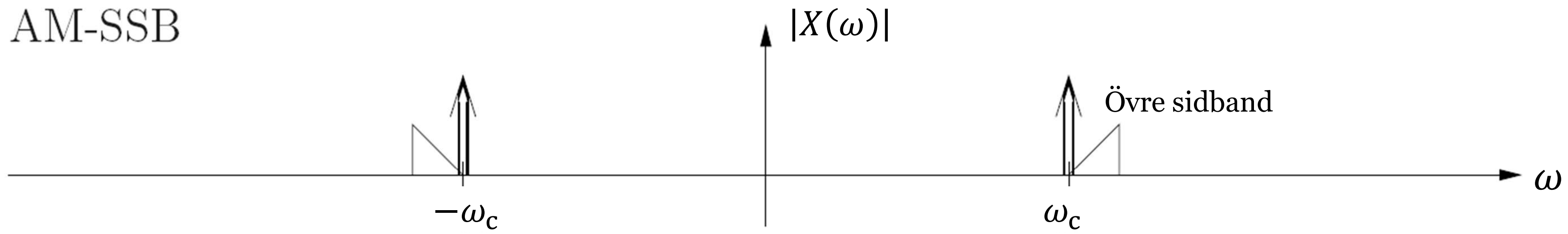


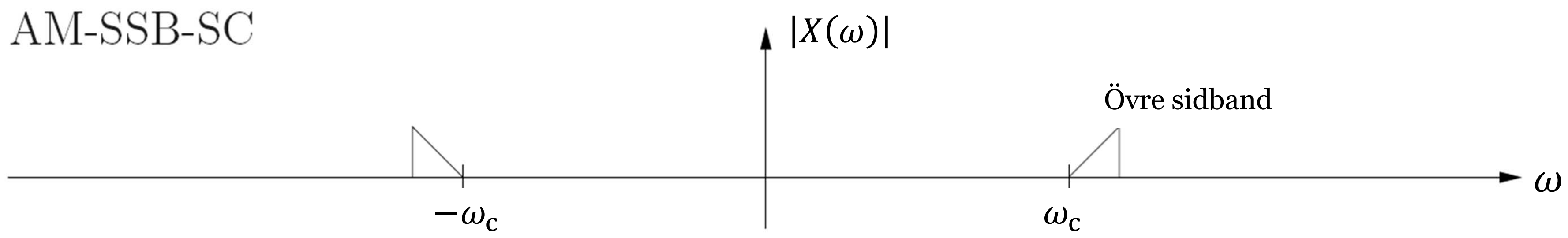
AM-SSB – Enkelt sidband

AM (SC) använder dubbelt så mycket bandbredd som behövs.

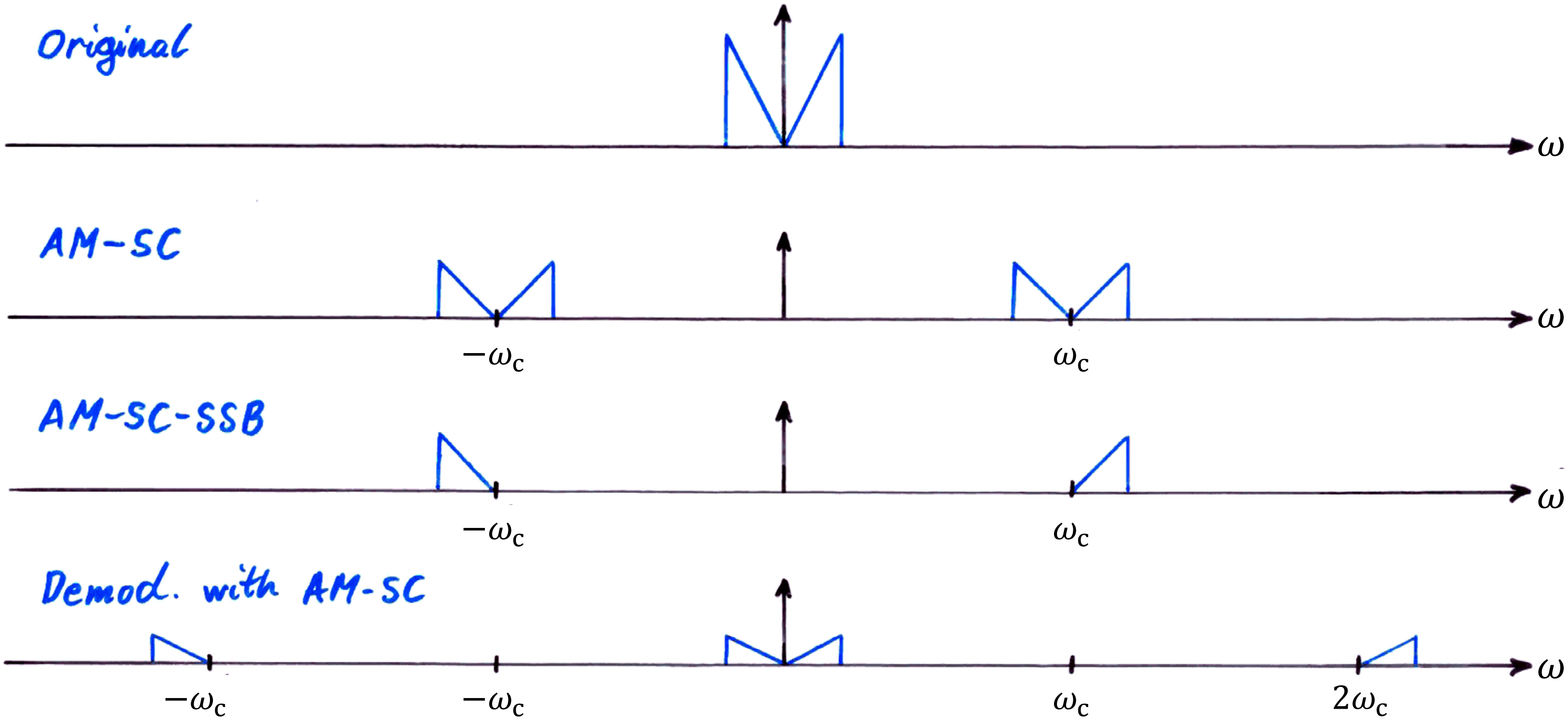
Vardera sidband innehåller all information.

Filtrera bort ett av sidbanden.





Demodulera SSB



LP-filter again!

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