Special relativity(2.1,2.7)

Inertial system(frame): A reference system where Newton´s first law (law of inertia) is valid.

An object moves with constant velocity if the net force is equal to zero.

Foundation of special relativity

2 postulates:

1. The laws of physics have the same (mathematical) form in all inertial systems.

2. The speed of light is the same relative to all inertial systems.

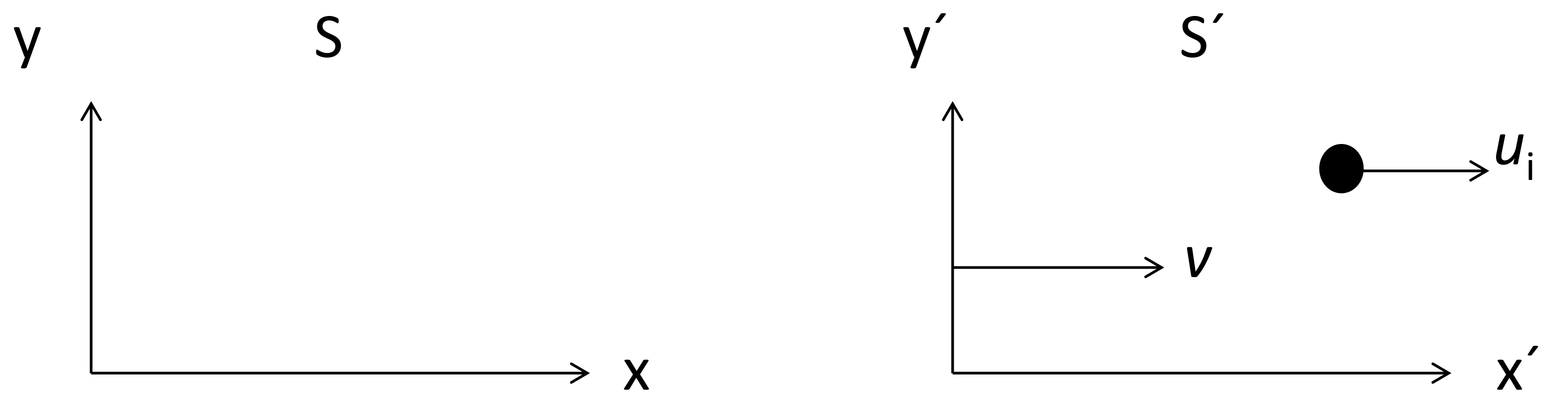
Focus in this course is on dynamics

From classical mechanics:

Momentum conservation law

The momentum $\overline{p}$ is constant if $\overline{F}_{ext} = \overline{0}$

$\overline{F}_{ext} = \frac{d\overline{p}}{dt}$ and $\overline{p} = m\overline{u}$ if speed *u* << c



Two inertial systems S and S´, particles moving in x direction

Speed u_i is relative to system S, speed relative to S´:

$u_i' = u_i - v$ **<u>if</u>** speed << c

$$\begin{cases} p_{tot} = \sum_i m_i u_i \\ p'_{tot} = \sum_i m_i u'_i \end{cases}$$

Lead to $p'_{tot} = p_{tot} - \sum_i m_i v$ Momentum in S´ is conserved if momentum in S is.

From experiments: The previous formulation of momentum conservation is not obeyed at high speeds!

Momentum p must be modified: $\overline{p} = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} m\overline{u}$

This will approach $m\overline{u}$ if u << c

$\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$ is called the gamma factor, γ or γ_u

The total energy E is conserved if

$$E = \frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} = \gamma_u mc^2$$ and if $u \to 0$, then $E \to mc^2$

In Harris: $E_{internal} = mc^2$

Physics Handbook (F-1.13) $E_{tot} = \gamma mc^2$

Kinetic energy $E_{kin} = \gamma_u mc^2 - mc^2$

A useful relation

$$\begin{cases} p = \gamma_u m u \\ E = \gamma_u m c^2 \end{cases}$$

Use these two equations (exercise) to eliminate u,result:

$$E^2 = (pc)^2 + \left(mc^2\right)^2$$

Special case: electromagnetic radiation(photons).No internal (rest) energy, total energy

$$E = pc$$

Check: Relativistic results should approach non-relativistic if

$u << c$

$$E_{kin} = \left(\frac{1}{\sqrt{1-\frac{u^2}{c^2}}} - 1 \right) mc^2$$

Compare with $\frac{1}{\sqrt{1-x^2}}$ for small values of x, is there a

suitable series expansion? Insert and keep the lowest order term, compare with non-relativistic E_{kin}.

When is relativistic dynamics necessary?

”Rule of thumb”: $\frac{E_{kin,rel} - E_{kin,cl}}{E_{kin,cl}} = 0.01$

1 % relative error

Insert $E_{kin,rel}$ and $E_{kin,cl}$

Conclusion:

Mass-energy equivalence

Electromagnetic radiation transports energy.

Energy passing a certain area during Δt :

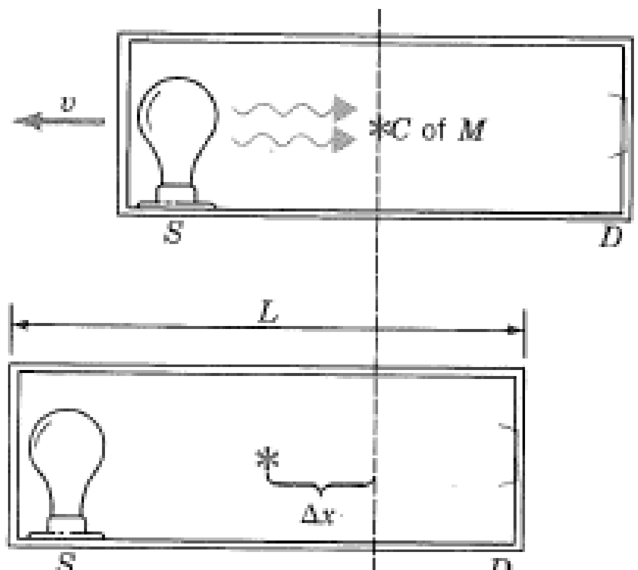
$E = P\Delta t$ (P : power, energy/time)

Linear momentum $p = \frac{P}{c}\Delta t$

"Force",cf *P=Fv*

Combine the relations above $\rightarrow E = pc$

Consider a box with a light source *S*, detector *D*, total mass *M*



”Recoil” when light is emitted, momentum is conserved:

$$\frac{E}{c} = Mv$$

If $v << c$ it takes a time $\Delta t = \frac{L}{c}$ for light to reach D. The box moves a distance $\Delta x = v\Delta t = \frac{EL}{Mc^2}$

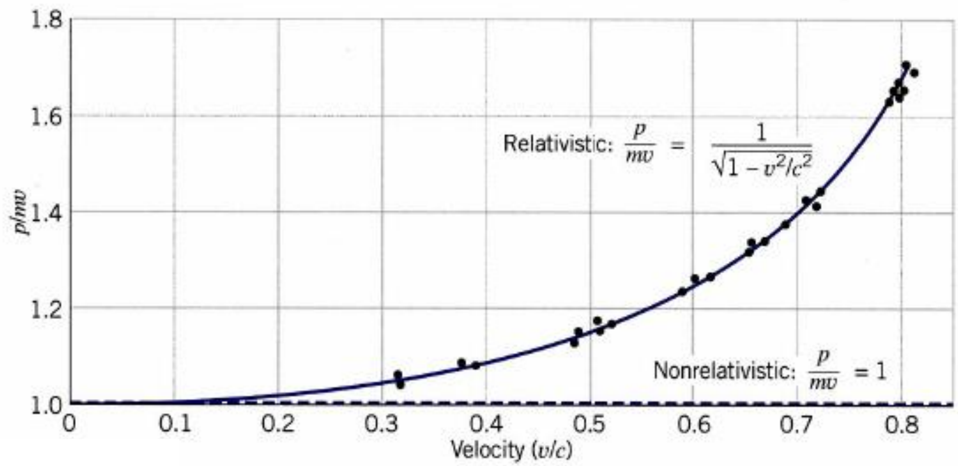
If the position of the center of mass is constant (no external forces), then

$$-M\Delta x + mL = 0$$

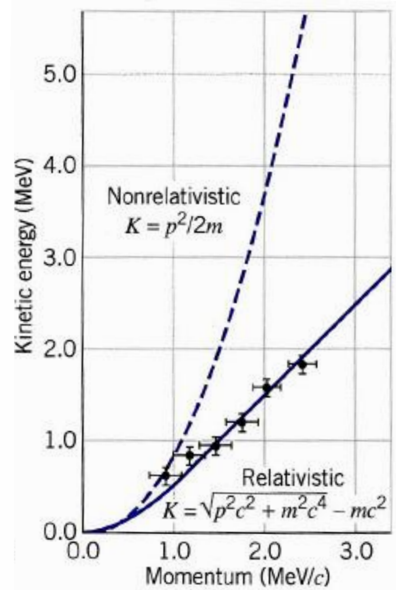
The ”mass” m is transferred and $m = \frac{M\Delta x}{L} = \frac{E}{c^2}$

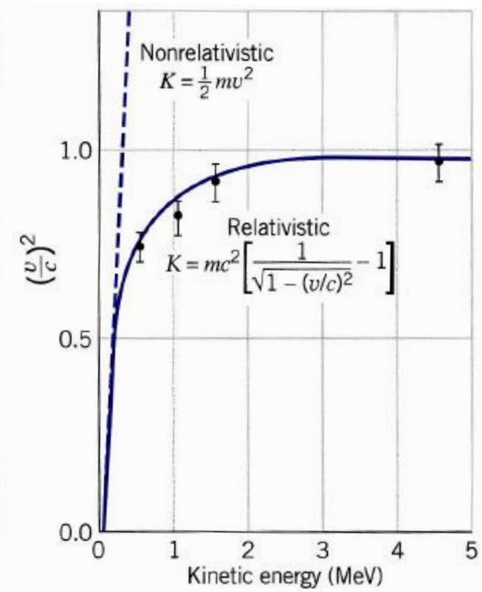
Result: $\boxed{E = mc^2}$

Relativistic momentum and energy, theory vs experiments



The ratio p/mv is plotted for electrons of various speeds. The data agree with the relativistic result and not at all with the nonrelativistic result ($p/mv = 1$).

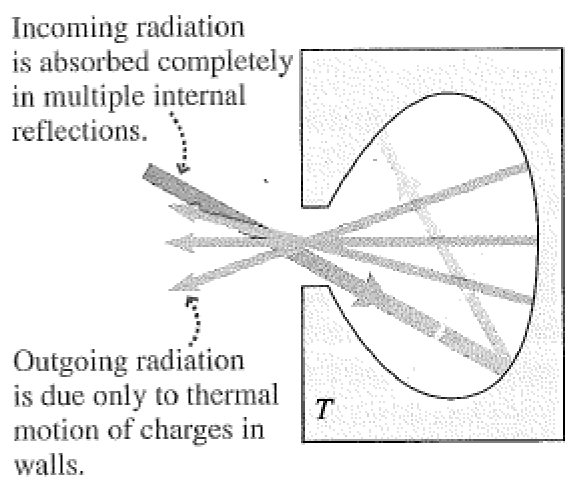




Examples of electromagnetic radiation behaving as particles (Chapter 3)

Blackbody radiation (3.1)

Figure 1 Radiation exits a cavity through a hole, which behaves as a blackbody.



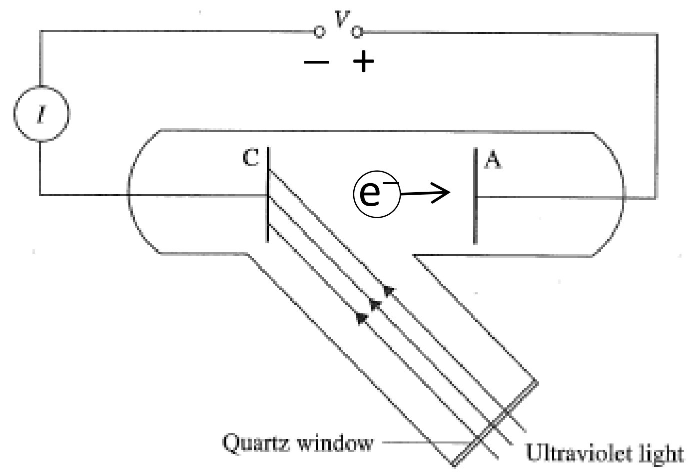
Blackbody: complete absorption of incoming radiation and all emitted radiation is due to thermal motion of charges.

The observed electromagnetic energy/frequency (spectral energy density) emitted cannot be explained by classical wave theory.

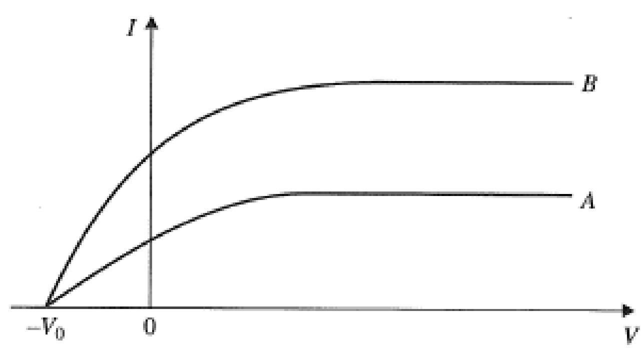
Assumption (Planck): Energy emitted $E = nhf$

n : integer h : Planck´s constant f : frequency

The photoelectric effect(3.2)



Schematic drawing of Lenard's apparatus for investigating the photoelectric effect.



Variation of the photoelectric current I with the potential difference V between the cathode and the anode, for two values $A < B$ of the intensity of the light incident on the cathode. No current is observed when V is less than $-V_0$; the stopping potential V_0 is found to be independent of the light intensity.

Classical wave theory

1. The maximum kinetic energy of the photoelectrons, $E_{kin,max}$, is proportional to the intensity of the incident light.

2. The photoelectric effect should in principle be observable for light of all frequencies (wavelengths).

3. There should be a measurable time delay before photoelectrons are ejected.

Experiments

1. $E_{kin,max}$ does not depend on the intensity.

2. No effect is observed if light frequency $f < f_{min}$ (Threshold frequency).

3. There is no measurable time delay.

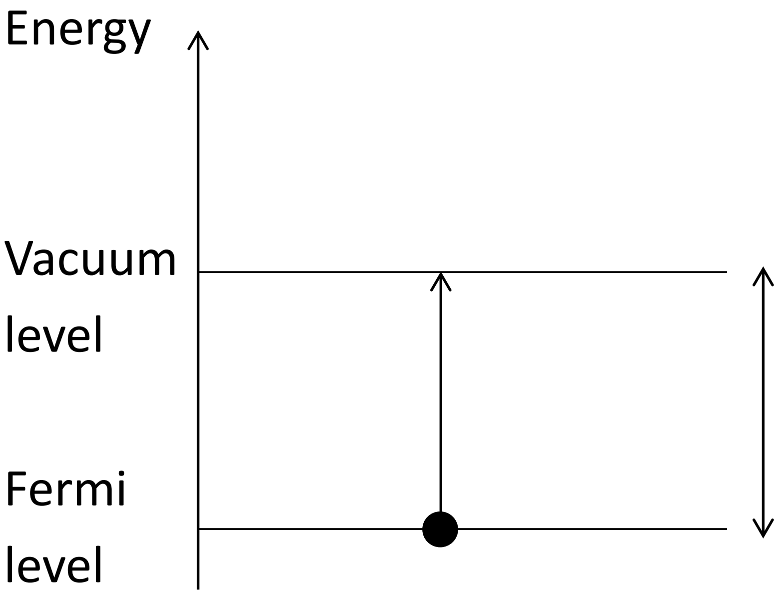
Conclusion: The classical wave model is not sufficient in order to explain experimental data.

Einstein 1905:

Photons (light quanta) with energy $E = hf$

$h = 6.626 \cdot 10^{-34} Js$, Planck´s constant

One photon ejects one electron. Not possible to ”collect” photons.



Work function ϕ

Typically a few eV

1 eV = $1.602 \cdot 10^{-19}$ J

Important relations:

$$E_{kin,max} = hf - \phi \qquad eV_0 = E_{kin,max}$$

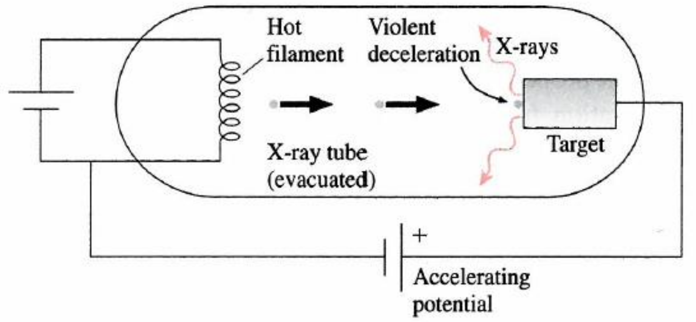
V_0 : Stopping voltage

$$V_0 = \frac{hf}{e} - \frac{\phi}{e}$$

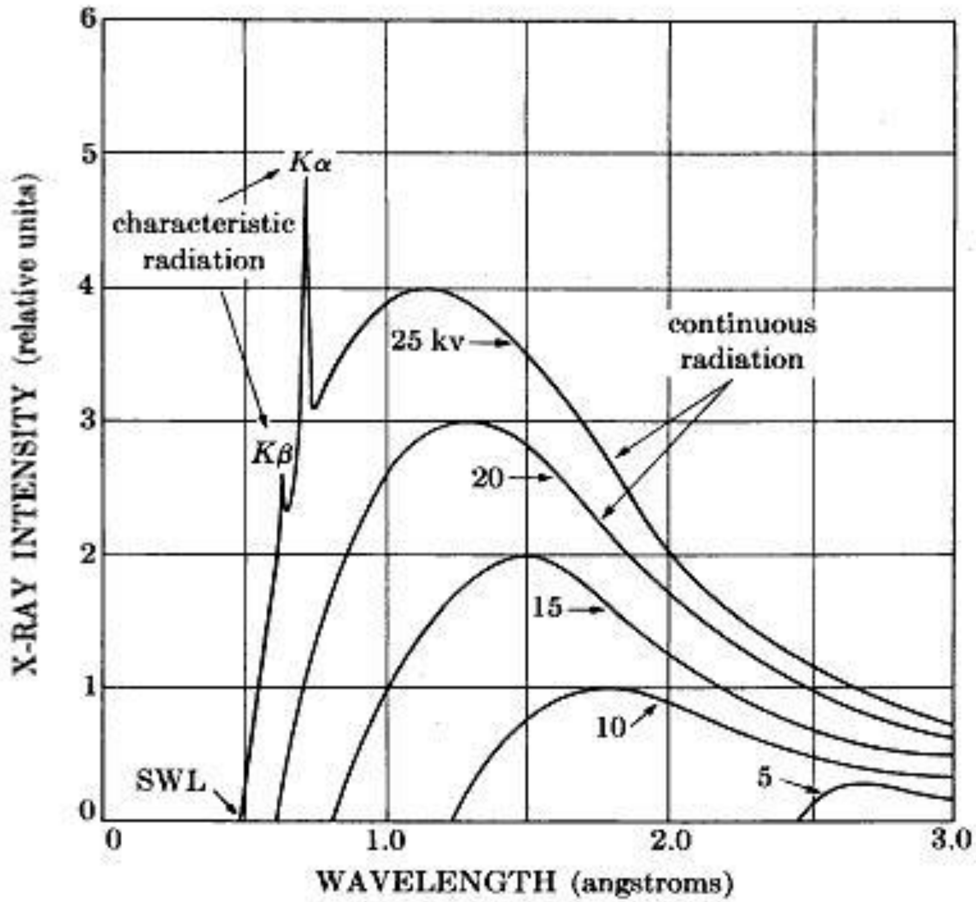
X-rays (röntgenstålning,3.3)

- Energetic electromagnetic radiation
- Emitted when electrons are accelerated and hit a metal target
- Kinetic energy of electrons typically several keV
- Continuous (bremsstrahlung) and characteristic radiation

Figure 6 X-rays are produced when electrons "boiled" off a hot filament are accelerated into a metal target.



Example of x-ray intensity vs wavelength (Molybdenum target)



Classically: No reason to expect a minimum (cutoff) wavelength.

Maximum photon energy: $E_{max} = hf_{max} = \frac{hc}{\lambda_{min}}$

An electron produces one photon, all kinetic energy is converted to photon energy.