TSEK03: Radio Frequency Integrated Circuits (RFIC)

Lecture 8 & 9: Oscillators

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Overview

- Razavi: Chapter 8:
  - 8.1 Performance Parameters
  - 8.2 Basic Principles
  - 8.3 Cross-coupled Oscillator
  - 8.4 Three-point Oscillators
  - 8.5 VCO
  - 8.7 Phase Noise (parts thereof)

- Lee: Chapter 17: Very different theoretical approach, many oscillator types.

CHAPTER 8 OSCILLATORS 497

8.1 Performance Parameters 497
8.2 Basic Principles 501
8.2.1 Feedback View of Oscillators 502
8.2.2 One-Port View of Oscillators 508
8.3 Cross-Coupled Oscillator 511
8.4 Three-Point Oscillators 517
8.5 Voltage-Controlled Oscillators 518
8.5.1 Tuning Range Limitations 521
8.5.2 Effect of Varactor Q 522
8.6 LC VCOs with Wide Tuning Range 524
8.6.1 VCOs with Continuous Tuning 524
8.6.2 Amplitude Variation with Frequency Tuning 532
8.6.3 Discrete Tuning 532
8.7 Phase Noise 536
8.7.1 Basic Concepts 536
8.7.2 Effect of Phase Noise 539
8.7.3 Analysis of Phase Noise: Approach I 544
8.7.4 Analysis of Phase Noise: Approach II 557
8.7.5 Noise of Bias Current Source 565
8.7.6 Figures of Merit of VCOs 570
8.8 Design Procedure 571
8.8.1 Low-Noise VCOs 573
8.9 LO Interface 575
8.10 Mathematical Model of VCOs 577
8.11 Quadrature Oscillators 581
8.11.1 Basic Concepts 581
8.11.2 Properties of Coupled Oscillators 584
8.11.3 Improved Quadrature Oscillators 589
8.12 Appendix I: Simulation of Quadrature Oscillators 592
References 593
Problems 594

8.1 Performance Parameters

- Performance parameters discussed in this section:
  - frequency range
  - output voltage swing
  - drive capability
  - phase noise (more in section 8.7)
  - output waveform
  - supply sensitivity
  - power dissipation

Performance Parameters: Frequency

- An RF oscillator must be designed such that its frequency can be varied (tuned) across a certain range. This range includes two components:

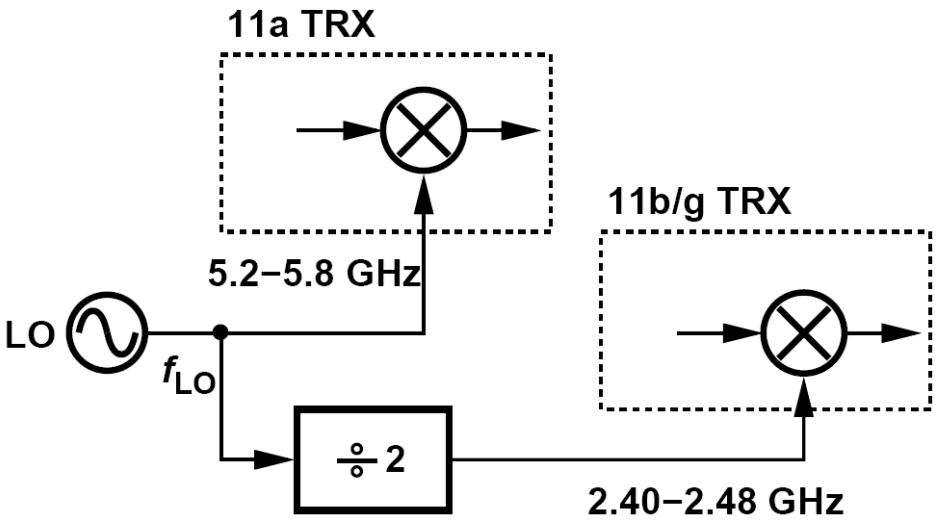
  (1) the system specification,

  (2) additional margin to cover process and temperature variations and errors due to modeling inaccuracies.

- IQ required? 2x frequency can be used, then ÷2.

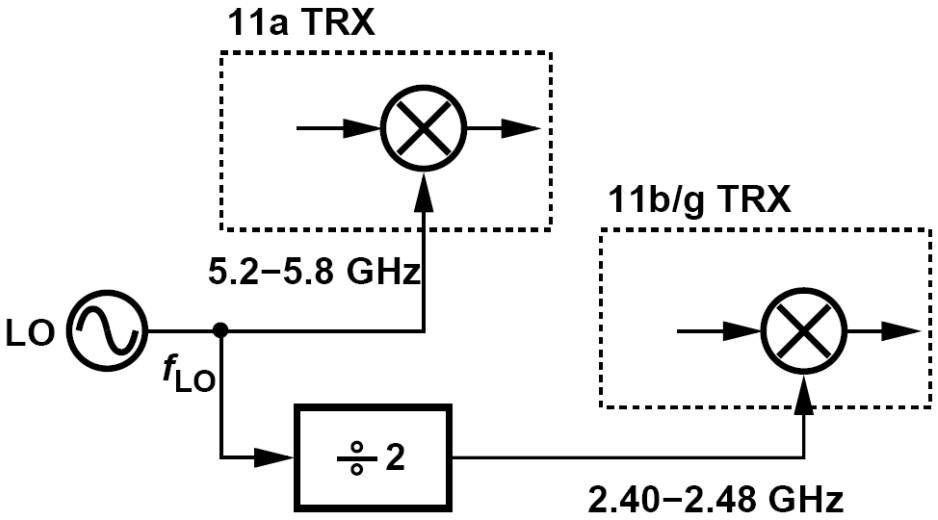
Example 8.1

- A direct-conversion transceiver is designed for the 2.4-GHz and 5-GHz wireless bands. If a single LO must cover both bands, what is the minimum acceptable tuning range?



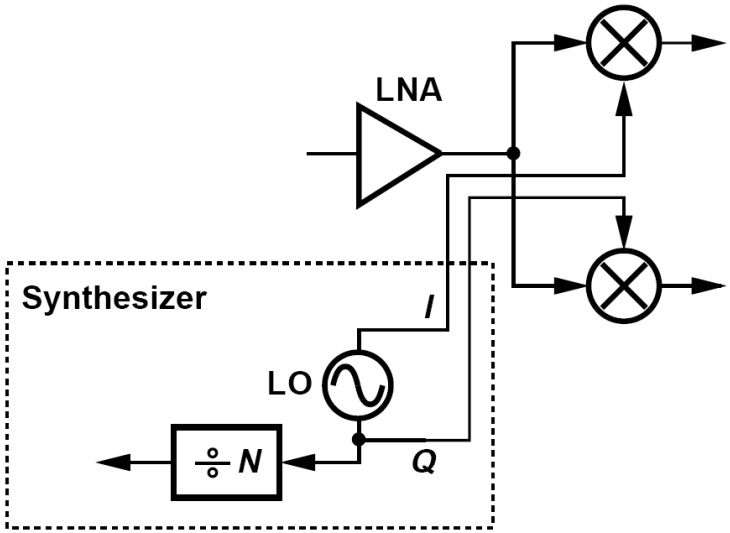
Example 8.1

- For the lower band, we must generate 4.8 GHz ≤ f_{LO} ≤ 4.96 GHz (2400-2480 MHz). Thus, we require a total tuning range of 4.8 GHz to 5.8 GHz, about 20 %. Such a wide tuning range is relatively difficult to achieve in LC oscillators.



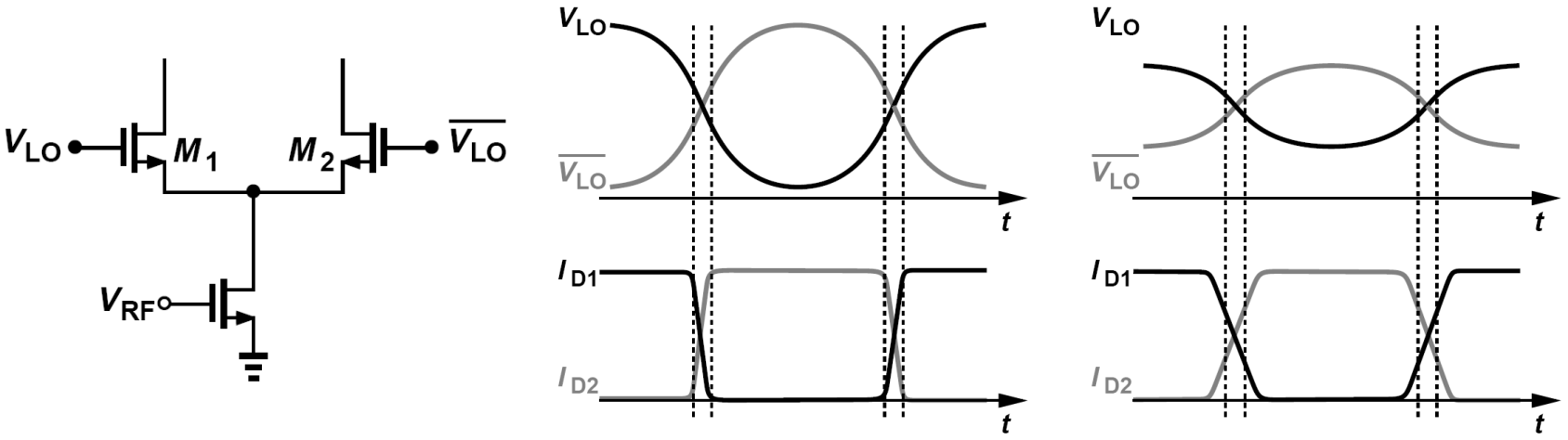
Performance Parameters: Output swing

- The oscillator must produce sufficiently large output swing to ensure nearly complete switching of the transistors in the subsequent stages.
- Excessively low output swing worsen the effect of the internal noise of the oscillator.
- Drive Capability is another important factor for an oscillator.
- In addition to a mixer, the oscillator must also drive a frequency divider, denoted by the ÷*N* block.



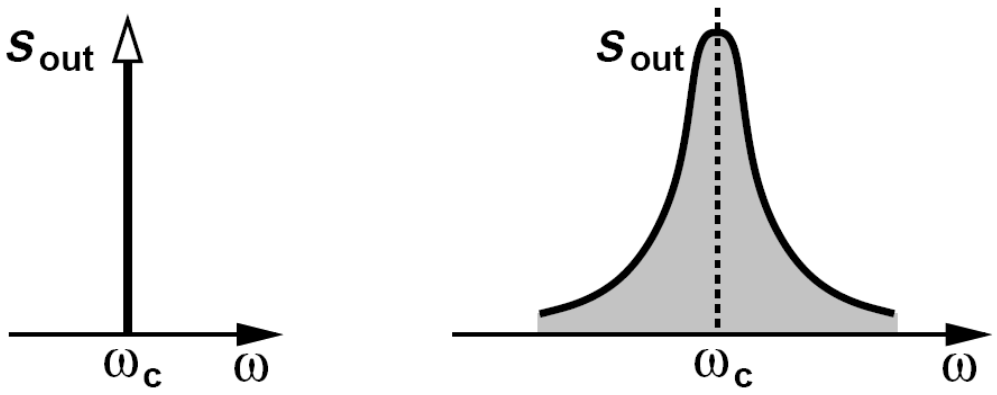
Performance Parameters: Drive Capability

- Typical mixers and dividers exhibit a trade-off between the minimum LO swing with which they can operate properly and the capacitance that they present at their LO port (large swing or wider transistors to steer higher current with small swing).
- Buffers may be inserted after LO stages (power, area)



Performance Parameters: Phase Noise

- The spectrum of an oscillator in practice deviates from an impulse and is “broadened” by the noise of its constituent devices, called phase noise.
- Phase noise bears direct trade-offs with the tuning range and power dissipation of oscillators, making the design more challenging.
- Phase noise is inversely proportional to the Q of LC oscillators.

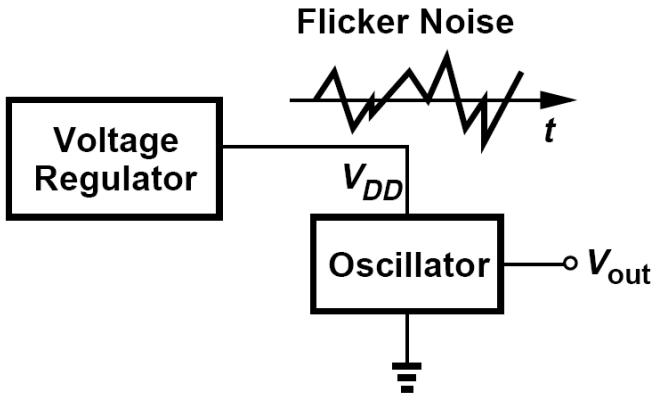


Performance Parameters: Output Waveforms

- What is the desired output waveform of an RF oscillator?
  - Abrupt LO transitions reduce the noise and increase the conversion gain.
  - Effects such as direct feedthrough are suppressed if the LO signal has a 50 % duty cycle.
  - Sharp transitions also improve the performance of frequency dividers.
- Thus, the ideal LO waveform in most cases is a square wave.
  - In practice, it is difficult to generate square LO waveforms (square waves contains many harmonics).
  - Use large LO amplitude or large transistors => abrupt current changes.

Performance Parameters: Supply and Power

- Supply sensitivity: The frequency of an oscillator may vary with the supply voltage, an undesirable effect because it translates supply noise to frequency (and phase) noise.



Flicker noise LF => not easily removed by filtering

- Power Dissipation: The power drained by the LO and its buffer(s) proves critical in some applications as it trades with the phase noise and tuning range.

Overview

- Razavi: Chapter 8, pp. 505-532, 544-551, 491-498.
  - 8.1 Performance Parameters
  - 8.2 **Basic Principles**
  - 8.3 Cross-coupled Oscillator
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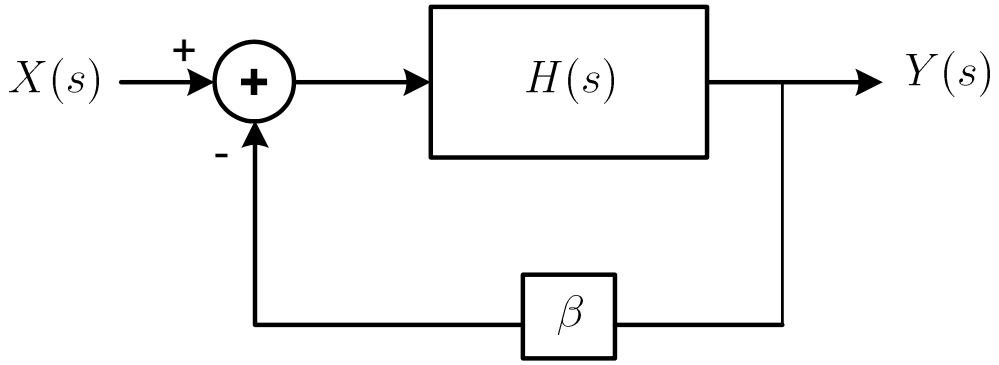
- Lee: Chapter 17: Very different theoretical approach, many oscillator types.

8.2.1 Feedback View

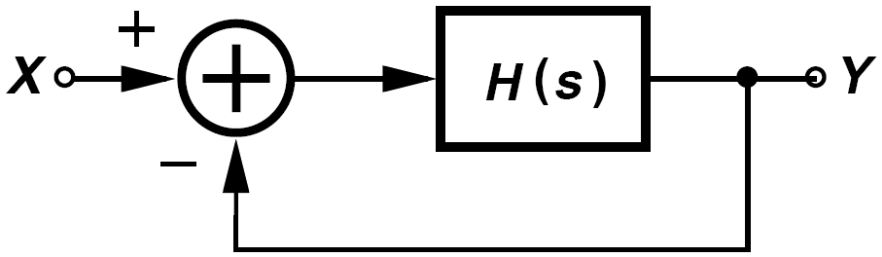
- An oscillator may be viewed as a “badly-designed” negative-feedback amplifier—so badly designed that it has a zero or negative phase margin.



$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1+\beta H(s)}$$

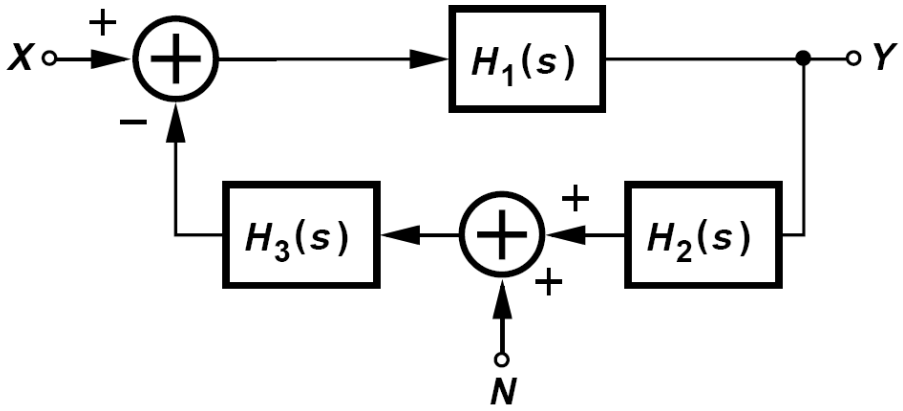
Example 8.3

- For the system to oscillate, must the noise at ω appear at the input?



Example 8.3

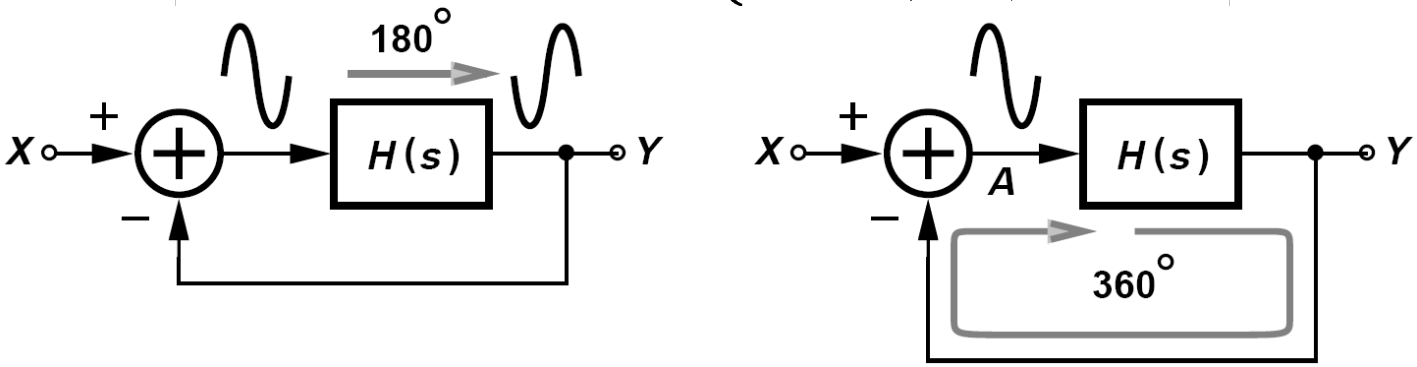
- No, the noise can be anywhere in the loop. For example, consider the system shown in figure below, where the noise N appears in the feedback path.



$$Y(s) = \frac{H_1(s)}{1 + H_1(s)H_2(s)H_3(s)}X(s) + \frac{H_1(s)H_3(s)}{1 + H_1(s)H_2(s)H_3(s)}N(s).$$

Barkhausen's Criteria

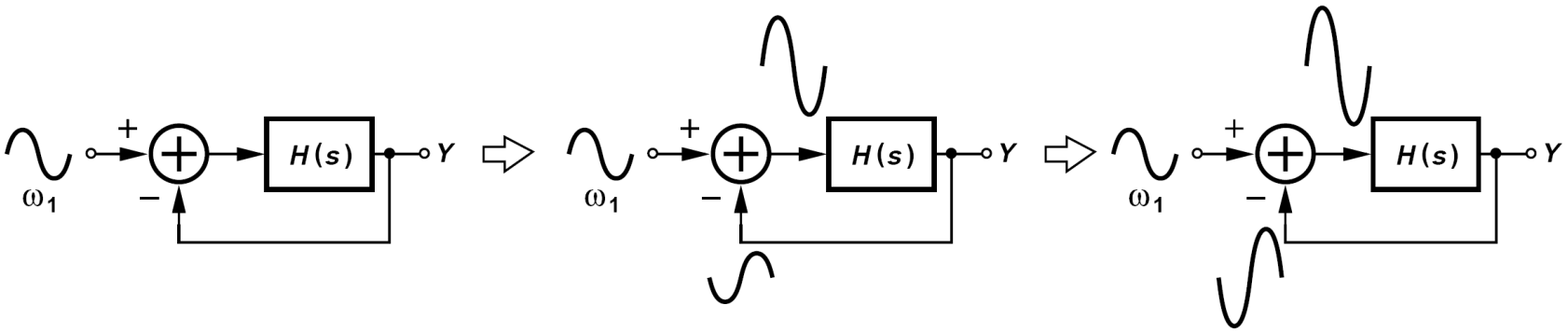
$$1 + \beta H(j\omega) = 0 \Rightarrow \begin{cases} |\beta H(j\omega_1)| = 1 \\ \angle \beta H(j\omega_1) = 180^o \end{cases}$$



- For the circuit to reach steady state, the signal returning to A must exactly coincide with the signal that started at A. We call $\angle H(j\omega_1)$ a "frequency-dependent" phase shift to distinguish it from the 180° phase due to negative feedback.
- Even though the system was originally configured to have negative feedback, H(s) is so "sluggish" that it contributes an additional phase shift of 180° at ω_1, thereby creating positive feedback at this frequency.

Significance of $|H(j\omega_1)| = 1$

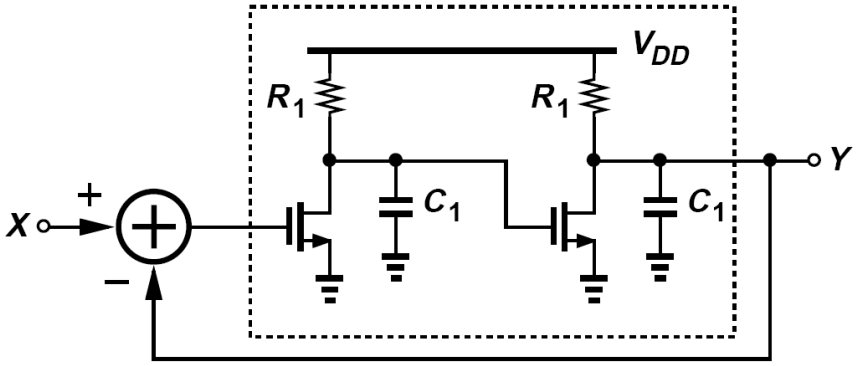
- For a noise component at ω_1 to "build up" as it circulates around the loop with positive feedback, the loop gain must be at least unity.
- We call $|H(j\omega_1)| = 1$ the "startup" condition.

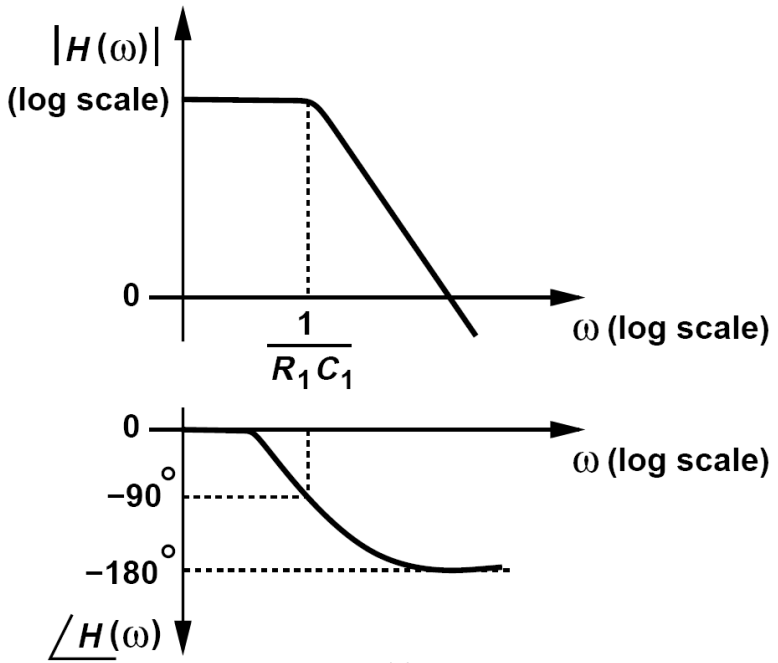


- What happens if $|H(j\omega_1)| > 1$ and $\angle H(j\omega_1) = 180°$? The growth shown in figure above still occurs but at a faster rate because the returning waveform is amplified by the loop.

Example 8.5

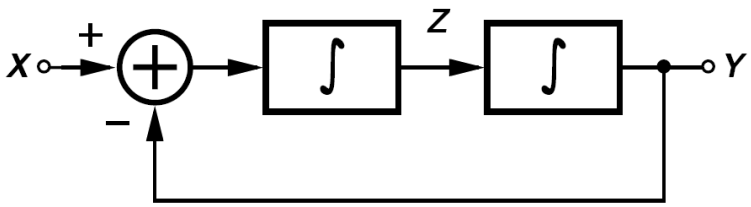
- Can a two-pole system oscillate?
- A two-pole system can not satisfy both of Barkhausen's criteria because the phase shift associated with each stage reaches 90° only at $\omega = \infty$ but $|H(\infty)| = 0$. Thus, the circuit cannot oscillate.

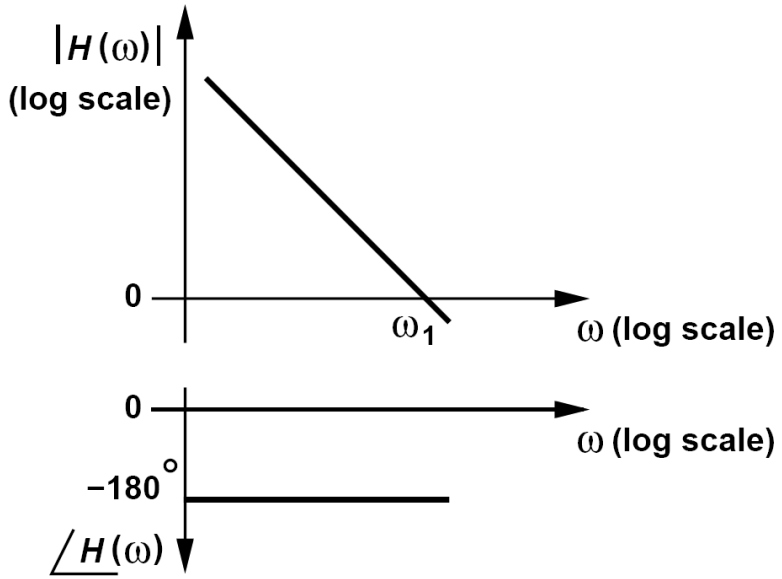




Example 8.5

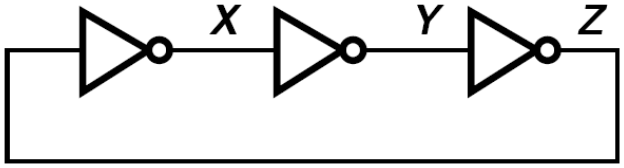
- But, what if both poles are located at the origin? Realized as two ideal integrators in a loop, such a circuit does oscillate because each integrator contributes a phase shift of -90° at any nonzero frequency.

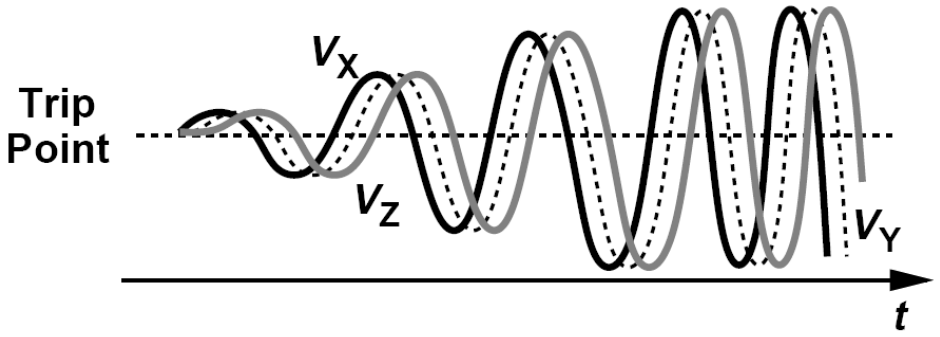




Oscillation Mechanism

- Other oscillators may begin to oscillate at a frequency at which the loop gain is higher than unity, thereby experiencing an exponential growth in their output amplitude.
- The growth eventually stops due to the saturating behavior of the amplifier(s) in the loop.
- Example, ring oscillator (RO):

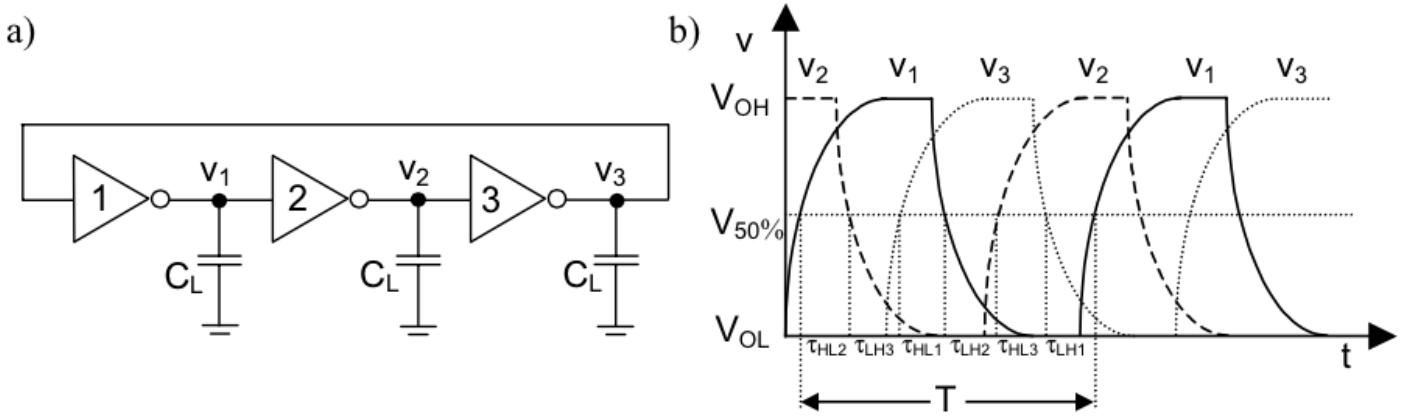




- 60° frequency-dependent phase shift for each inverter stage

Ring Oscillator

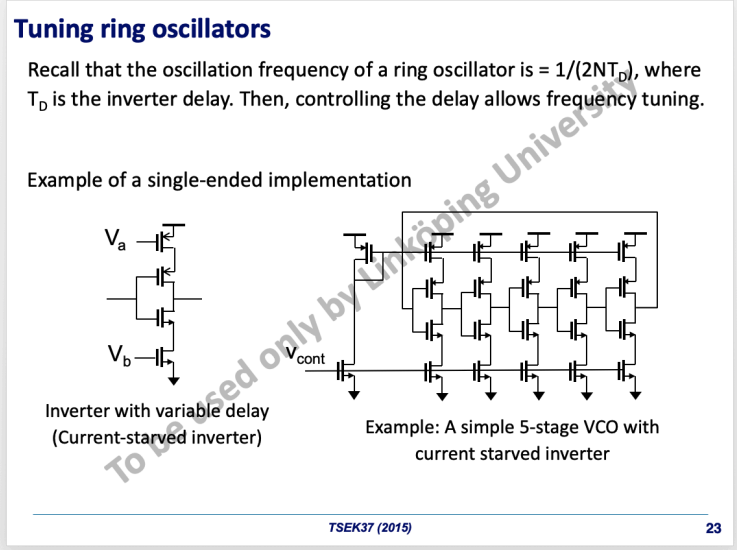
- Usually inverters
- Usually odd number of elements
- Rings/oscillates with a period equal to twice the sum of the individual delays of all stages.



$$T = \tau_{HL2} + \tau_{LH3} + \tau_{HL1} + \tau_{LH2} + \tau_{HL3} + \tau_{LH1} = 2 \cdot n \cdot \tau_p$$

Ring Oscillator steering

- As frequency of oscillation depends on the delay introduced by each inverter stage, delay can be voltage controlled.
- One way to control the delay is to control the amount of current available to charge or discharge the capacitive load of each stage.
- This type of circuit is called a current-starved ring VCO.

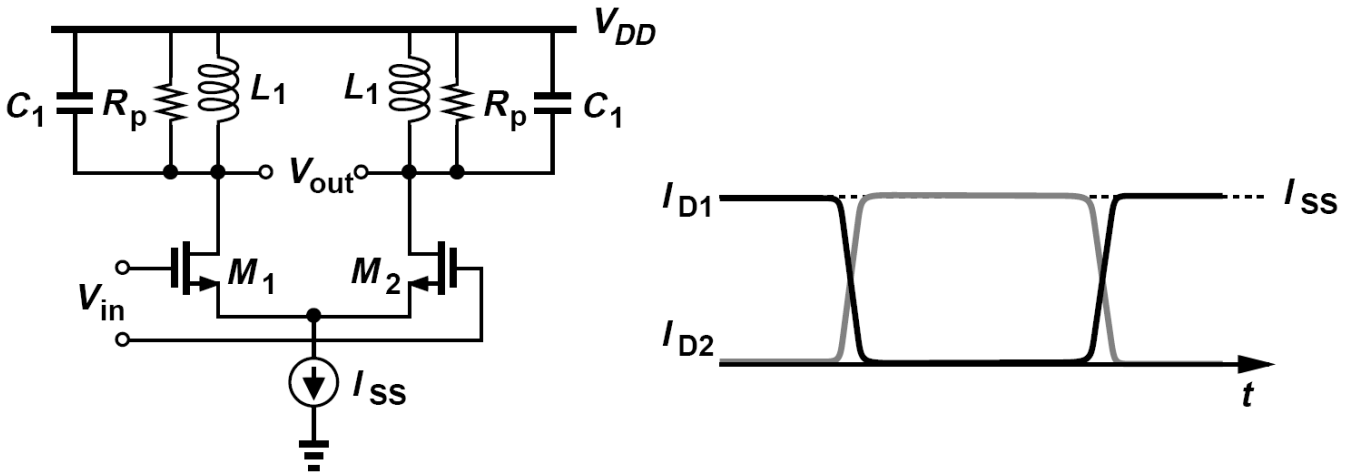


Example 8.7

- The inductively-loaded differential pair shown in figure below is driven by a large input sinusoid at

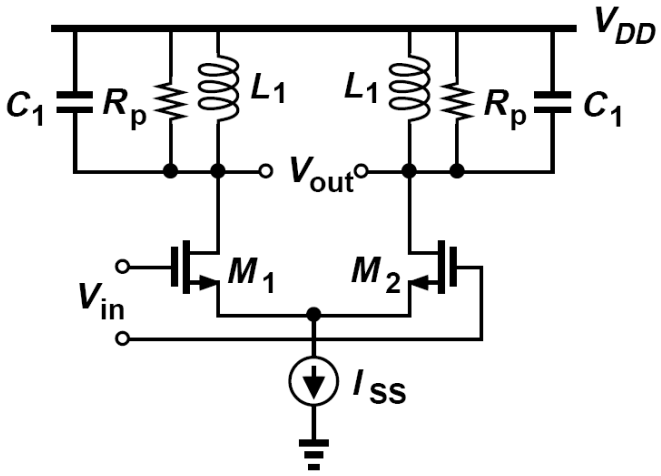
$$\omega_0 = 1/\sqrt{L_1 C_1}$$

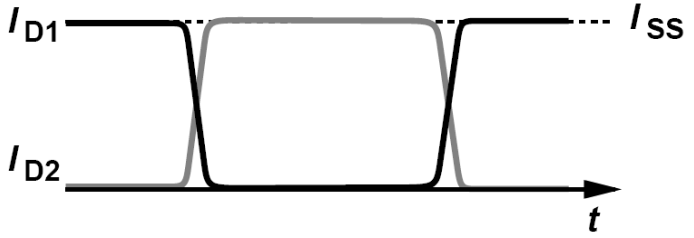
- Plot the output waveforms and determine the output swing.



Example 8.7

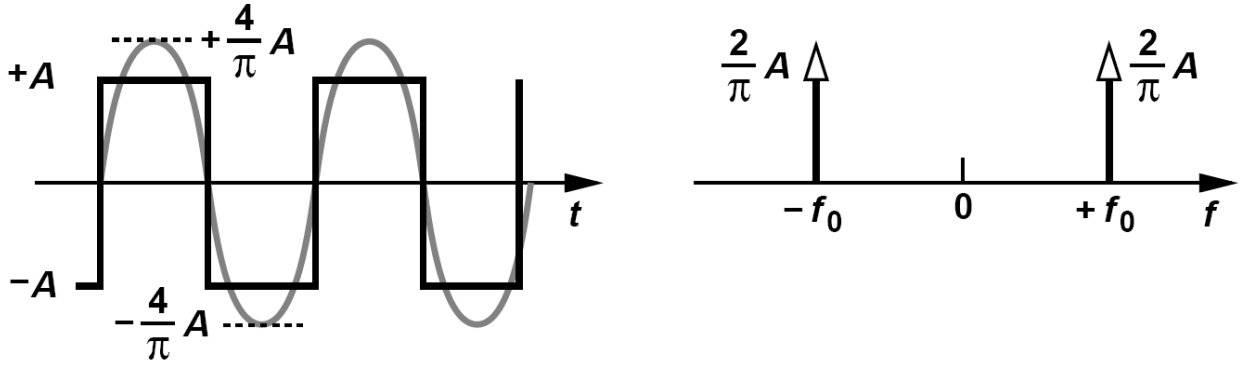
- With large input swings, M_1 and M_2 experience complete switching in a short transition time, injecting nearly square current waveforms into the tanks. Each drain current waveform has an average of $I_{SS}/2$ and a peak amplitude of $I_{SS}/2$. The first harmonic of the current is multiplied by R_p whereas higher harmonics are attenuated by the tank selectivity.





Example 8.7

- Recall from the Fourier expansion of a square wave of peak amplitude A (with 50% duty cycle) that the first harmonic exhibits a peak amplitude of (4/π)A (slightly greater than A).

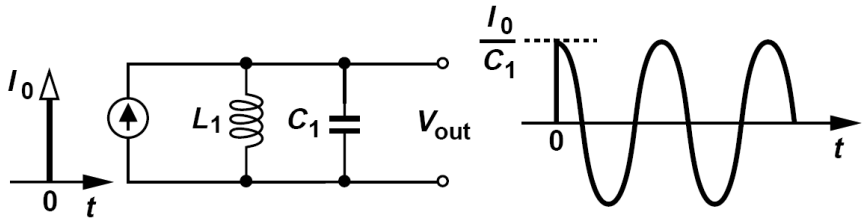


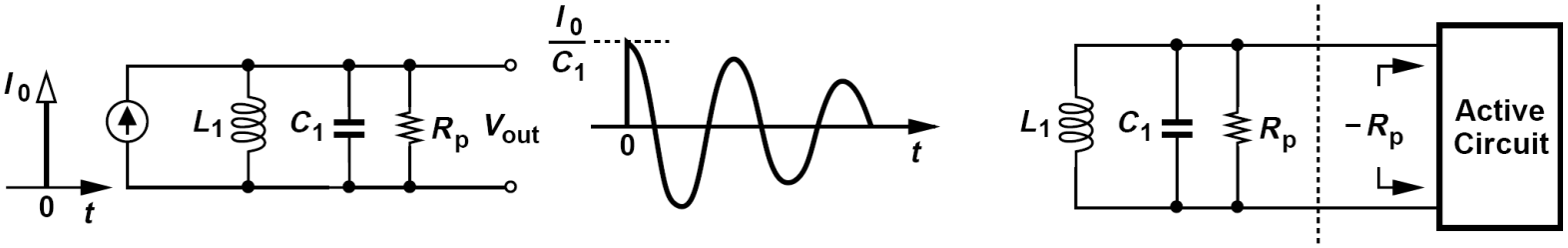
- The peak single-ended output swing will be $2*I_{SS}R_P/\pi$.
- The peak differential output swing will be:

$$V_{out} = \frac{4}{\pi} I_{SS} R_p$$

8.2.2 One-Port View of Oscillators

- An alternative perspective views oscillators as two one-port components, namely, a lossy resonator and an active circuit that cancels the loss.

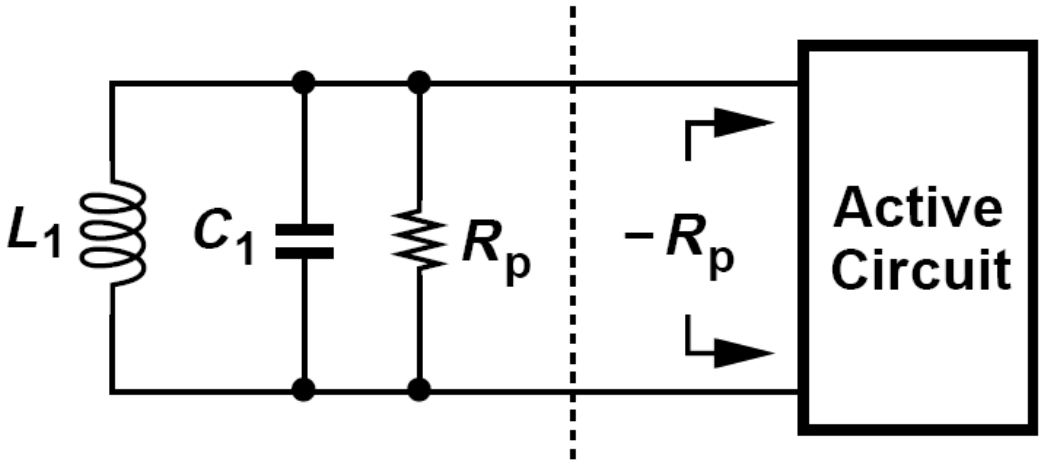




- If an active circuit refills the energy lost in each period, then the oscillation can be sustained.
- An active circuit exhibiting an input resistance of $-R_p$ can be attached across the tank to cancel the effect of R_p.

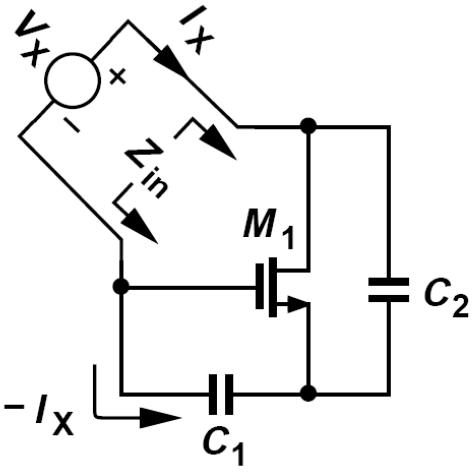
Example 8.8

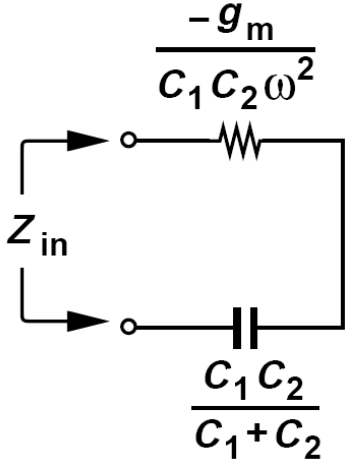
- A student who remembers that loss in a tank results in noise postulates that, if the circuit below resembles the ideal lossless topology, then it must also exhibit zero noise. Is that true?



One-Port View of Oscillators

- How can a circuit present a negative (small-signal) input resistance?





- Impedance: $$\frac{V_X}{I_X}(j\omega) = \frac{1}{jC_1\omega} + \frac{1}{jC_2\omega} - \frac{g_m}{C_1C_2\omega^2}$$

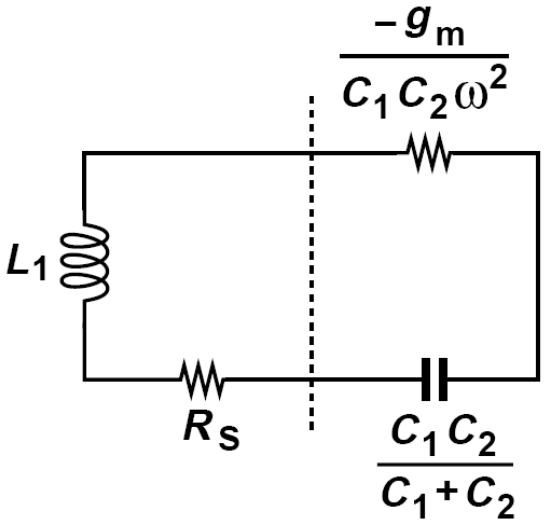
Negative Resistance

One-Port View of Oscillators

- Since the capacitive component can become part of the tank, we simply connect an inductor to the negative-resistance port.

$$R_S = \frac{g_m}{C_1 C_2 \omega^2}$$

$$\omega_{osc} = \frac{1}{\sqrt{L_1 \frac{C_1 C_2}{C_1 + C_2}}}$$



Quality Factor

- Definitions useful with circuit design involving inductors:

$$Q_1 = \frac{L\omega}{R_S}$$

$$Q_2 = \frac{R_p}{L\omega}$$

$$Q_3 = \frac{Im\{Z\}}{Re\{Z\}}.$$

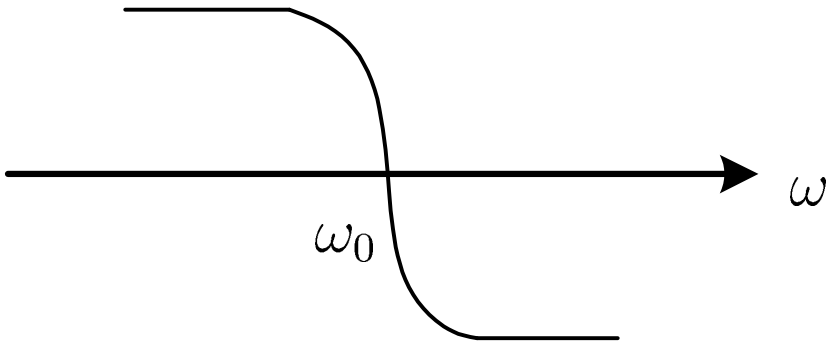
- Basic physics: $$Q_4 = 2\pi \frac{\text{Energy Stored}}{\text{Energy Dissipated per Cycle}}.$$

Quality Factor

- For oscillators without inductive components, we need another definition

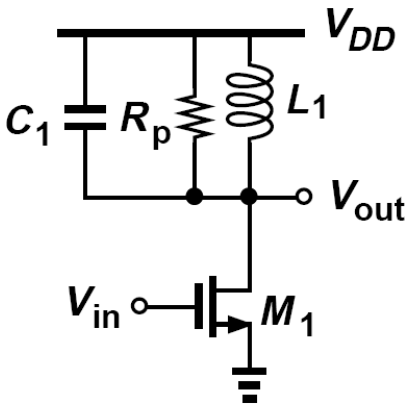
$$Q = \frac{\omega_0}{2}\left|\frac{d\phi}{d\omega}\right|$$

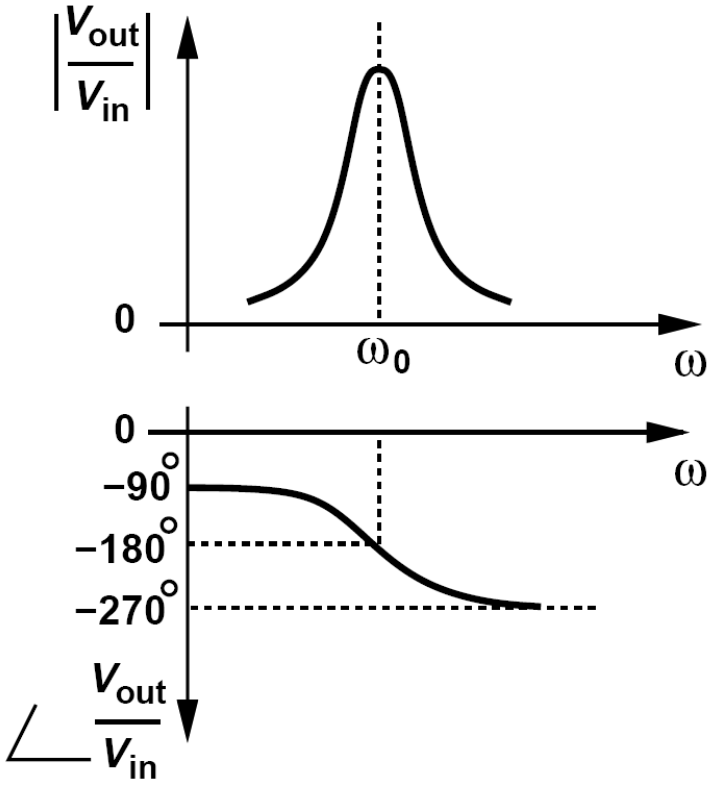
$\phi = \angle H(j\omega)$ Open-loop phase shift



- Higher Q means makes the output spectrum closer to an ideal impulse.

Quality Factor





Example 8.9

- Express the oscillation condition in terms of inductor's parallel equivalent resistance, R_P, rather than R_S.

$$R_S = \frac{g_m}{C_1 C_2 \omega^2} \qquad \omega_{osc} = \frac{1}{\sqrt{L_1 \dfrac{C_1 C_2}{C_1 + C_2}}}$$

- If Q > 3 then

$$\frac{L_1\omega}{R_S} \approx \frac{R_p}{L_1\omega} \Rightarrow \frac{L_1^2\omega^2}{R_p} = \frac{g_m}{C_1 C_2 \omega^2}$$

$$\begin{aligned} g_m R_p &= \frac{(C_1 + C_2)^2}{C_1 C_2} \\ &= \frac{C_1}{C_2} + \frac{C_2}{C_1} + 2 \end{aligned}$$

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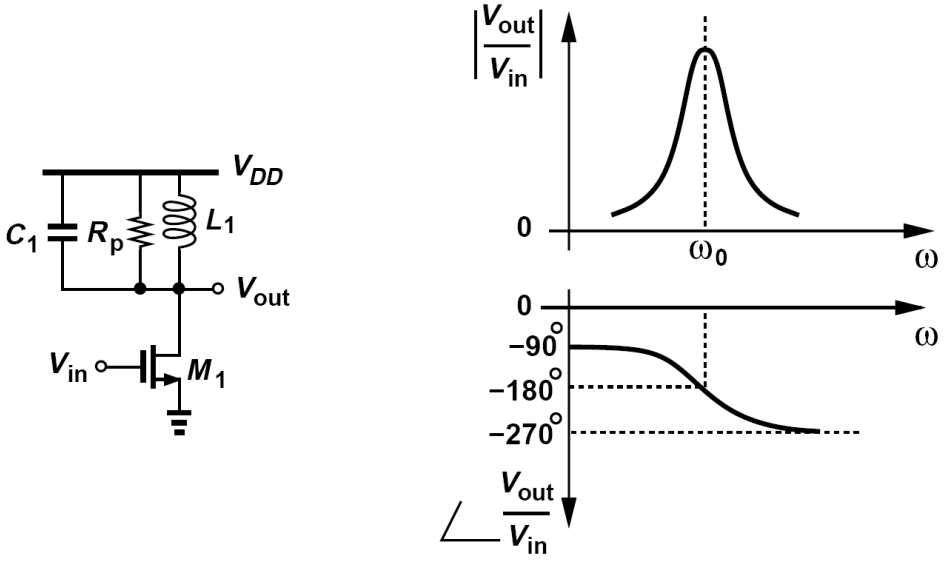
- Lee: Chapter 17: Very different theoretical approach, many oscillator types.

8.3 Cross-Coupled Oscillators

- We wish to build a negative-feedback oscillatory system using “LC-tuned” amplifier stages.
- Consider an RL-circuit connected as a load for an amplifier stage (C_1 denotes total cap seen at output).



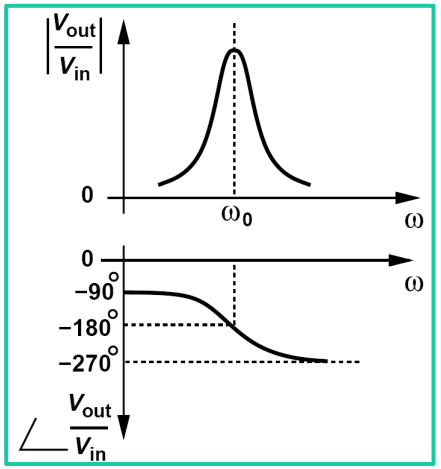
Cross-Coupled Oscillators

- At very low frequencies, L_1 dominates the load, and $\frac{V_{out}}{V_{in}} \approx -g_m L_1 s$
- $|V_{out}/V_{in}|$ is very small and $\angle$(Vout/Vin) remains around -90°.

- At the resonance frequency $\frac{V_{out}}{V_{in}} = -g_m R_p$

- The phase shift from the input to the output is thus equal to 180°.

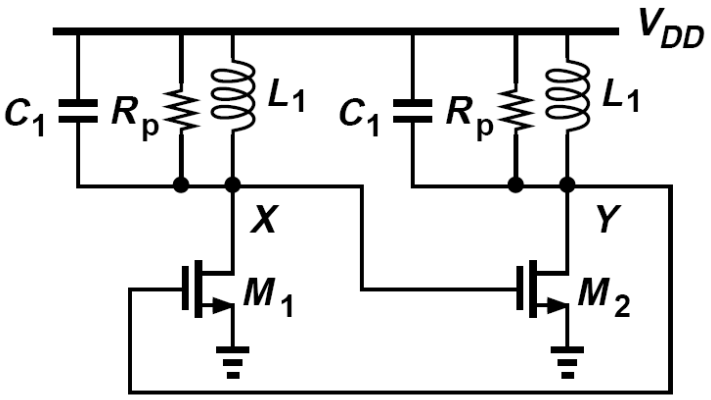
$$\frac{V_{out}}{V_{in}} \approx -g_m \frac{1}{C_1 s}$$

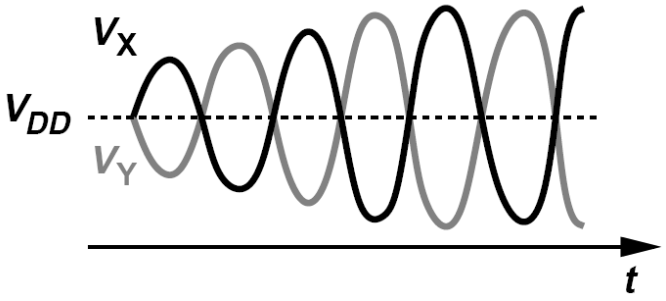


- At very high frequencies, $|V_{out}/V_{in}|$ dinimishes, $\angle$(Vout/Vin) approaches +90°.

Cross-Coupled Oscillators

- Can the previous circuit oscillate if input and output shorted? No.
- Since the circuit provides a phase shift of 180° with possibly adequate gain ($g_m R_p$) at ω_0, we simply need to increase the phase shift to 360° by adding another similar stage in cascade.

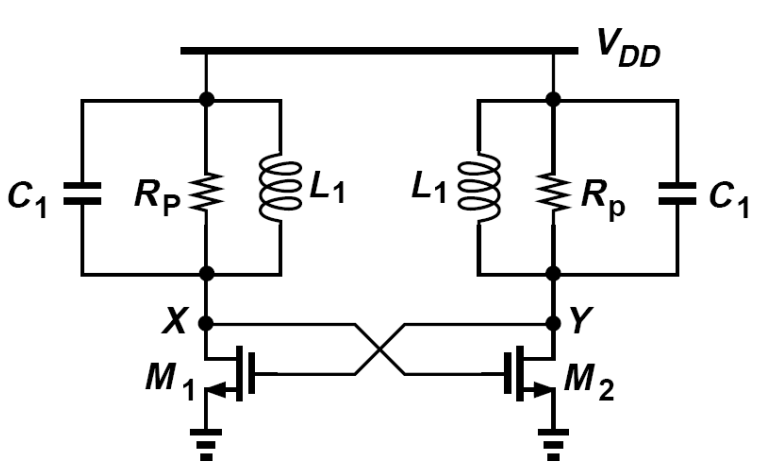


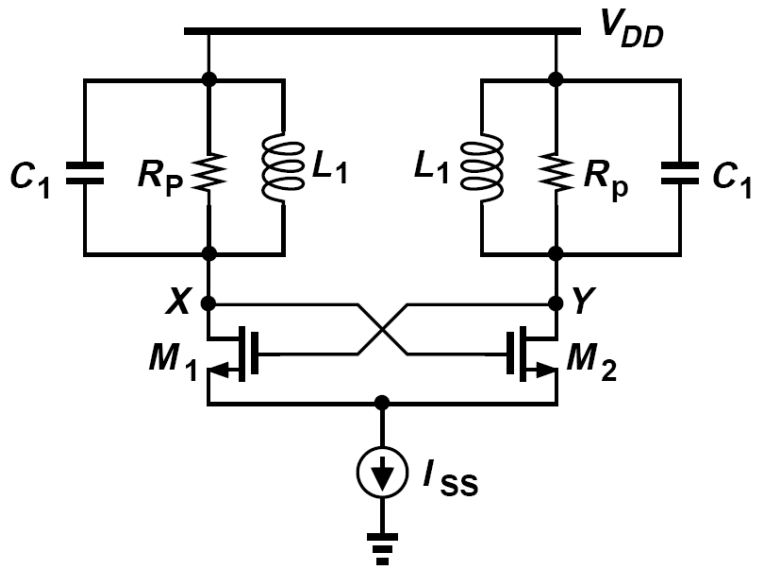


- To oscillate: $(g_m R_p)^2 \geq 1$

Cross-Coupled Oscillators

- We redraw the circuit and call it “cross-coupled” oscillator due to the connection of M_1 and M_2.
- Adding I_{SS} makes it robust with well-defined bias.

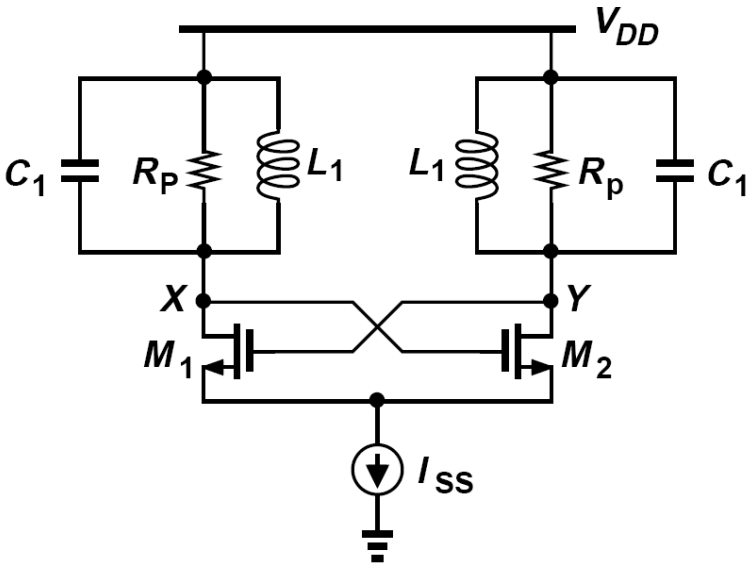




$$\omega_{osc} = \frac{1}{\sqrt{L_1(C_{GS2} + C_{DB1} + 4C_{GD} + C_1)}}$$

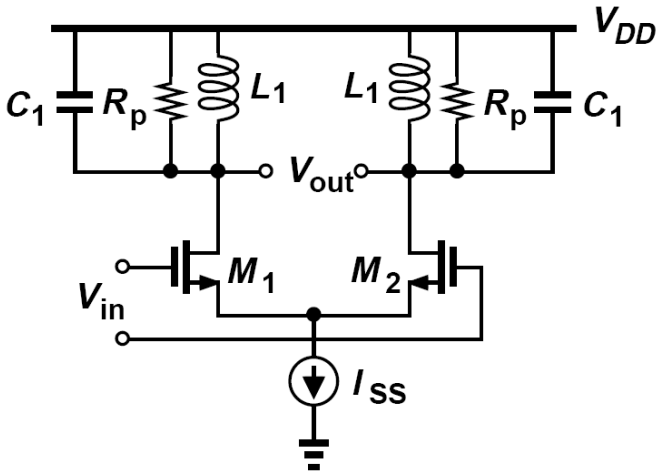
Example 8.11

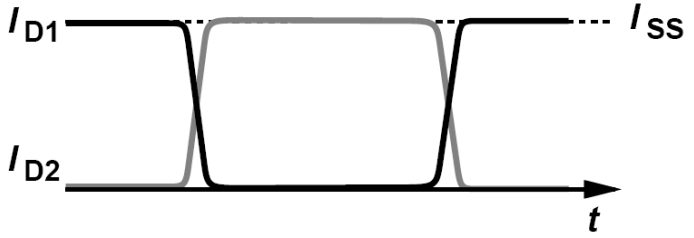
- Compute the voltage swings in the circuit if M_1 and M_2 experience complete current switching with abrupt edges.



Example 8.7

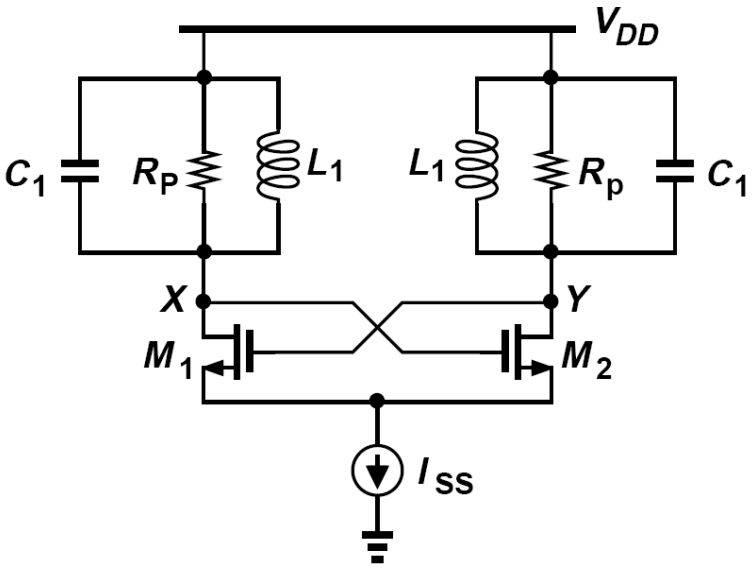
- With large input swings, M_1 and M_2 experience complete switching in a short transition time, injecting nearly square current waveforms into the tanks. Each drain current waveform has an average of $I_{SS}/2$ and a peak amplitude of $I_{SS}/2$. The first harmonic of the current is multiplied by R_p whereas higher harmonics are attenuated by the tank selectivity.





Example 8.11

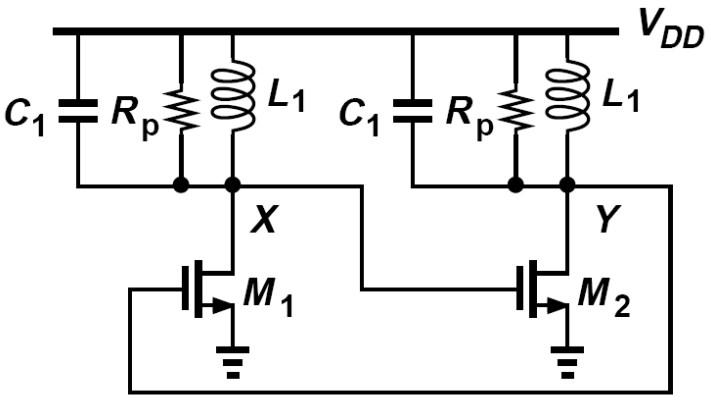
- Compute the voltage swings in the circuit if M_1 and M_2 experience complete current switching with abrupt edges.

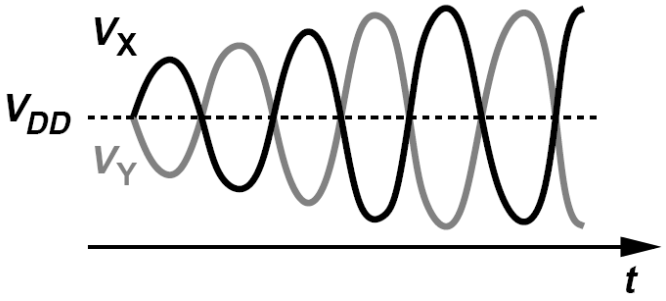


$$V_{XY} \approx \frac{4}{\pi} I_{SS} R_p$$

Cross-Coupled Oscillators

- Can the previous circuit oscillate if input and output shorted? No.
- Since the circuit provides a phase shift of 180° with possibly adequate gain ($g_m R_p$) at ω_0, we simply need to increase the phase shift to 360° by adding another similar stage in cascade.





- To oscillate: $(g_m R_p)^2 \geq 1$

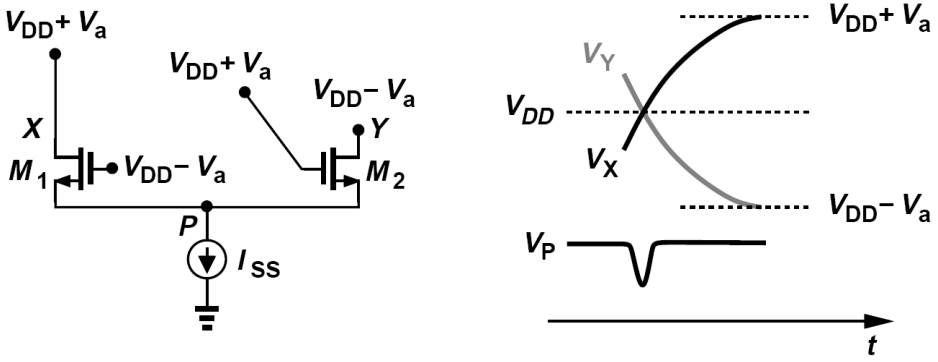
Above-Supply Swings in Cross-Coupled Oscillator

- Each transistor may experience stress under the following conditions:

(1) The drain reaches $V_{DD} + V_a$. The transistor remains off but its drain-gate voltage is equal to $2V_a$ and its drain-source voltage is greater than $2V_a$.

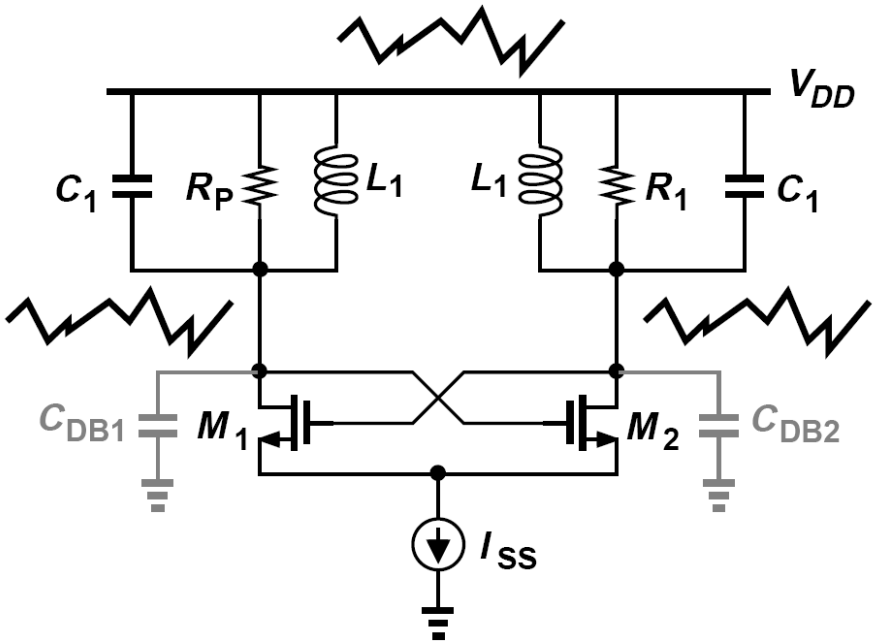
(2) The drain falls to V_{DD} - Va while the gate rises to $V_{DD} + V_a$. Thus, the gate-drain voltage reaches $2V_a$ and the gate-source voltage exceeds $2V_a$.

- Proper choice of V_a, I_{SS}, and device dimensions avoids stressing the transistors

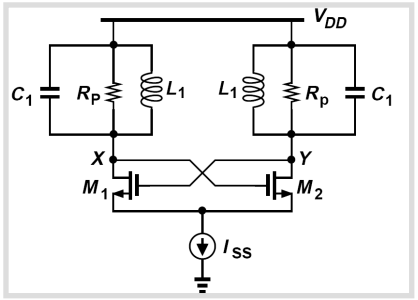


Example 8.12

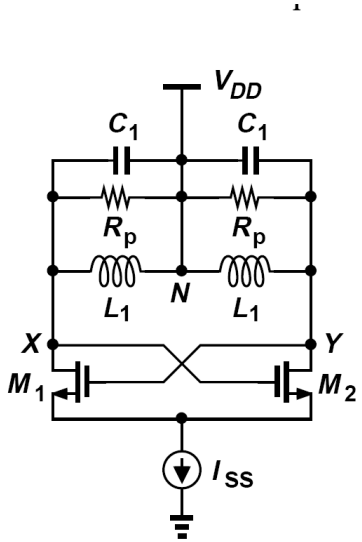
- A student claims that the cross-coupled oscillator below exhibits no supply sensitivity if the tail current source is ideal. Is this true?

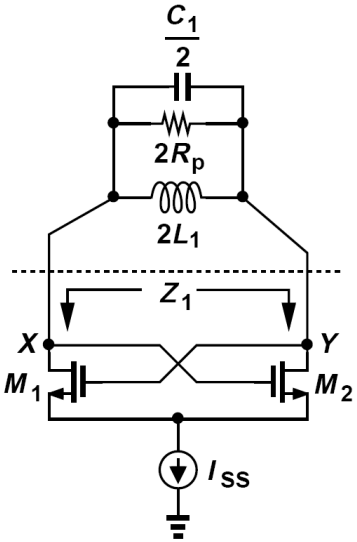


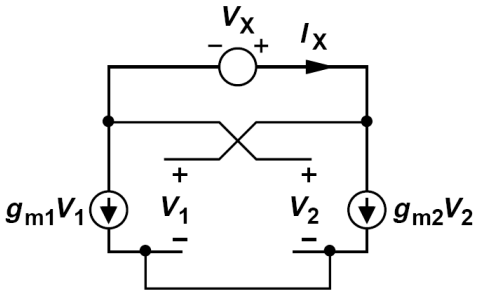
Cross-Coupled Oscillators



- One-port view:







$$I_X = -g_{m1}V_1 = g_{m2}V_2 \quad \Rightarrow \quad \frac{V_X}{I_X} = -\left(\frac{1}{g_{m1}} + \frac{1}{g_{m2}}\right)$$

$$\frac{V_X}{I_X} = -\frac{2}{g_m} \qquad \frac{2}{g_m} \leq 2R_p \quad \Rightarrow \quad g_m R_p \geq 1$$

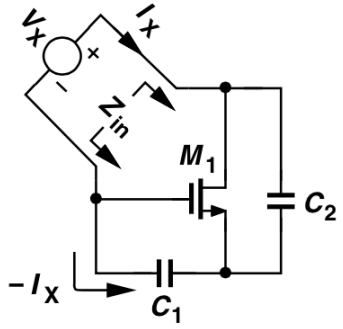
Overview

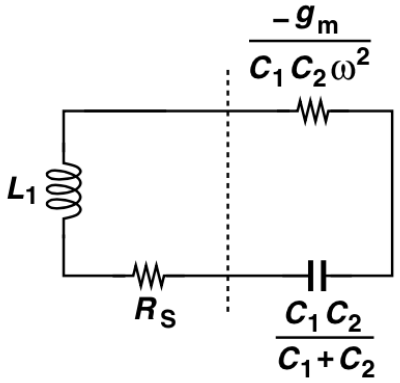
- Razavi: Chapter 8, pp. 505-532, 544-551, 491-498.
  - 8.1 Performance Parameters
  - 8.2 Basic Principles
  - 8.3 Cross-coupled Oscillator
  - 8.4 **Three-point Oscillators**
  - 8.5 VCO
  - 8.7 Phase Noise (parts thereof)

- Lee: Chapter 17: Very different theoretical approach, many oscillator types.

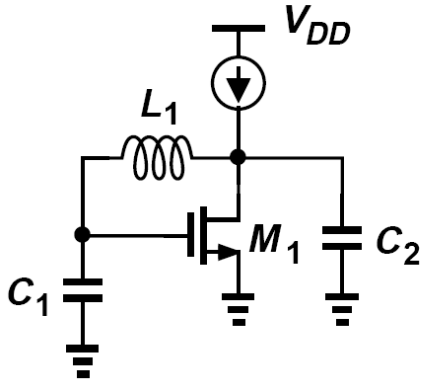
8.4 Three-Point Oscillators

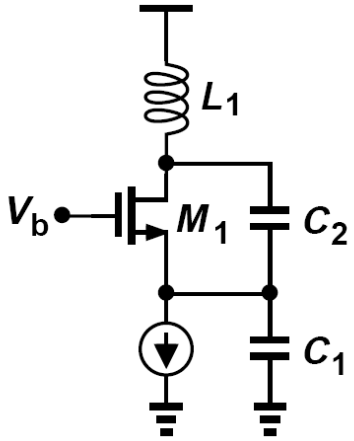
- Remember the one-port oscillator and how to achieve negative resistance?

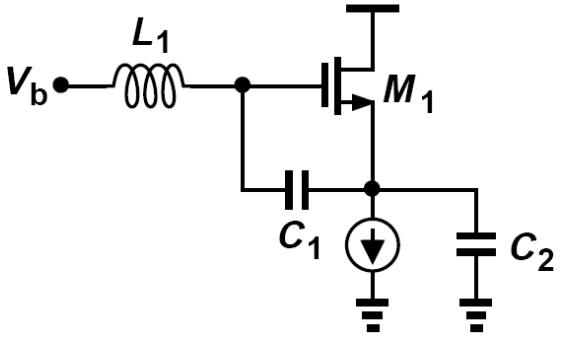




- Three different oscillator topologies can be obtained by grounding each of the transistor terminals.

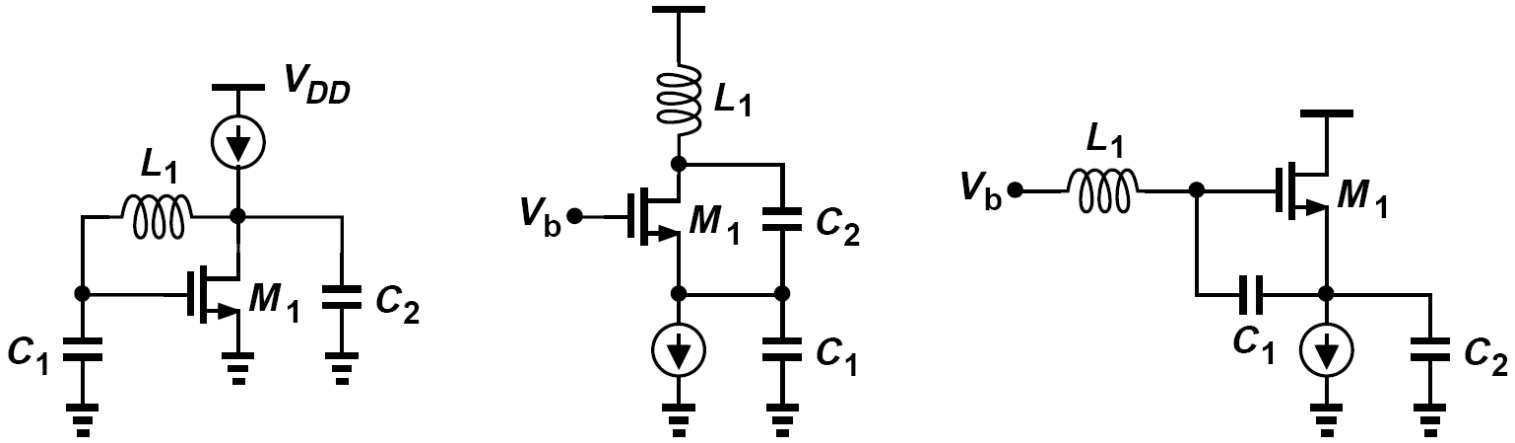






Three-Point Oscillators

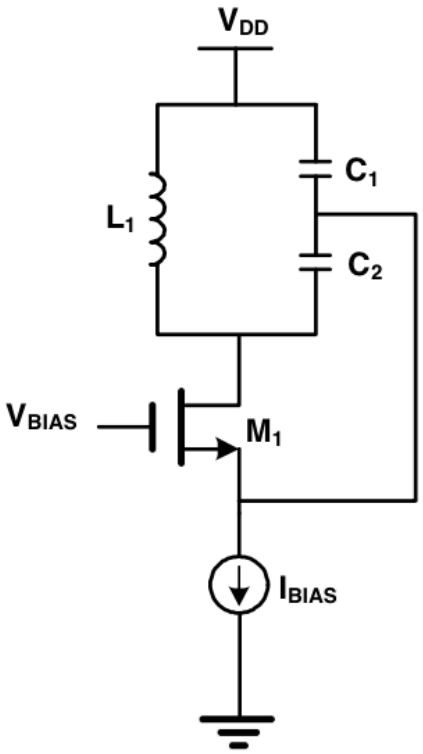
- For C_1=C_2: $g_m R_p \geq 4$ to ensure the oscillation (eq. 8.26 + 8.30)
- Not so popular because increased start-up condition (">4") and single-ended output.
- Also require high-Q L to oscillate.



Three-Point Oscillator: Colpitts

- With a capacitive voltage divider, a positive feedback is applied to a single-transistor amplifier.

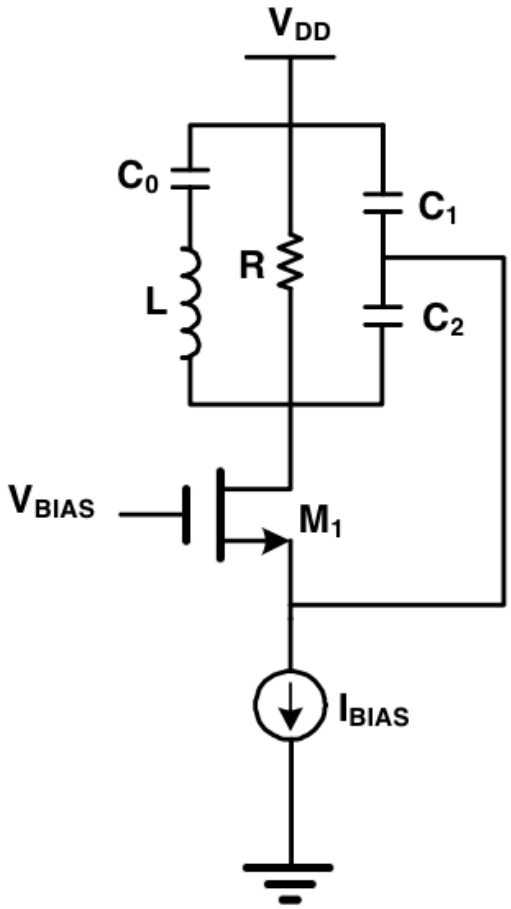
$$f_{osc} = \frac{1}{2\pi\sqrt{L_1 \frac{C_1 C_2}{C_1 + C_2}}}$$



Three-Point Oscillator: Clapp

- Clapp oscillator is a modified form of Colpitts oscillator. The inductor is replaced by a series LC circuit.
- Extra capacitor helps to overcome L noise by increasing the swing.

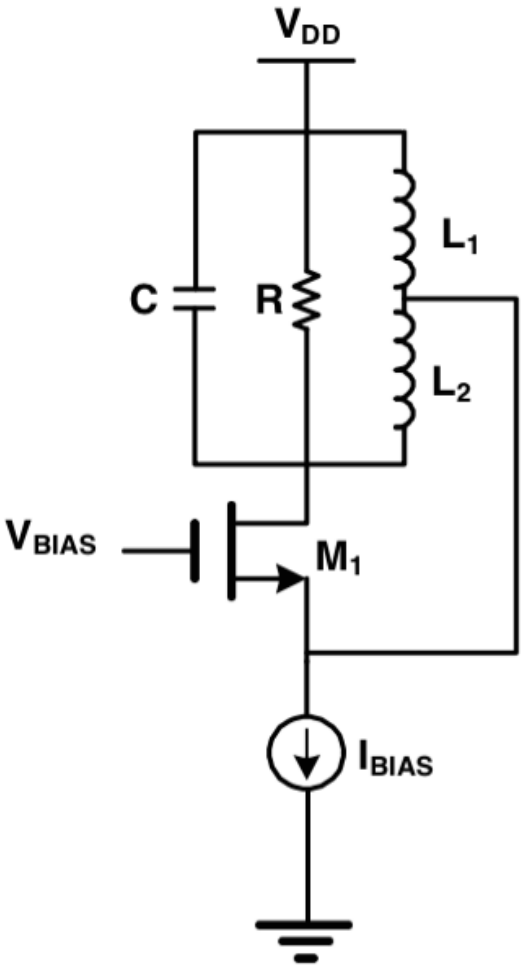
$$f_{osc} = \frac{1}{2\pi}\sqrt{\frac{1}{L}\left(\frac{1}{C_0} + \frac{1}{C_1} + \frac{1}{C_2}\right)}$$



Three-Point Oscillator: Hartley

- A similar structure to Colpitts oscillator is used. The capacitive voltage divider is replaced by an inductive divider.

$$f_{osc} = \frac{1}{2\pi\sqrt{C(L_1 + L_2)}}$$



Quarter-Wave Resonators

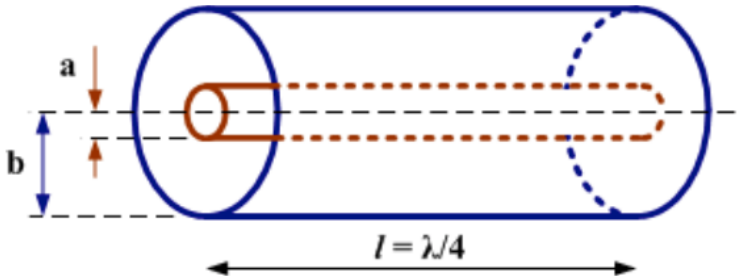
- By shorting a quarter-wave-length transmission line, we get an open end with reflection coefficient equal to 1.

$$Z(l) = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

$$\beta = \frac{2\pi}{\lambda}$$

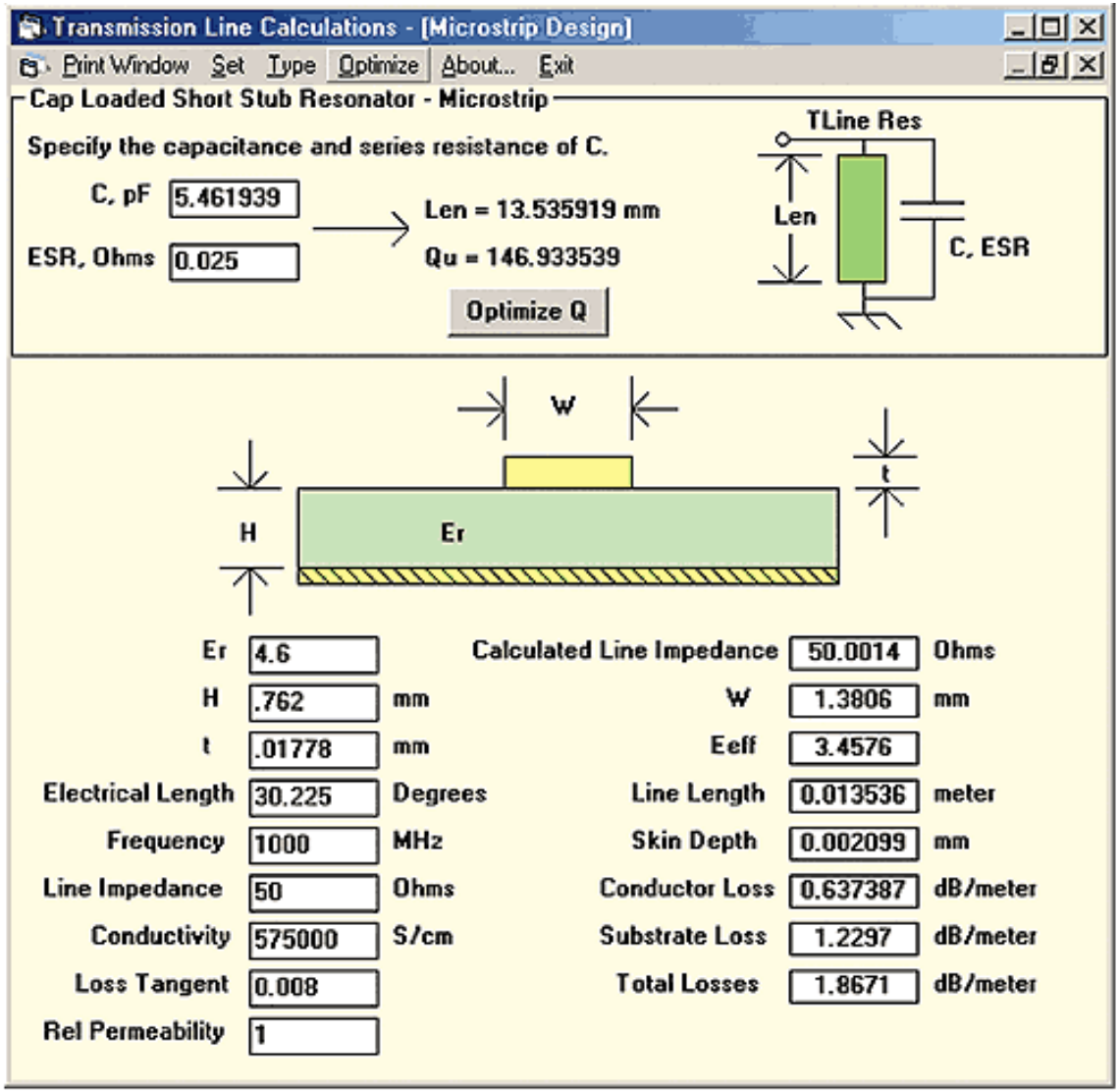
$$l = \frac{\lambda}{4} \Rightarrow Z(\frac{\lambda}{4}) = Z_0 \frac{jZ_0}{jZ_L} = \frac{Z_0^2}{Z_L}$$

$$Z_L = 0 \Rightarrow Z(\frac{\lambda}{4}) = \infty$$



Quarter-Wave Resonators

- At frequencies below the resonant frequency the line behaves like an inductor.
- At frequencies above the resonant frequency it is like a capacitor.
- Around the resonant frequency it behaves like a parallel RLC circuit.
- By filling the resonator with a dielectric material, the required length shrinks to be used at PCB level design.
- The quality factor of these resonators depends on their dimensions and dielectric material used inside the line. Typically, the quality factor is very high (e.g. 20 000).

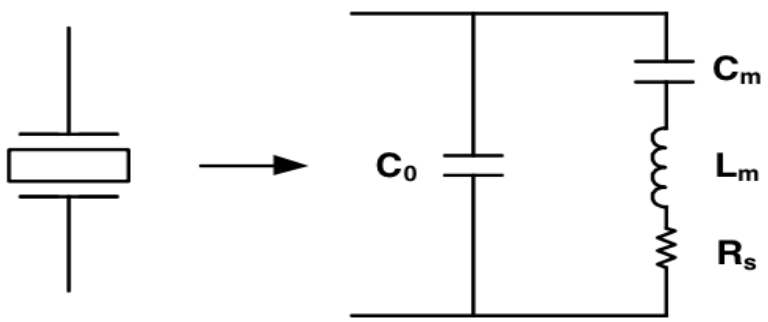

Transmission Line Calculations - [Microstrip Design]
Print Window
Set
Type
Optimize
About...
Exit
Cap Loaded Short Stub Resonator - Microstrip
Specify the capacitance and series resistance of C.
C, pF
5.461939
ESR, Ohms
0.025
Len = 13.535919 mm
Qu = 146.933539
Optimize Q
TLine Res
Len
C, ESR
W
t
H
Er
Er
4.6
H
.762
mm
t
.01778
mm
Electrical Length
30.225
Degrees
Frequency
1000
MHz
Line Impedance
50
Ohms
Conductivity
575000
S/cm
Loss Tangent
0.008
Rel Permeability
1
Calculated Line Impedance
50.0014
Ohms
W
1.3806
mm
Eeff
3.4576
Line Length
0.013536
meter
Skin Depth
0.002099
mm
Conductor Loss
0.637387
dB/meter
Substrate Loss
1.2297
dB/meter
Total Losses
1.8671
dB/meter

Quartz Crystals

- Quartz (crystalline SiO_2) is used in this kind of resonators. Mechanical vibrations are translated into electrical signal with a precise frequency.
- Roughly, the oscillation frequency is inversely proportional to the thickness of the quartz piece.
- These resonators are very stable with low temperature dependency and high quality factors.
- Inside a vacuum it is possible to get a Q-value between 10^4 to 10^6

Quartz Crystals

- In the electrical model of a quartz resonator, C_0 is the parallel plate capacitance, C_m and L_m represent the mechanical energy storage, and R_s accounts for loss in the resonator.



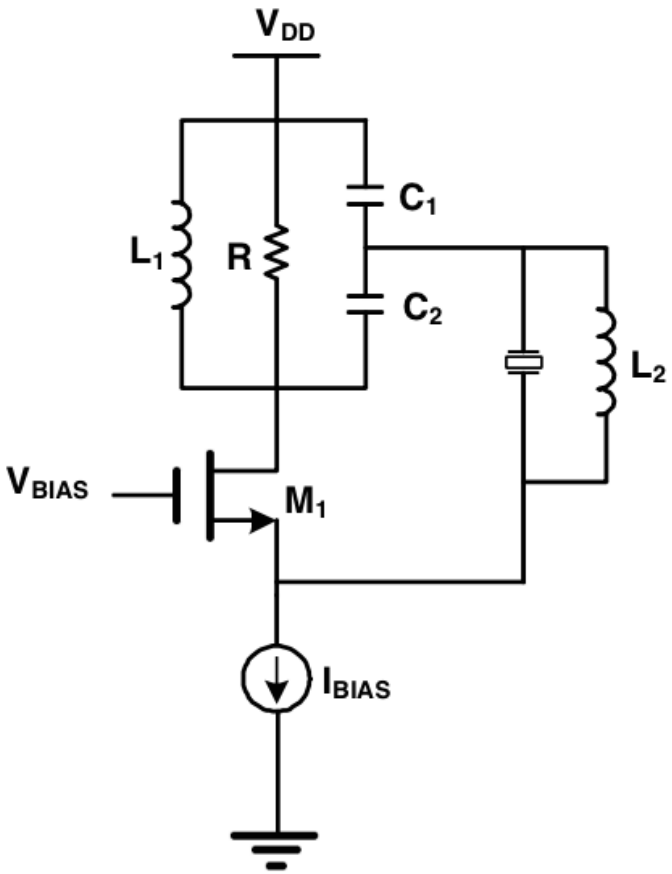
$$R_s \approx \frac{5 \times 10^8}{f_0}$$

- Quality factor can be calculated based on a series RLC circuit formed by C_m, L_m, and R_s.

$$Q = \frac{1}{R_s}\sqrt{\frac{L_m}{C_m}}$$

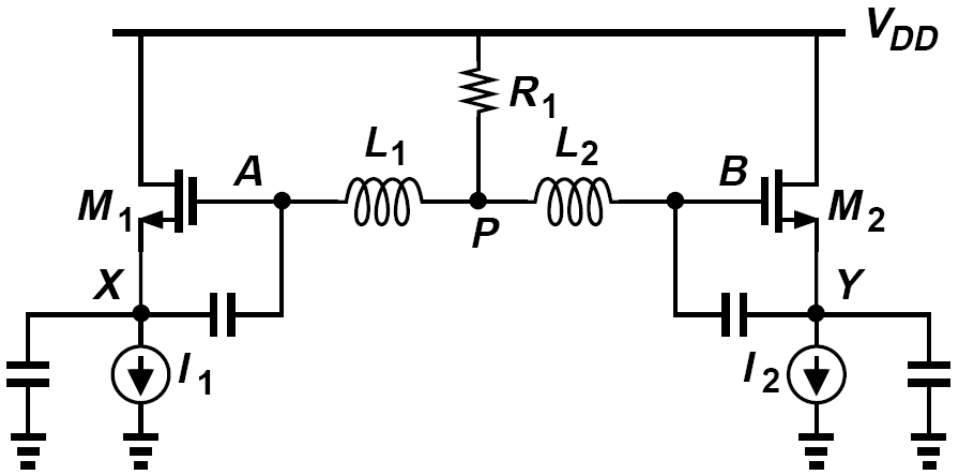
Crystals in LC Configurations

- Crystal oscillators can be derived from LC oscillators.
- As an example, a crystal oscillator can be inserted in the feedback loop in a Colpitts oscillator.
- L_2 resonates with parallel capacitance of the crystal and feedback is V_{BIAS} controlled by series RLC arm.
- Feedback is closed only at the desired frequency.



Differential three-point oscillators

- It is possible to couple two copies of one oscillator so that they operate differentially.
- If chosen properly, the resistor R_1 prohibits common-mode oscillation.



Overview

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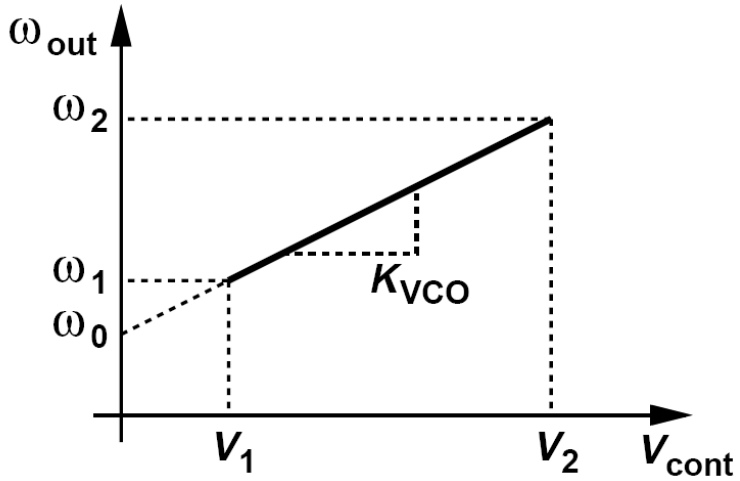
  - 8.1 Performance Parameters
  - 8.2 Basic Principles
  - 8.3 Cross-coupled Oscillator
  - 8.4 Three-point Oscillators
  - 8.5 **VCO**
  - 8.7 Phase Noise (parts thereof)

- Lee: Chapter 17: Very different theoretical approach, many oscillator types.

8.5 Voltage-Controlled Oscillators

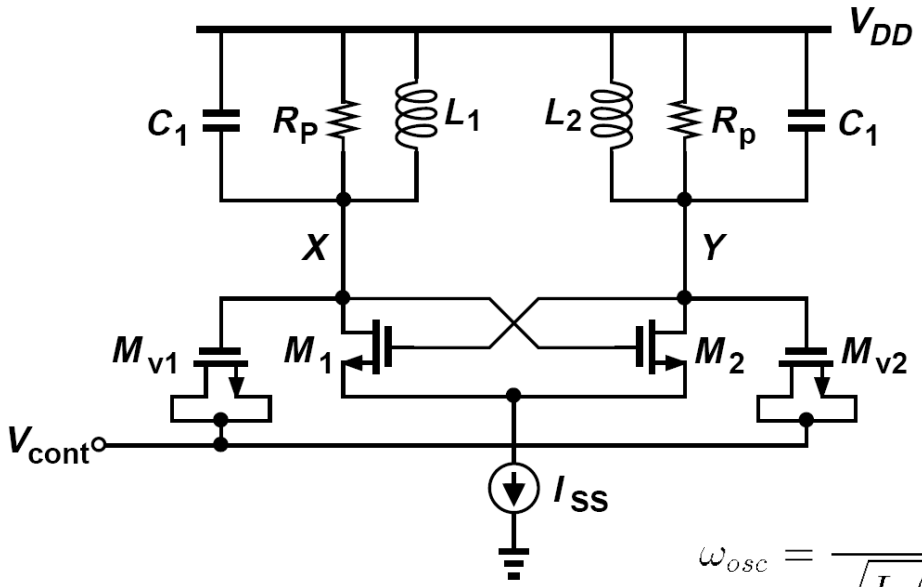
- The output frequency varies from ω_1 to ω_2 (the required tuning range) as the control voltage, V_{cont}, goes from V_1 to V_2.
- The slope of the characteristic, K_{VCO}, is called the “gain” or “sensitivity” of the VCO and expressed in rad/Hz/V.

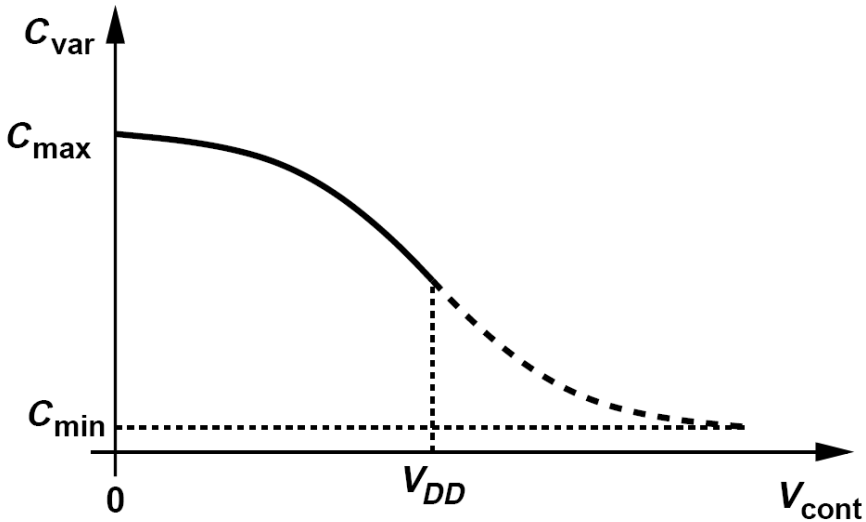


$$\omega_{out} = K_{VCO} V_{cont} + \omega_0$$

Voltage-Controlled Oscillators

- Since it is difficult to vary the inductance, we only vary the capacitance by means of a **varactor**.
- C_1 is a fixed capacitance due to transistor and inductor parasitics.

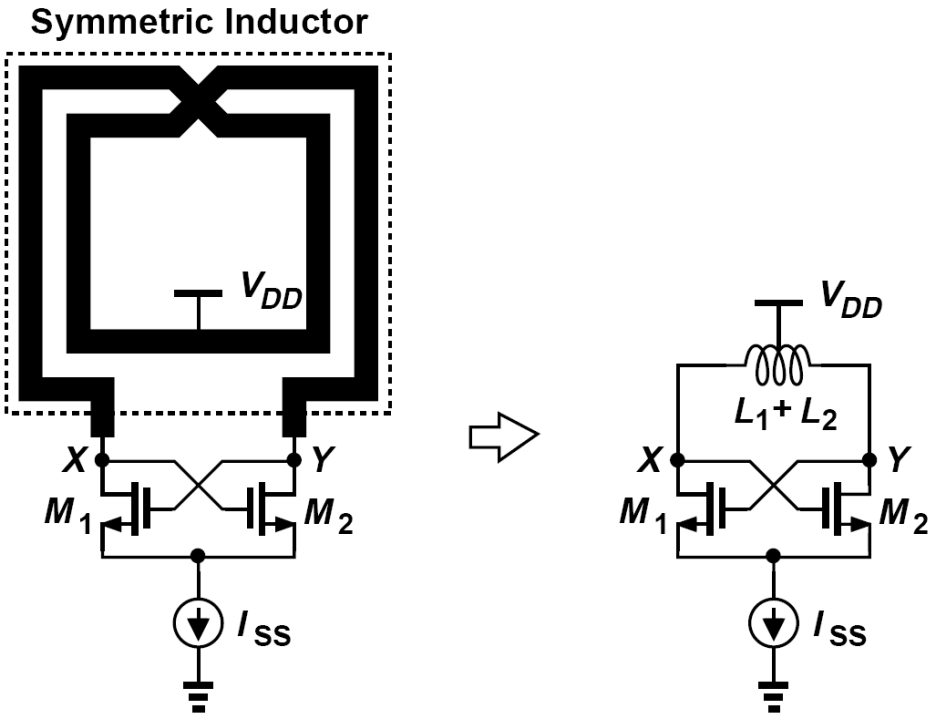




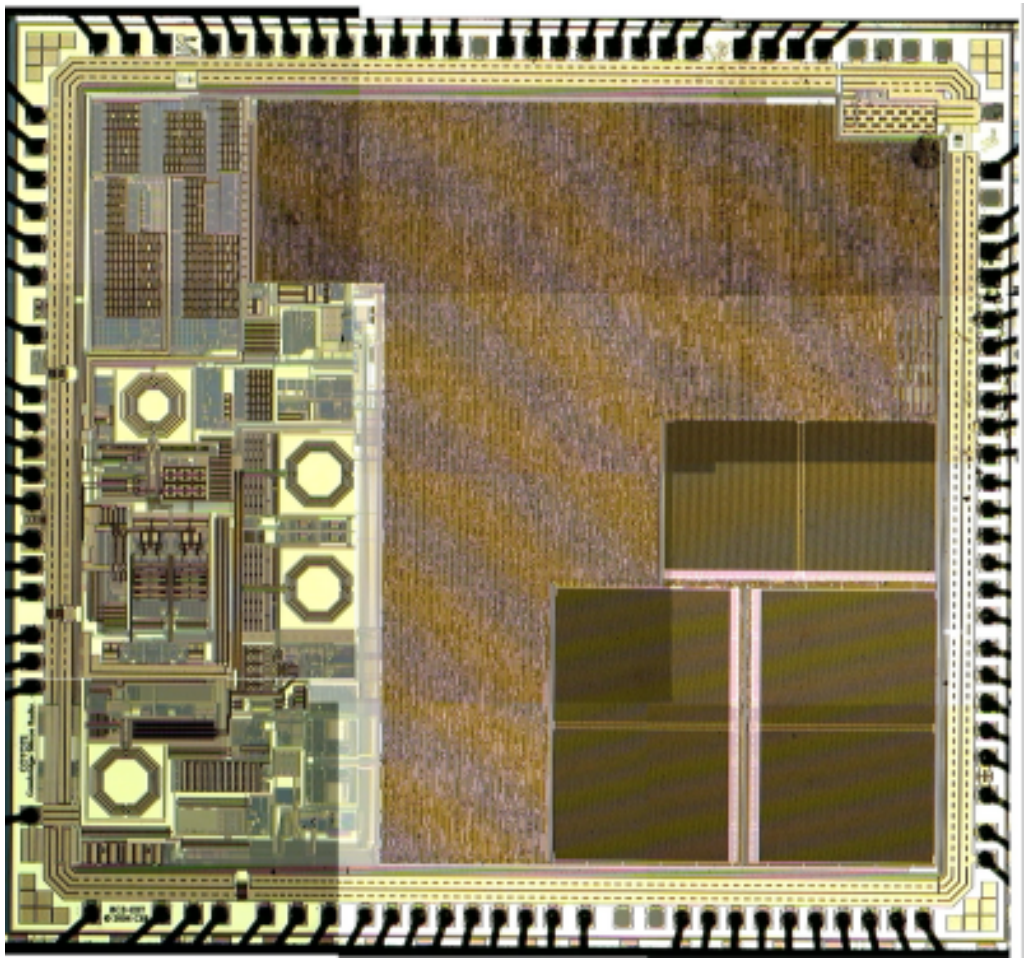
$$\omega_{osc} = \frac{1}{\sqrt{L_1(C_1 + C_{var})}}$$

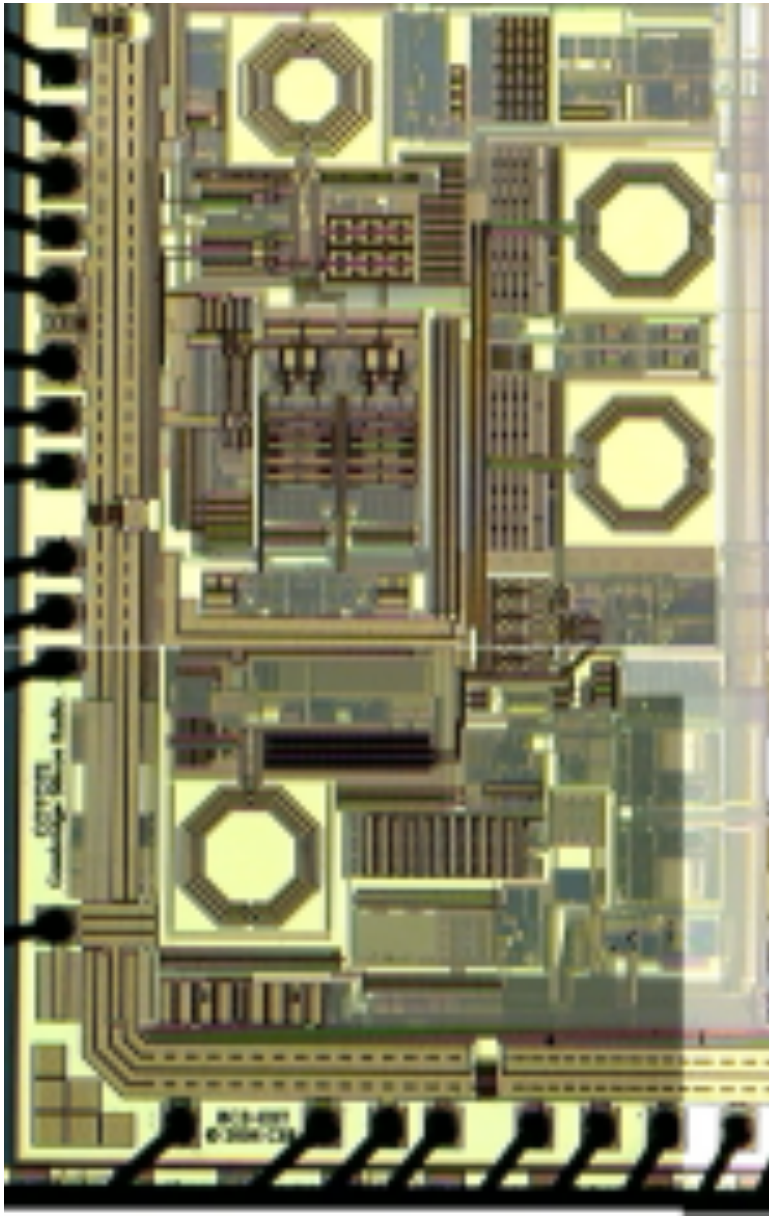
Voltage-Controlled Oscillators

- Symmetric spiral inductors excited by differential waveforms exhibit a higher *Q* than their single-ended counterparts.



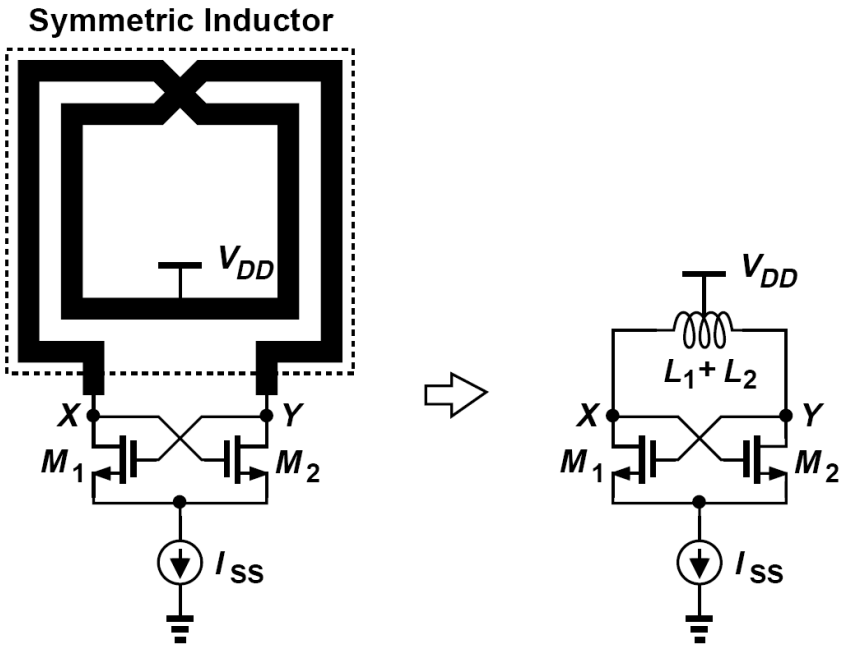
Bluetooth chip 2004 (Bluetooth 4)





Example 8.14

- The symmetric inductor in the figure has a value of 2 nH and a Q of 10 at 10 GHz. What is the minimum required transconductance of M_1 and M_2 to guarantee startup?



Example 8.14

- The symmetric inductor in the figure has a value of 2 nH and a Q of 10 at 10 GHz. What is the minimum required transconductance of M_1 and M_2 to guarantee startup?

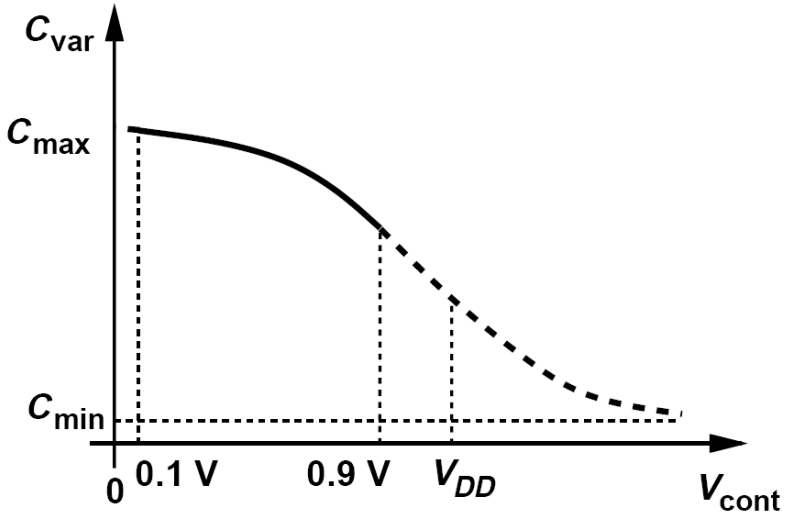
The parallel equivalent resistance of $L_1 + L_2 = 2$ nH is equal to $Q(L_1 + L_2)\omega = 1.26$ kΩ. From Eq. (8.40), we have

$$g_{m1,2} \geq (630\ \Omega)^{-1}. \tag{8.51}$$

Alternatively, we can decompose the inductor into L_1 and L_2 and return to the circuit of Fig. 8.18(b). In this case, $R_p = QL_1\omega = QL_2\omega = 630\ \Omega$, and $g_{m1,2}R_p \geq 1$. Thus, $g_{m1,2} \geq (630\ \Omega)^{-1}$. The point here is that, for frequency and startup calculations, we can employ the one-port model with $L_1 + L_2$ as one inductor or the feedback model with L_1 and L_2 belonging to two stages.

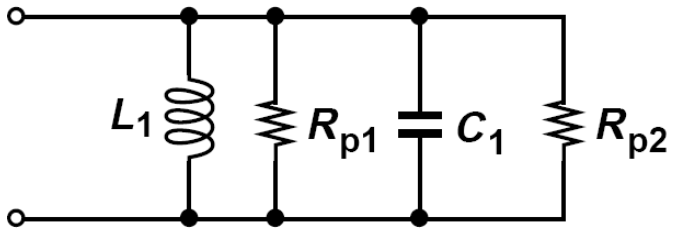
8.5.1 Tuning Range Limitations

- If we assume $C_{var} \ll C_1$: $\omega_{osc} \approx \frac{1}{\sqrt{L_1 C_1}} \left(1 - \frac{C_{var}}{2C_1}\right)$
- If the varactor capacitance varies from C_{var1} to C_{var2}, then the tuning range is $\Delta\omega_{osc} \approx \frac{1}{\sqrt{L_1 C_1}} \frac{C_{var2} - C_{var1}}{2C_1}$
- The tuning range trades with the overall tank *Q.*
- Another limitation on C_{var2} - C_{var1} is the available range for the control voltage of the oscillator, V_{cont}.



8.5.2 Effect of Varactor Q

- If a lossy inductor and a lossy capacitor form a parallel tank, then the overall *Q* in terms of the quality factor of each is (Example 8.15)

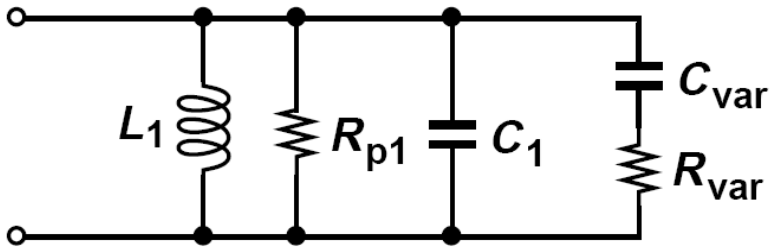


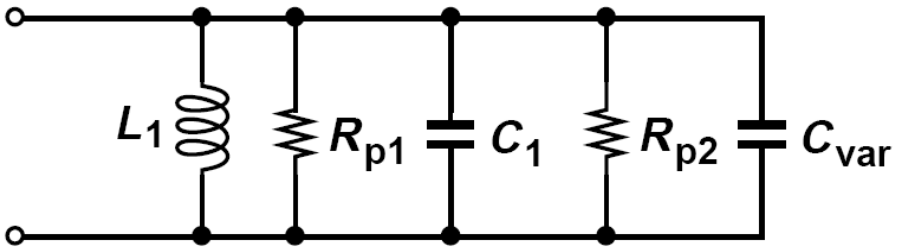
$$Q_L = \frac{R_{p1}}{L_1\omega}$$
$$Q_C = R_{p2}C_1\omega$$

$$\frac{1}{Q_{tot}} = \frac{1}{Q_L} + \frac{1}{Q_C}$$

Tank using lossy varactor

- Now let us consider the effect of varactor Q:





- Transforming the series combination of C_{var} and R_{var} to a parallel combination:
- The Q associated with C_1+C_{var} is

$$R_{p2} = \frac{1}{C_{var}^2 \omega^2 R_{var}}$$

$$\begin{aligned} Q_C &= R_{P2}(C_1 + C_{var})\omega \\ &= \frac{C_1 + C_{var}}{C_{var}^2 \omega R_{var}}. \end{aligned}$$

$$\Rightarrow \quad Q_C = \left(1 + \frac{C_1}{C_{var}}\right) Q_{var}$$

Tank using lossy varactor

- Thus the overall tank Q is therefore given by

$$\frac{1}{Q_{tot}} = \frac{1}{Q_L} + \frac{1}{\left(1 + \frac{C_1}{C_{var}}\right) Q_{var}}$$

Q is boosted by a factor of $1+C_1/C_{var}$

- Equation above can be generalized if the tank consists of an ideal capacitor, C_1, and lossy capacitors, C_2-C_n, that exhibit a series resistance of R_2-R_n, respectively:

$$\frac{1}{Q_{tot}} = \frac{1}{Q_L} + \frac{C_2}{C_{tot}}\frac{1}{Q_2} + \cdots + \frac{C_n}{C_{tot}}\frac{1}{Q_n}$$

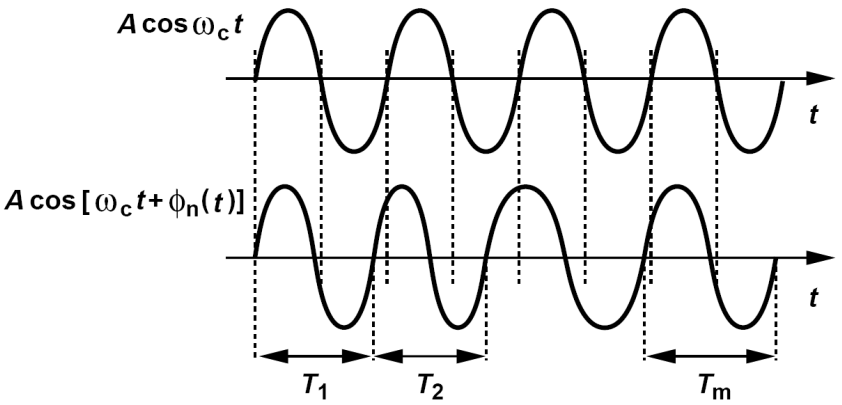
Overview

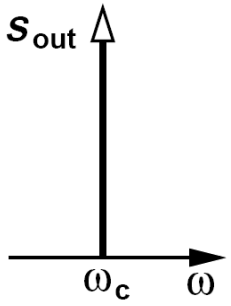
- Razavi: Chapter 8, pp. 505-532, 544-551, 491-498.
  - 8.1 Performance Parameters
  - 8.2 Basic Principles
  - 8.3 Cross-coupled Oscillator
  - 8.4 Three-point Oscillators
  - 8.5 VCO
  - 8.7 **Phase Noise (parts thereof)**

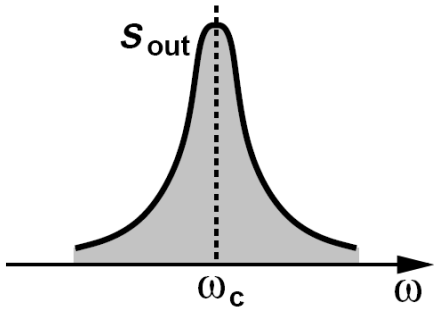
- Lee: Chapter 17: Very different theoretical approach, many oscillator types.

8.7 Phase noise

- The phase of the oscillator varies as $A\cos(\omega_c(t)+\Phi_n(t))$.
- The term $\Phi_n(t)$ is called the “phase noise.”
- Can also be viewed as a random frequency variation, leading to a broadening of the spectrum called phase noise.

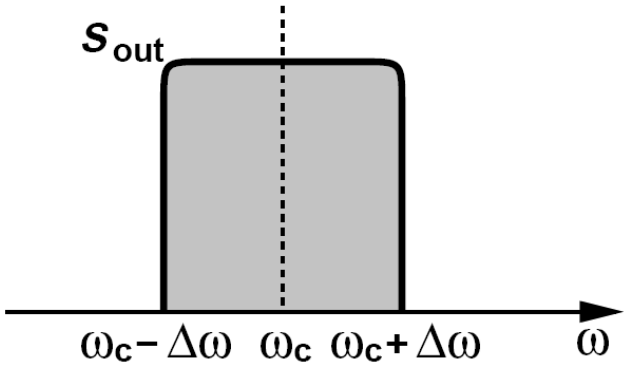






Example 8.23

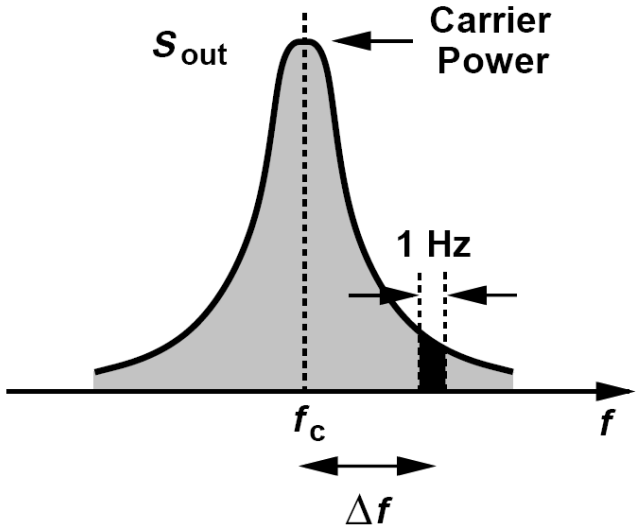
- Explain why the broadened impulse cannot assume the shape shown below.



- This spectrum occurs if the oscillator frequency has equal probability of appearing anywhere between ω_c - $\Delta\omega$ and ω_c + $\Delta\omega$.
  However, we intuitively expect that the oscillator prefers ω_c to other frequencies, thus spending lesser time at frequencies that are farther from ω_c.

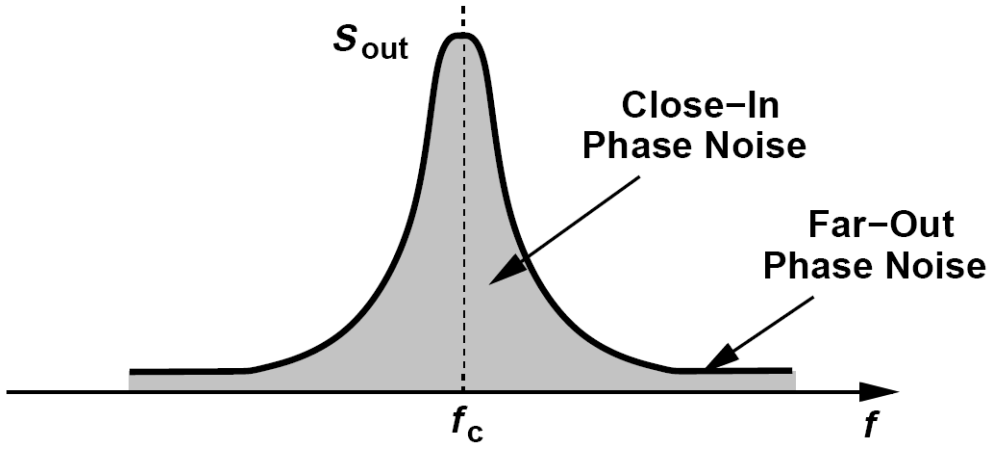
Phase noise

- Since the phase noise falls at frequencies farther from ω_c, it must be specified at a certain “frequency offset”, i.e., at a certain difference with respect to ω_c.
- We consider a 1-Hz bandwidth of the spectrum at an offset of Δf, measure the power in this bandwidth, and normalize the result to the “carrier power”, called “dB with respect to the carrier”, **dBc**.



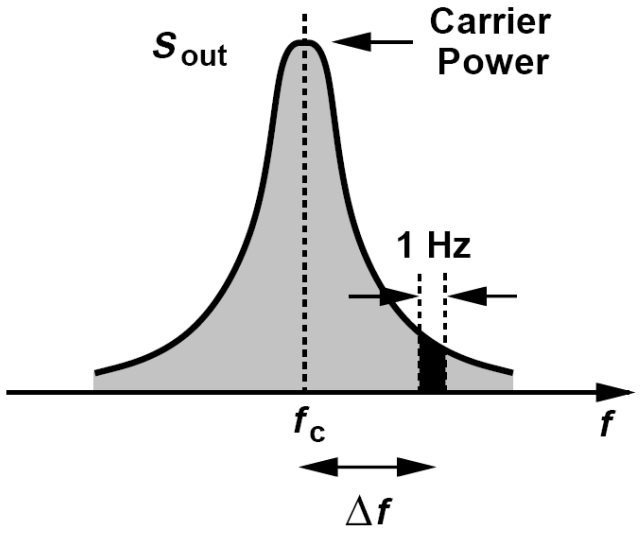
Phase noise

- In practice, the phase noise reaches a constant floor at large frequency offsets (beyond a few megahertz).
- We call the regions near and far from the carrier the “close-in” and the “far-out” phase noise, respectively.



Example 8.23

- At high carrier frequencies, it is difficult to measure the noise power in a 1-Hz bandwidth. Suppose a spectrum analyzer measures a noise power of -70 dBm in a 1-kHz bandwidth at 1-MHz offset. How much is the phase noise at this offset if the average oscillator output power is -2 dBm?



Example 8.23

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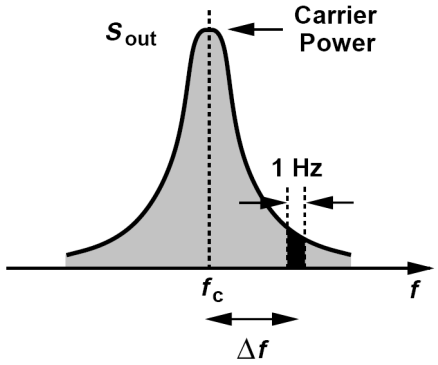
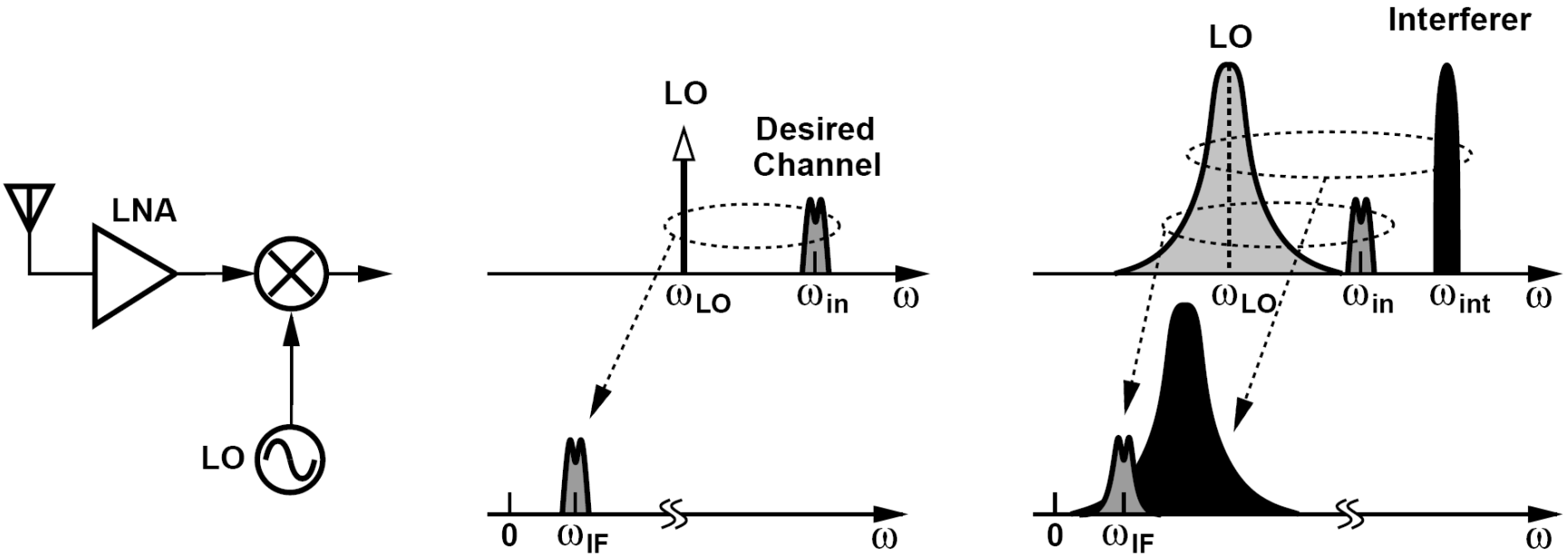


Since a 1-kHz bandwidth carries $10\log(1000\text{ Hz}) = 30$ dB higher noise than a 1-Hz bandwidth, we conclude that the noise power in 1 Hz is equal to -100 dBm. Normalized to the carrier power, this value translates to a phase noise of -98 dBc/Hz.

8.7.2 Effect of phase noise: RX

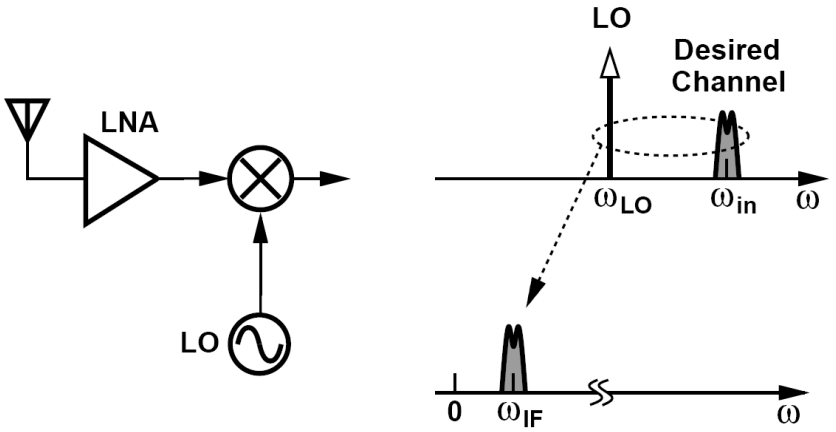
- Phase noise + interferer lead to reciprocal mixing.



8.7.2 Effect of phase noise: RX

- Phase noise + interferer lead to reciprocal mixing.

(a)

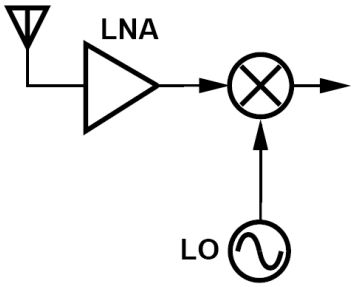


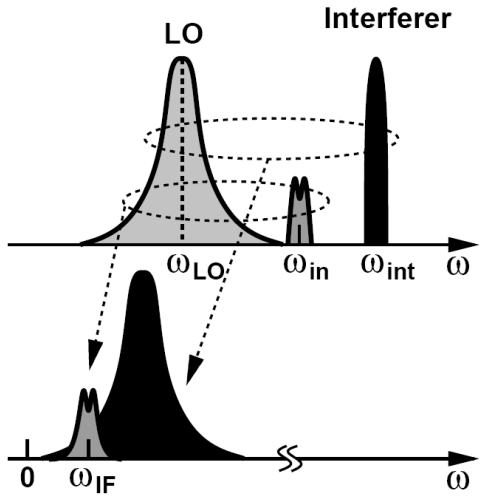
- Referring to the ideal case depicted above, we observe that the desired channel is convolved with the impulse at ω_{LO}, yielding an IF signal at $\omega_{IF} = \omega_{in} - \omega_{LO}$.

8.7.2 Effect of phase noise: RX

- Phase noise + interferer lead to reciprocal mixing.

(b)

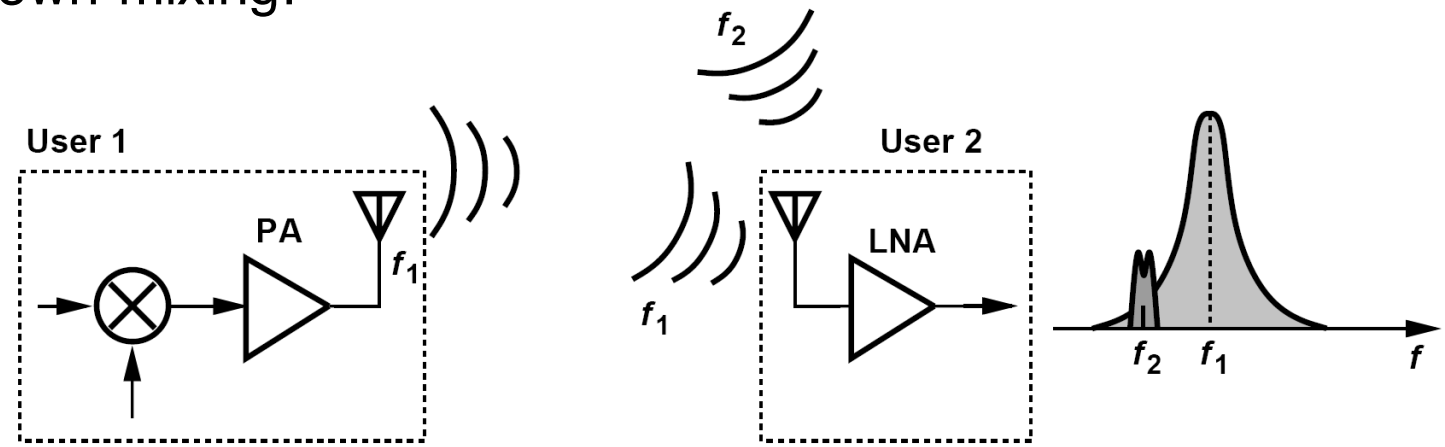




- Now, suppose the LO suffers from phase noise and the desired signal is accompanied by a large interferer. The convolution of the desired signal and the interferer with the noisy LO spectrum results in a broadened downconverted interferer whose noise skirt corrupts the desired IF signal.

Effect of phase noise: TX

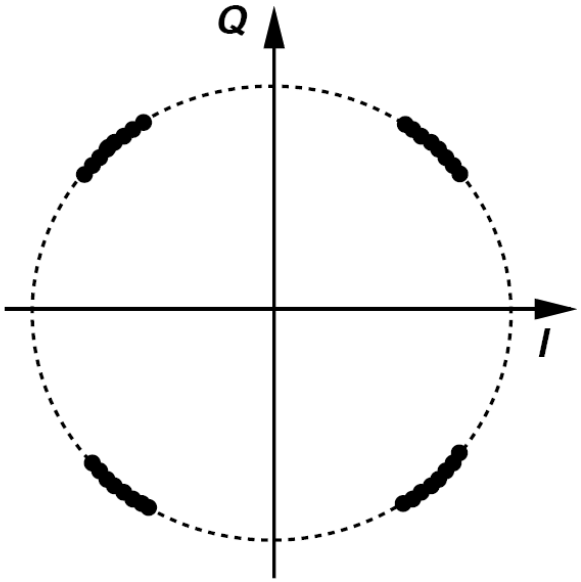
- TX phase noise can corrupt the receiver signal even before down-mixing!



- In the figure, two users are located in close proximity, with user #1 transmitting a high-power signal at f_1 and user #2 receiving this signal and a weak signal at f_2. If f_1 and f_2 are only a few channels apart, the phase noise skirt masking the signal received by user #2 greatly corrupts it even before downconversion.

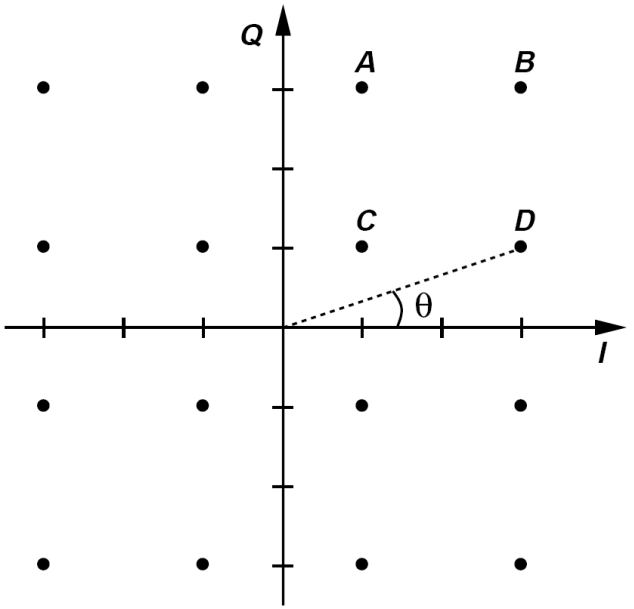
Effect of phase noise: TX

- Since the phase noise is indistinguishable from phase (or frequency) modulation, the mixing of the signal with a noisy LO in the TX or RX path corrupts the information carried by the signal.
- The constellation points experience only random rotation around the origin. If large enough, phase noise and other nonidealities move a constellation point to another quadrant, creating an error.



Example 8.26

- Which points in a 16-QAM constellation are most sensitive to phase noise?
- Consider the four points in the top right quadrant. Points B and C can tolerate a rotation of 45° before they move to adjacent quadrants. Points A and D, on the other hand, can rotate by only θ = tan-1(1/3) = 18.4°.
- Thus, the eight outer points near the I and Q axes are most sensitive to phase noise.



Types of oscillators

- Oscillators may be divided into different groups
  (the list below from Lee is maybe not fully consistent with Razavi)
- Resonators:
  - e.g. quartz crystal oscillators
- Tuned Oscillators
  - e.g. colpitts oscillators
- Negative Resistance Oscillator
  - e.g. differential LC oscillators
- Nonlinear Oscillators
  - Inverter-based ring oscillators