LINKÖPINGS UNIVERSITET

Institutionen för teknik och naturvetenskap
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Exam for TNE041, **Modern Physics**, 20 March 2023, kl. 8.00 – 12.00.

Allowed examination material: Physics handbook (Studentlitteratur)
calculator (with no wifi)
additional formulae (attached)
one hand-written sheet (A4, not copied, with notes on one side)

Define all quantities you use and give a clear answer, including unit if a numerical value is given. No points are given if only the answer is submitted, with the exception of true/false questions. The maximum score is 24 points (6x4). The limits for different grades given below is with bonus included. The solutions may be given in English or in Swedish.

The following limits for grades apply:

Grade 3	$\geq$ 10 points
Grade 4	$\geq$ 15 points
Grade 5	$\geq$ 19 points

Questions are answered by Michael Hörnquist who will visit the exam room around 9.00 and 10.30. Answers and short solutions will be available at Studieinfo at 2 pm at the latest. Results will be reported not later than 15 working days after the exam.

Good luck!

1. (a) Are the following statements true or false?
    i. The probability to find a QM-particle in 1D between a and b along the x-axis is given by $\int_a^b |\psi(x)|\, dx$, where ψ is the wave-function of the particle.
    ii. The Schrödinger equation is *not* consistent with relativity theory.
    iii. If electrons had spin $s = 3/2$ (instead of $s = 1/2$), the maximum number of electrons in the $1s$ subshell would increase from 2 to 3.
    iv. A typical band gap for a semiconductor is of the magnitude a few keV (kilo-electronvolts)

    Only the answers (true/false) are required. (2p)

   (b) What is the minimum potential required to stop a beam of electrons ejected from a metal surface of silver (Ag) by incoming light with an intensity of 58 W/m^2 and a wavelength of 250 nm. Necessary data can be found in Physics Handbook. (2p)

2. Consider the phenomenon of "pair production", i.e., when a photon of sufficient energy turns into two particles, let's say an electron and a positron. Show that this process cannot take place in vacuum, i.e., with only the photon, the electron and the positron present.
   Hint: Use suitable conservation laws in their relativistic forms.

3. A QM-particle och energy E is affected by a potential V in one dimension. The potential has the form

$$V(x) = \begin{cases} 0 & \text{if } x < -2a \text{ or } x > 2a \\ V_0 & \text{if } -2a \leq x < -a \text{ or } a < x \leq 2a \\ -2V_0 & \text{if } -a \leq x \leq a, \end{cases}$$

   where a is a length on the nanoscale, $a > 0$, and V_0 is a potential on the eV-scale, $V_0 > 0$. The position of the particle is measured and found to be between $x = -a$ and $x = a$.

   (a) Draw the potential. For which interval(s) of energies is the particle absolutely confined? (2p)

   (b) For which interval(s) of energies is the particle free, i.e., not at all affected by the potential? (2p)

   Motivate your answer clearly, no calculations neccessary.

4. An electron in a hydrogen atom is in a superposition with the quantum numbers (n, l, m) (ignoring spin) given by $(3, 2, 2)$ and $(3, 2, 0)$. The time-independent wave function is described by

$$\psi(r, \theta, \phi) = \frac{1}{\sqrt{5}} \left(R_{3,2}(r) Y_{2,2}(\theta, \phi) + 2R_{3,2}(r) Y_{2,0}(\theta, \phi) \right).$$

   (a) What is the time-dependent wave function for this electron? (1p)

   (b) What is the probability of finding the particle above the xy-plane? (3p)

5. Determine the density of states, $D(E)$, for non-interacting *electrons* in a one-dimensional well with length L and infinite walls, exchanging energy with each other. (4p)

6. (a) The semiconductor gallium arsenide (GaAs) has a band gap of 1.4 eV. Charge carriers, i.e., electron-hole pairs, can be generated by absorption of incoming photons. What is the maximum wavelength for a photon to create such a pair? (2p)

   (b) Describe briefly with words what is meant with "effective mass" for a charge-carrier within a semiconductor. Calculate the effective mass for an electron in the free-electron-model. (2p)