

TSEK02: Radio Electronics

Lecture 8:

RX Nonlinearity Issues, Demodulation

Ted Johansson, EKS, ISY

# RX Nonlinearity Issues, Demodulation

- **RX nonlinearities (parts of 2.2)**
- System Nonlinearity
- Sensitivity and Dynamic Range
- The Quadrature Demodulator
- Bit and Symbol Error Rate and  $E_b/N_0$

# RX Nonlinearity Issues

- Nonlinearities that dominates at the TX:
  - harmonic distortion,
  - gain compression,
  - intermodulation, ...
- At RX side, similar and some additional effects are also relevant:
  - desensitization,
  - cross modulation.

# Harmonic distortion

- Consider a nonlinear system

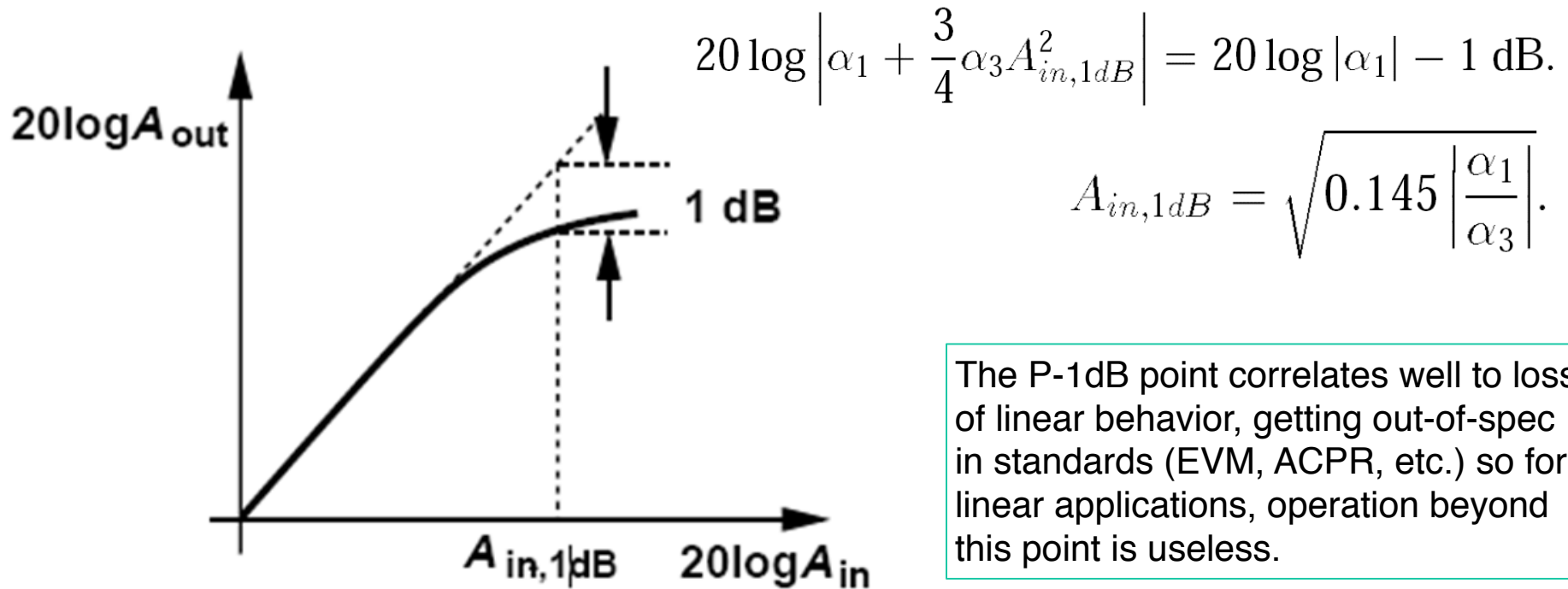
$$x(t) \rightarrow \left[ \text{Nonlinear System} \right] \rightarrow y(t) = \alpha_1 V_{in} + \alpha_2 V_{in}^2 + \alpha_3 V_{in}^3 + \dots$$

Let us apply a single-tone ( $A \cos \omega t$ ) to the input and calculate the output:

$$\begin{aligned}
 y(t) &= \alpha_1 A \cos \omega t + \alpha_2 A^2 \cos^2 \omega t + \alpha_3 A^3 \cos^3 \omega t \\
 &= \alpha_1 A \cos \omega t + \frac{\alpha_2 A^2}{2} (1 + \cos 2\omega t) + \frac{\alpha_3 A^3}{4} (3 \cos \omega t + \cos 3\omega t) \\
 &= \underbrace{\frac{\alpha_2 A^2}{2}}_{\text{DC}} + \underbrace{\left( \alpha_1 A + \frac{3\alpha_3 A^3}{4} \right)}_{\text{Fundamental}} \cos \omega t + \underbrace{\frac{\alpha_2 A^2}{2}}_{\text{Second Harmonic}} \cos 2\omega t + \underbrace{\frac{\alpha_3 A^3}{4}}_{\text{Third Harmonic}} \cos 3\omega t.
 \end{aligned}$$

# Gain Compression (1dB, $P_{1dB}$ )

- Eventually at large enough signal levels, output power does not follow the input power



The P-1dB point correlates well to loss of linear behavior, getting out-of-spec in standards (EVM, ACPR, etc.) so for linear applications, operation beyond this point is useless.

# Intermodulation

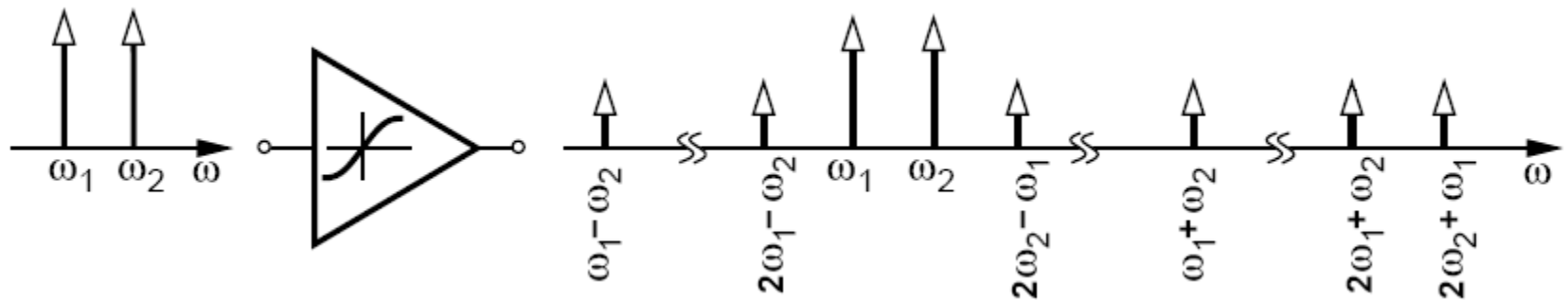
## Fundamental components:

$$\omega = \omega_1, \omega_2 : \left( \alpha_1 A_1 + \frac{3}{4} \alpha_3 A_1^3 + \frac{3}{2} \alpha_3 A_1 A_2^2 \right) \cos \omega_1 t + \left( \alpha_1 A_2 + \frac{3}{4} \alpha_3 A_2^3 + \frac{3}{2} \alpha_3 A_2 A_1^2 \right) \cos \omega_2 t$$

## Intermodulation products:

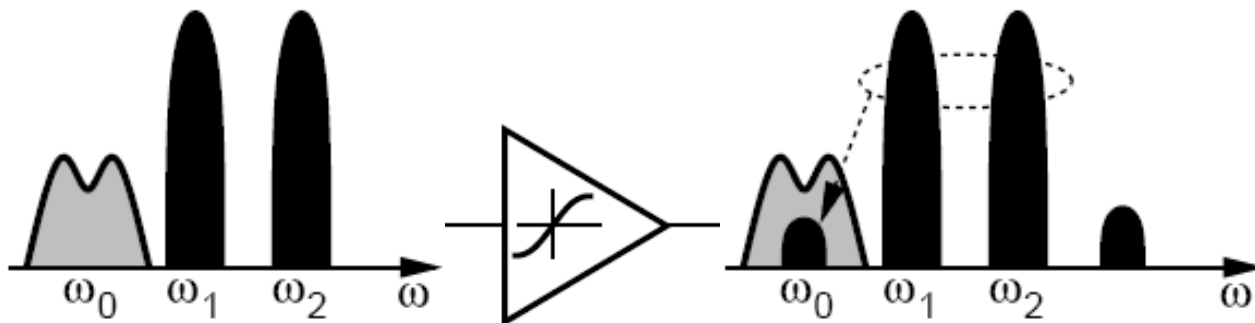
$$\omega = 2\omega_1 \pm \omega_2 : \frac{3\alpha_3 A_1^2 A_2}{4} \cos(2\omega_1 + \omega_2)t + \frac{3\alpha_3 A_1^2 A_2}{4} \cos(2\omega_1 - \omega_2)t$$

$$\omega = 2\omega_2 \pm \omega_1 : \frac{3\alpha_3 A_1 A_2^2}{4} \cos(2\omega_2 + \omega_1)t + \frac{3\alpha_3 A_1 A_2^2}{4} \cos(2\omega_2 - \omega_1)t$$



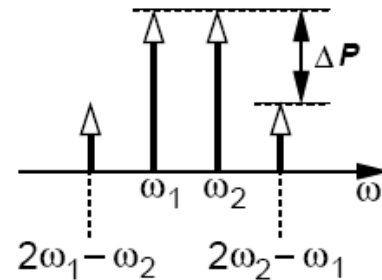
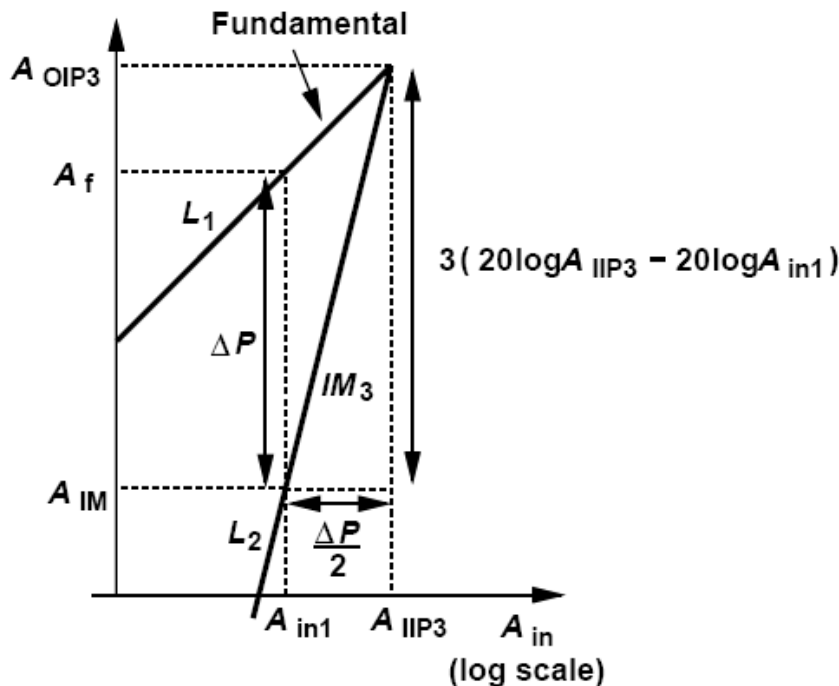
# Intermodulation

- IM3 products do not interfere with main tones, so why should we be worried?
  - They interfere with adjacent channels!
- Intermodulation products are troublesome both in the transmitter and in the receiver.



# Intermodulation – Intercept Point

For a given input level (well below P1dB), the IIP3 can be calculated by halving the difference between the output fundamental and IM levels and adding the result to the input level, where all values are expressed as logarithmic quantities.

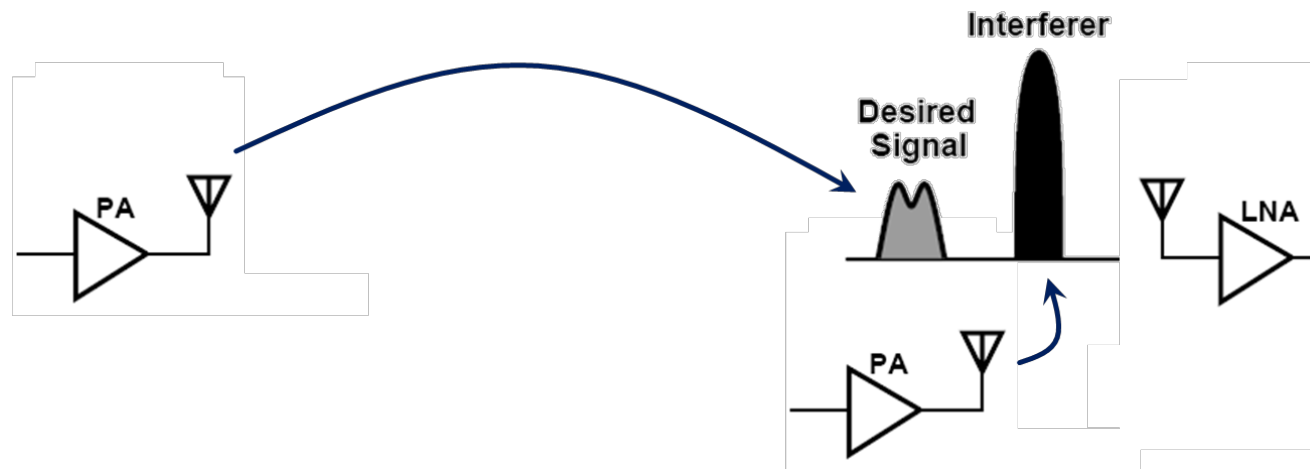


$$IIP3|_{dBm} = \frac{\Delta P|_{dB}}{2} + P_{in}|_{dBm}$$



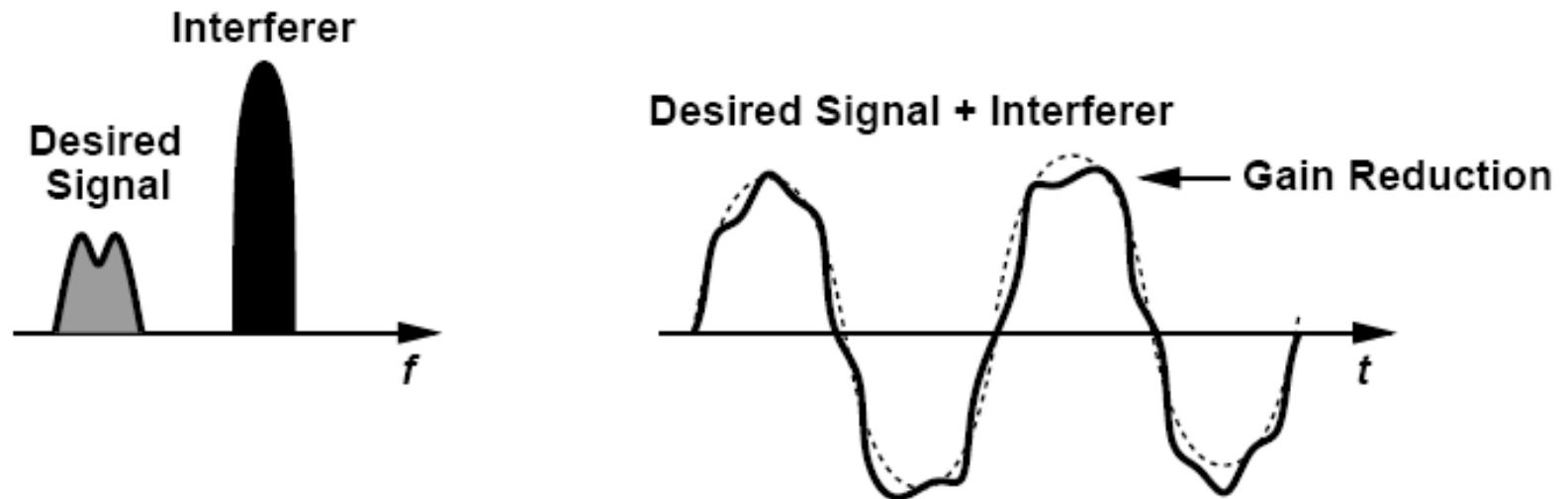
# Desensitization (p. 19)

- At the input of the receiver, a strong interference may exist close to the desired signal

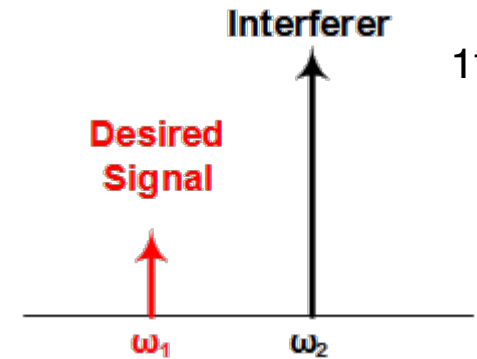


# Desensitization: related to gain compression

- The small signal is superimposed on the large signal (time domain). If the large signal compresses the amplifiers, it will also affect the small signal.



# Desensitization



- Assume  $x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$   
where  $A_1$  is the desired component at  $\omega_1$ ,  $A_2$  the interferer at  $\omega_2$ .
- When in compression  $y(t) = \left( \alpha_1 + \frac{3}{4} \alpha_3 A_1^2 + \frac{3}{2} \alpha_3 A_2^2 \right) A_1 \cos \omega_1 t + \dots$
- For  $A_1 \ll A_2$ :  $y(t) = \left( \alpha_1 + \frac{3}{2} \alpha_3 A_2^2 \right) A_1 \cos \omega_1 t + \dots$

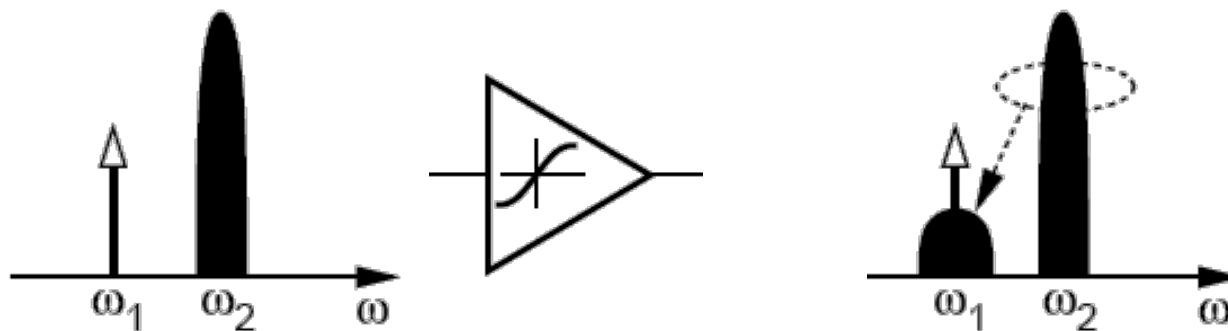
If  $\alpha_1 \alpha_3 < 0$ , the receiver may not sufficiently amplify the small signal  $A_1$  due to the strong interferer  $A_2$

- Also called "blocker". Creates problems when trying to keep the number of filters low.

## Cross-Modulation (2.2.3)

- Any amplitude variation (AM) of the strong interferer  $A_2$  will also appear on the amplitude of the signal  $A_1$  at the desired frequency and distort the signal.
- Interferer:**  $A_2(1 + m \cos \omega_m t) \cos \omega_2 t$  results in:

$$y(t) = \left[ \alpha_1 + \frac{3}{2} \alpha_3 A_2^2 \left( 1 + \frac{m^2}{2} + \frac{m^2}{2} \cos 2\omega_m t + 2m \cos \omega_m t \right) \right] A_1 \cos \omega_1 t + \dots$$

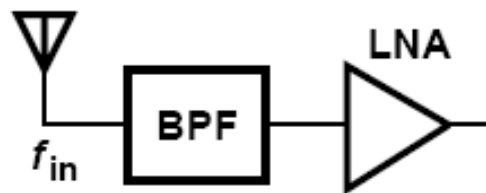


# Cross-Modulation

- Cross modulation commonly arises in amplifiers that must simultaneously process many independent signal channels.
- Examples include cable television transmitters and systems employing OFDM such as WLAN and 4G LTE.

# Band Filter

- In order to limit the input power to the receiver, a band pass filter covering the RX frequency band of interest is inserted after the antenna.
  - Since BW is large compared to center frequency, a moderate Q-factor is enough for the filter.
  - Due to its loss, it however adds noise to the system.
  - It is always desirable to design more linear low-noise amplifiers and remove this filter.

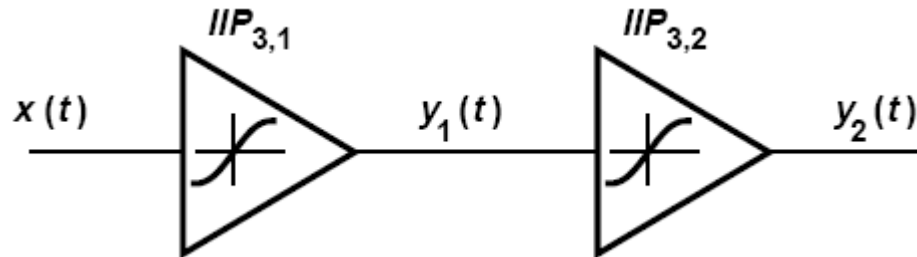


# RX Nonlinearity Issues, Demodulation

- RX nonlinearities
- **System Nonlinearity**
- Sensitivity and Dynamic Range
- The Quadrature Demodulator
- Bit and Symbol Error Rate and  $E_b/N_0$

# Intercept point of Cascaded Stages (Le5)

- Nonlinearity of each stage contributes to the overall system linearity



$$y_1(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

$$y_2(t) = \beta_1 y_1(t) + \beta_2 y_1^2(t) + \beta_3 y_1^3(t)$$

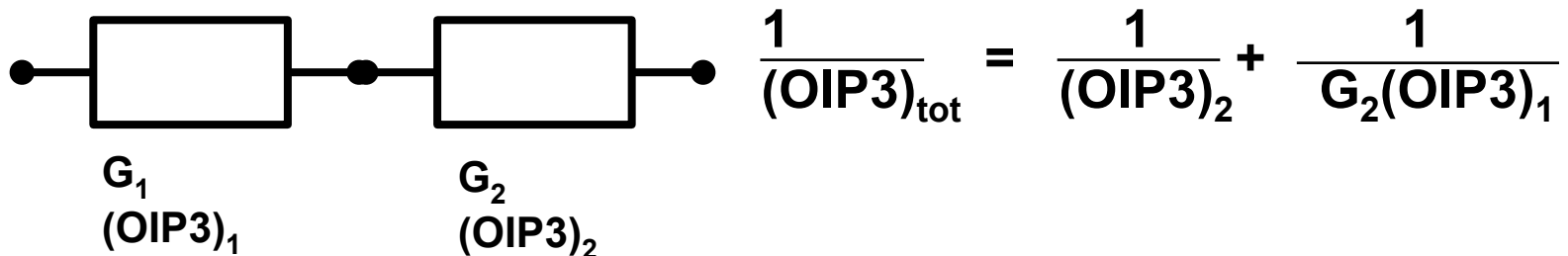
$$y_2(t) = \beta_1[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)] + \beta_2[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^2 + \beta_3[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^3.$$



# Intercept Point of Cascaded Stages

In order to calculate the total intercept point:

1. Slide all intercept points of all stages to one side of the chain (input or output)
  - Note that IIP3 and OIP3 are related through gain
2. Calculate the total intercept point at that point like a parallel resistor calculation



# Receiver Linearity and Noise: Summary

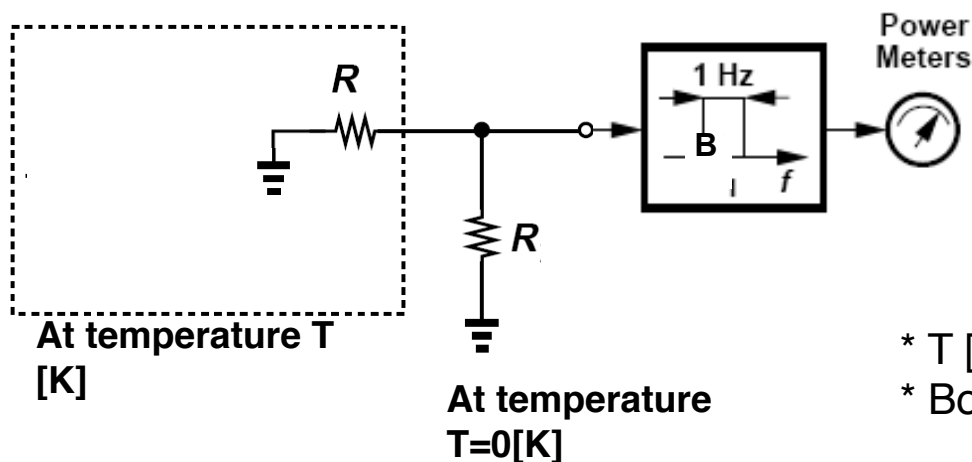
- For a receiver we would like to have a high IIP3:
  - Less gain at earlier stages and more linearity at later stages.
- We also need to decrease the total noise:
  - More gain and less noise at earlier stages.
- Conflict!
  - Always do calculations and see how gain, noise and linearity of each stage affect the overall RX performance.

# RX Nonlinearity Issues, Demodulation

- RX nonlinearities
- System Nonlinearity
- **Sensitivity and Dynamic Range (2.4)**
- The Quadrature Demodulator
- Bit and Symbol Error Rate and  $E_b/N_0$

# Thermal Noise

- Due to random movements of electrons, a resistor ( $R$ ) at temperature  $T$  [K] generates noise.
- Average Power of this noise *across a matched load* ( $R_L=R$ ) measured over  $B$  Hz at *any frequency* is given by  $kTB^*$



Noise power is:

- Only determined by  $T$
- Independent of value of  $R$
- Independent of frequency

\*  $T$  [K]

\* Boltzmann constant  $k = 1.38 \times 10^{-23}$  [J/K]

# Sensitivity (2.4.1)

- The *sensitivity* is defined as the minimum signal level that a receiver can detect with “acceptable quality”.

$$\begin{aligned}
 NF &= \frac{SNR_{in}}{SNR_{out}} \\
 &= \frac{P_{sig}/P_{RS}}{SNR_{out}}
 \end{aligned}$$

$P_{sig}$  = input signal power  
 $P_{RS}$  = noise power from the source resistance

$$P_{sig} = P_{RS} \cdot NF \cdot SNR_{out}$$

Total signal power

$$P_{sig,tot} = P_{RS} \cdot NF \cdot SNR_{out} \cdot B$$

Overall signal power is distributed over bandwidth B

# Sensitivity

- Going to dB and dBm for a minimum input signal gives the sensitivity:

$$P_{sen}|_{dBm} = P_{RS}|_{dBm/Hz} + NF|_{dB} + SNR_{min}|_{dB} + 10 \log B$$

$$P_{sen} = \underbrace{-174 \text{ dBm/Hz} + NF + 10 \log B}_{\text{Total integrated noise of the system ("noise floor")}} + SNR_{min}$$

Total integrated noise of the system ("noise floor")  
(receiver matched to the antenna)

### Example 2.25

A GSM receiver requires a minimum  $SNR$  of 12 dB and has a channel bandwidth of 200 kHz. A wireless LAN receiver, on the other hand, specifies a minimum  $SNR$  of 23 dB and has a channel bandwidth of 20 MHz. Compare the sensitivities of these two systems if both have an  $NF$  of 7 dB.

### Example 2.25

A GSM receiver requires a minimum  $SNR$  of 12 dB and has a channel bandwidth of 200 kHz. A wireless LAN receiver, on the other hand, specifies a minimum  $SNR$  of 23 dB and has a channel bandwidth of 20 MHz. Compare the sensitivities of these two systems if both have an  $NF$  of 7 dB.

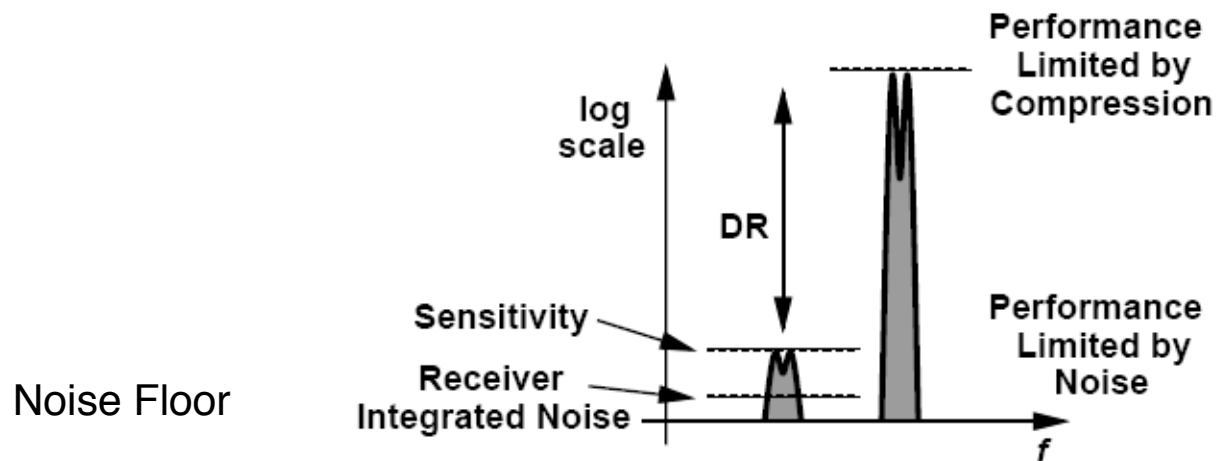
#### **Solution:**

For the GSM receiver,  $P_{sen} = -102$  dBm, whereas for the wireless LAN system,  $P_{sen} = -71$  dBm. Does this mean that the latter is inferior? No, the latter employs a much wider bandwidth and a more efficient modulation to accommodate a data rate of 54 Mb/s. The GSM system handles a data rate of only 270 kb/s. In other words, specifying the sensitivity of a receiver without the data rate is not meaningful.



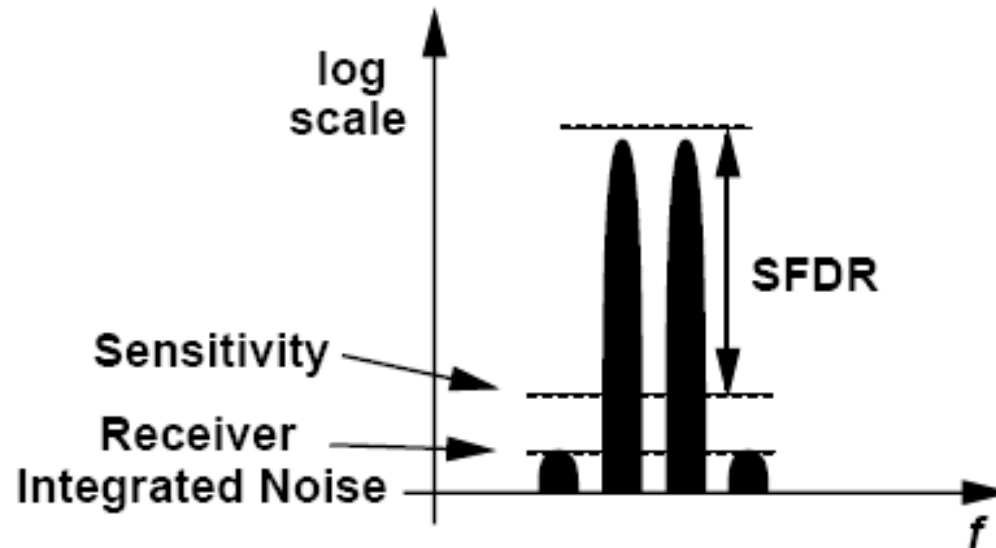
# Dynamic Range (2.4.2)

- Range of signals which could be processed by the receiver is limited:
  - Lower end: Signals should be strong enough to provide the desired SNR.
  - Higher end: Signals should not push the receiver into nonlinear operation.
- Dynamic Range: Maximum tolerable desired signal power / minimum tolerable desired signal power. Expressed in [dB].

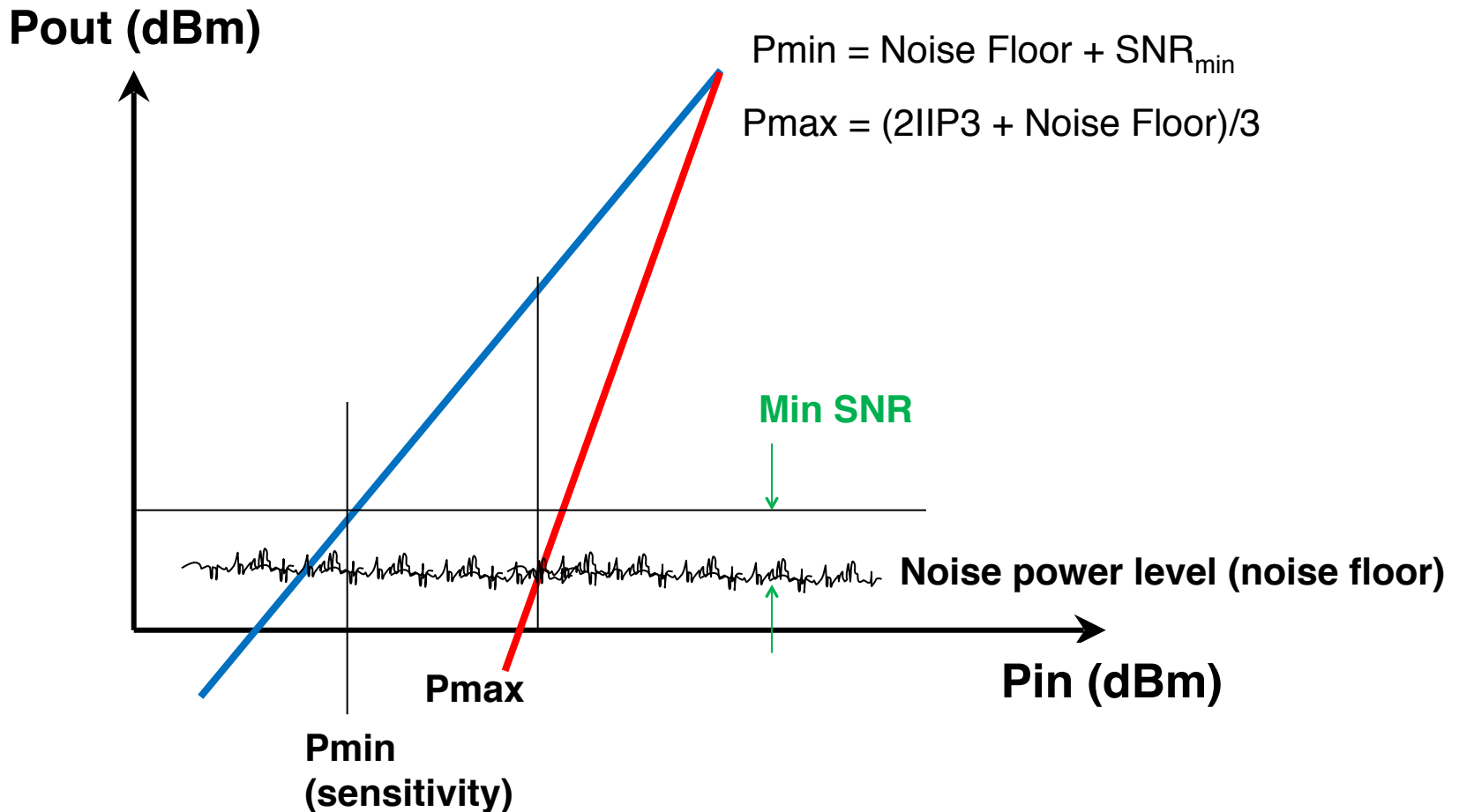


# Spurious-Free Dynamic Range (SFDR)

- Lower end: Equal to sensitivity.
- Higher end: Maximum input level in a two-tone test for which the third-order IM products do not exceed the integrated noise of the receiver.



# Spurious-Free Dynamic Range (SFDR)



# Spurious-Free Dynamic Range (SFDR)

$$SFDR = P_{in,max} - (-174 \text{ dBm} + NF + 10 \log B + SNR_{min}) \quad (2.157)$$

$$= \frac{2(P_{IIP3} + 174 \text{ dBm} - NF - 10 \log B)}{3} - SNR_{min} \quad (2.158)$$

- The SFDR represents the maximum relative level of interferers that a receiver can tolerate while producing an acceptable signal quality from a small input level.

# Automatic Gain Control (AGC)

- At some point along the receiver chain, circuits should operate with fixed signal levels (or range of levels).
- An example is an Analog-to-Digital Converter which operates with a fixed input voltage.
- As the input signal level varies, gain of the receiver should also be variable to maintain the fixed voltage at the input of an ADC.
- This is achieved by an AGC, a closed-loop regulating circuit, providing a controlled output signal amplitude, despite variations in the input signal.

# Automatic Gain Control (AGC)

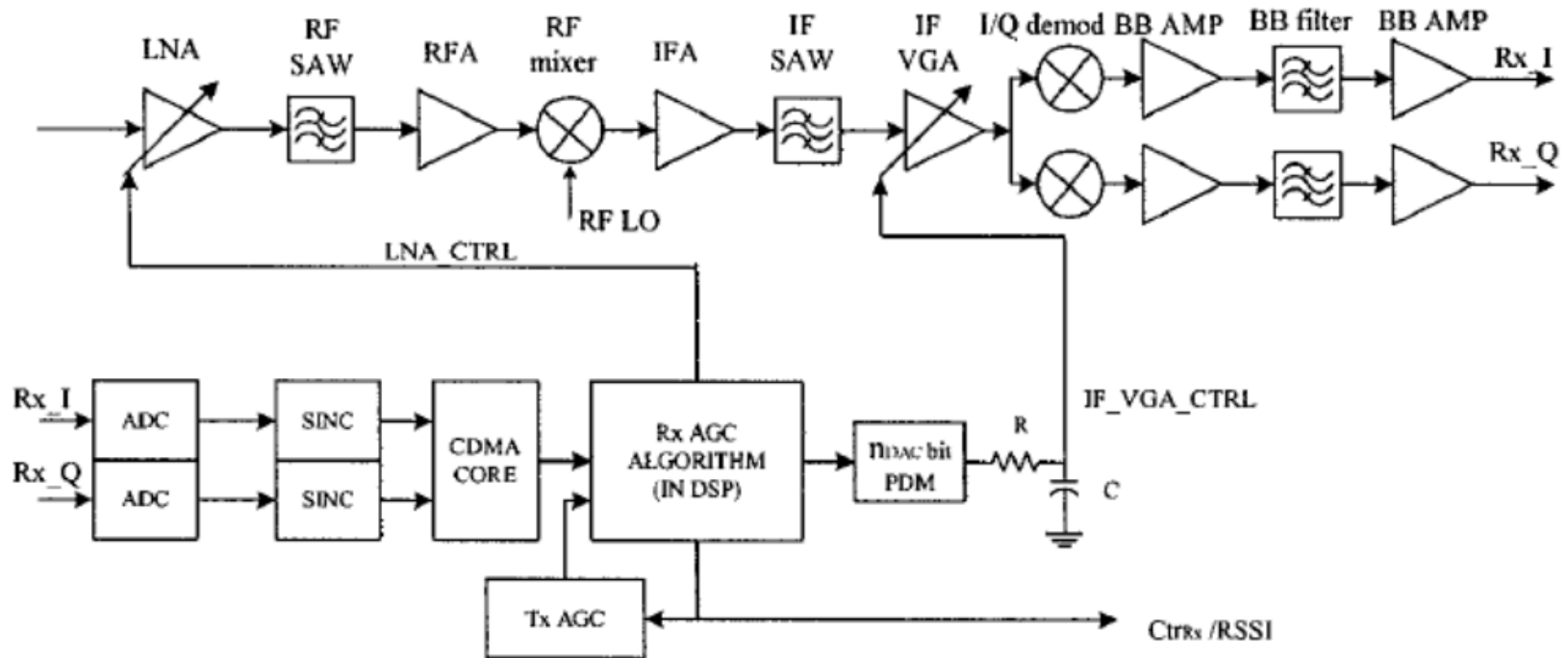


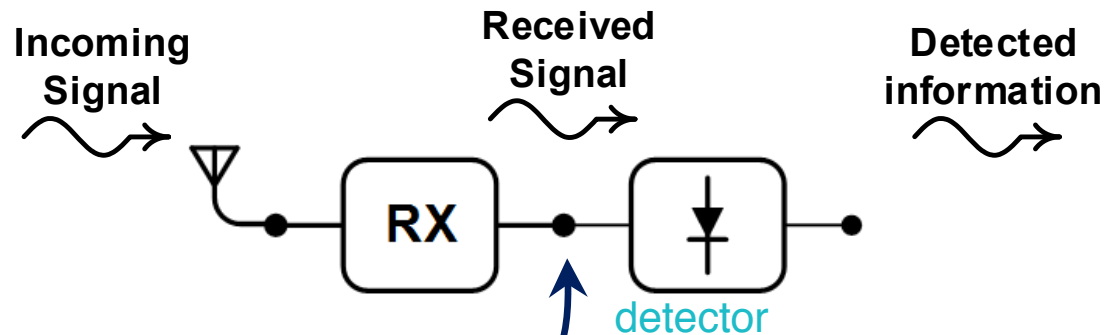
Figure 4.13. CDMA receiver AGC system block diagram

# RX Nonlinearity Issues, Demodulation

- RX nonlinearities
- System Nonlinearity
- Sensitivity and Dynamic Range
- **The Quadrature Demodulator**
- Bit and Symbol Error Rate and  $E_b/N_0$

# Amplitude and Phase Information

Remember our receiver-detector arrangement:



Depending on the modulation format, the received signal may contain information on both amplitude and phase

The detector should be able to detect both amplitude and phase



# Amplitude Detection

- Amplitude detection, often referred to as envelope detection, is relatively simple and may be performed:
  - incoherently (we only need to know the carrier frequency)
  - coherently (the detector must also include the phase of the signal)

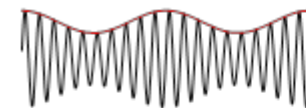
# Incoherent Envelop Detection

- By passing the modulated signal through a nonlinear transfer function (e.g. a diode), the envelop of the signal may be detected.
- Condition for successful detection: the signal contains a component at the carrier frequency
  - Advantage : simplicity
  - Disadvantage : limited application and higher error

$$V_{in} = [A(t) + k] * \cos \omega_c t$$

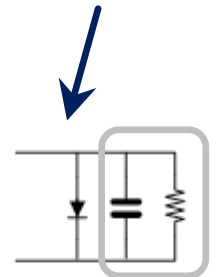
$$V_{out} = V_{in}^2 = [A^2(t) + k^2]/2 + \boxed{kA(t)} + [\dots] * \cos 2\omega_c t$$

Extracted by a filter



$A(t) \cos \omega_c t$

A diode has a square-like characteristic



Parallel RC acts as LPF

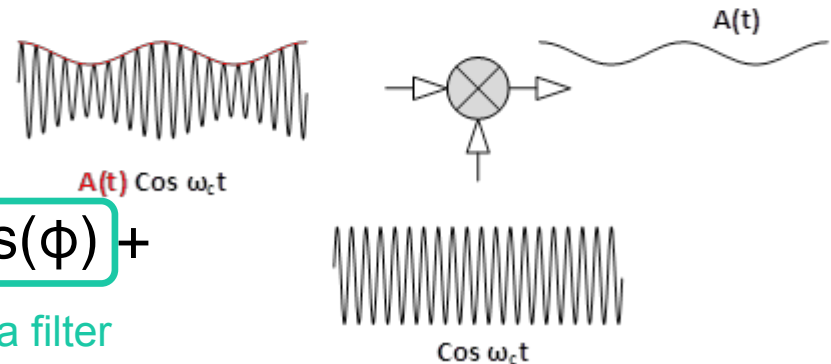
# Coherent Envelope Detection

- Mixing the signal with a reference signal at the carrier frequency.
- The reference signal may be generated by a local oscillator or extracted from the signal itself.
  - Advantage: superior accuracy and wider application
  - Disadvantage: complex

$$V_{in} = A(t) \cos \omega_c t$$

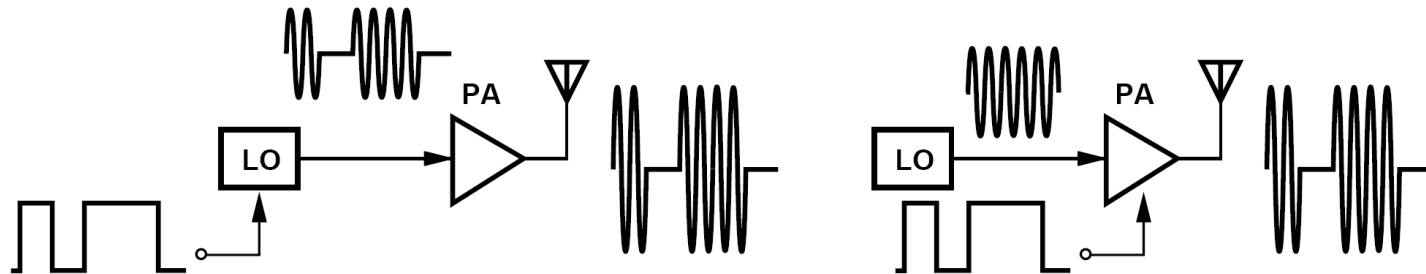
$$V_{out} = V_{in} \cos (\omega_c t + \phi) = \boxed{\frac{1}{2} A(t) \cos(\phi)} + [\dots] \cos(2\omega_c t + \phi)$$

Extracted by a filter

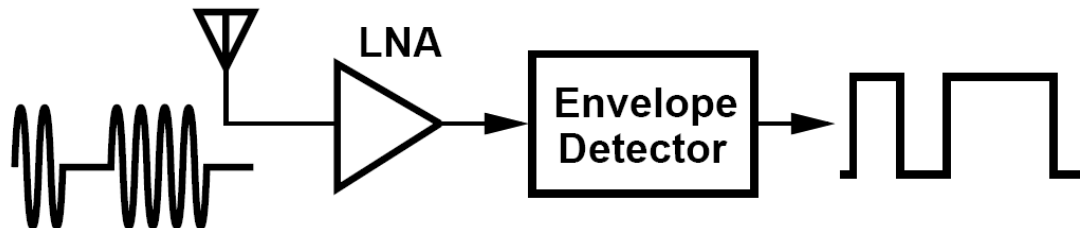


# Example of an envelope detector (4.4)

- “On-off keying” (OOK) modulation is a special case of ASK where the carrier amplitude is switched between zero and maximum.



- An LNA followed by an envelope detector can recover the binary data.




# Phase Detection

- Phase detection is however more complex.
- We wish to detect phase of a signal with an envelop detector!

# Quadrature Demodulator

- A signal with variable amplitude and phase may be expressed as  $s(t) = A(t) \cos [\omega_c t + \phi(t)]$ .
- When expanded:

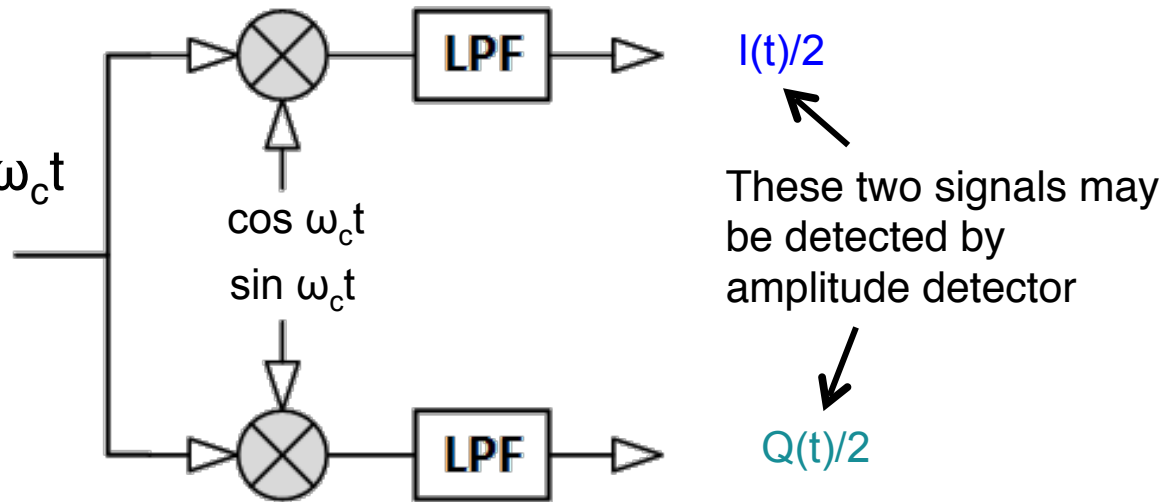
$$\begin{aligned}
 s(t) &= A(t) \cos [\omega_c t + \phi(t)] \\
 &= A(t) \cos \omega_c t \cos \phi(t) - A(t) \sin \omega_c t \sin \phi(t) \\
 &= A(t) \cos \phi(t) \cos \omega_c t - A(t) \sin \phi(t) \sin \omega_c t \\
 &= I(t) \cos \omega_c t + Q(t) \sin \omega_c t
 \end{aligned}$$


We call these the **In-phase** and **Quadrature** components of the signal.

# Quadrature Demodulator

$$s(t) = A(t) \cos [\omega_c t + \phi(t)]$$

$$= I(t) \cos \omega_c t + Q(t) \sin \omega_c t$$



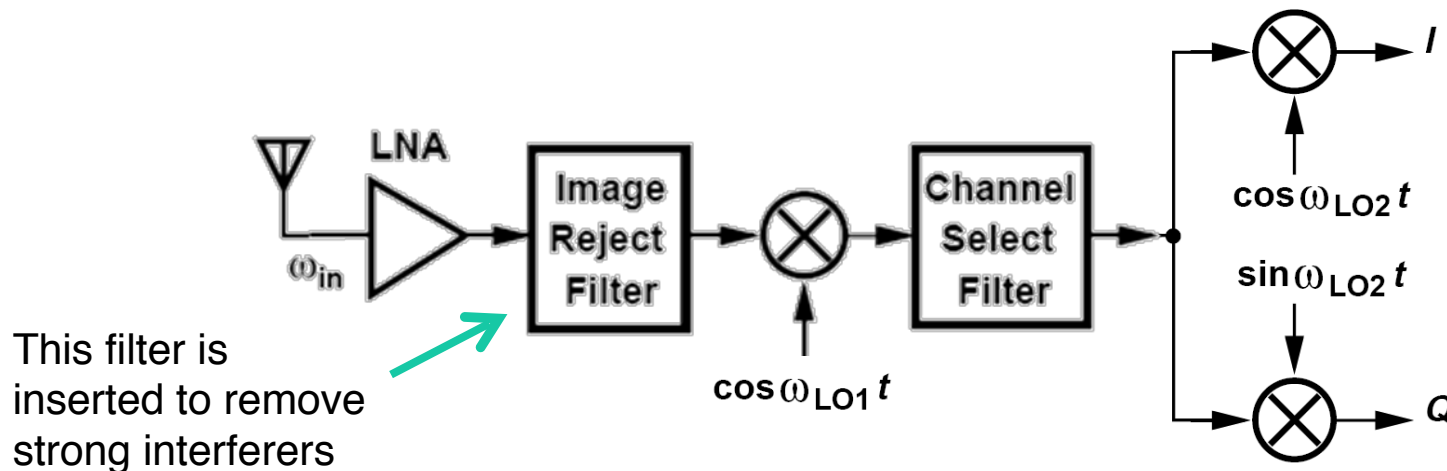
Once  $I(t)$  and  $Q(t)$  are detected, the amplitude and phase of the signal can be recalculated:

$$A(t) = \sqrt{I^2(t) + Q^2(t)}$$

$$\phi(t) = \tan^{-1} \frac{Q(t)}{I(t)}$$

# Putting everything together...

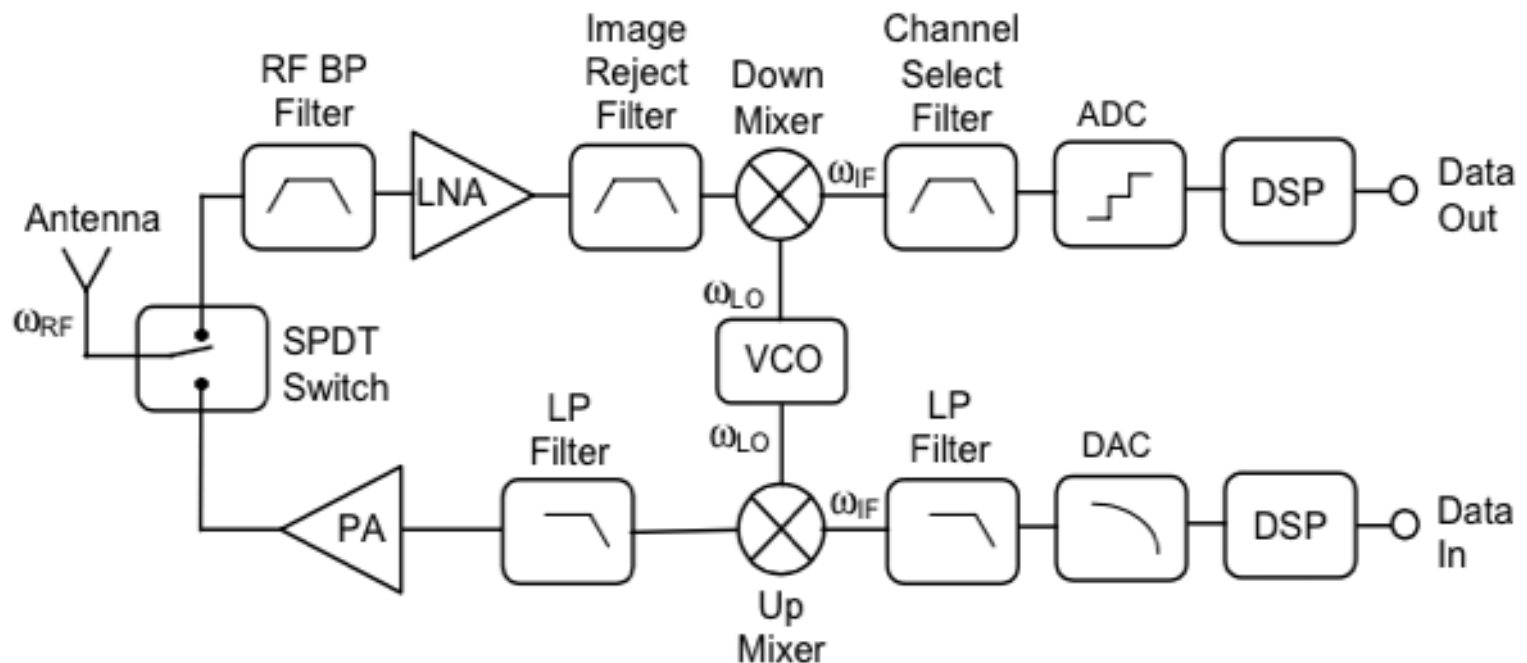
- A quadrature-mixer can be placed after the frequency of the signal is reduced to IF and channel selection is performed.
  - I and Q signals are baseband, so  $\omega_{in} = \omega_{LO1} + \omega_{LO2}$
  - Channel selection may be more effectively performed on I & Q.
  - Images may have to be taken care of.





# "Real" heterodyne "sampling-IF" (TDD)

- More advanced variants include sampling the signal at IF and then doing the rest digitally.



# RX Nonlinearity Issues, Demodulation

- RX nonlinearities
- System Nonlinearity
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- The Quadrature Demodulator
- **Bit and Symbol Error Rate and  $E_b/N_0$**

# Motivation

- Different modulation formats have different number of symbols and occupy different bandwidth.
- To be fair when comparing performance of different modulation formats, we would like to base our judgment on:
  - Bit Error Rate (instead of symbol error rate),
  - Bit Energy (instead of signal energy),
  - Noise spectral density (instead of noise power).

# Bit error vs. Symbol Error

- A symbol consists of  $k$  bits
- Symbol error is related to the bit error by

$$\textit{Bit Error} = \textit{Symbol Error} / k = \textit{Symbol Error} / \log_2 M$$

where  $M$  is the number of symbols.

# Bit Energy ( $E_b$ ) vs. Signal Power

- A signal consists of bits.
- Signal power is energy.

*Average Signal Power = bit energy ( $E_b$ ) \* number of bits per second (bitrate,  $R_b$ ) =  $E_b * R_b$*

*Bit energy ( $E_b$ ) = the power in one bit ( $P$ ) multiplied by the bit time  $t_b$*

# Noise spectral density ( $N_0$ ) vs. noise power

- Noise power density is constant over frequency, so

$$\text{Noise power} = N_0 * B$$

where  $N_0$  is the noise spectral density (kT) and  $B$  is the bandwidth.

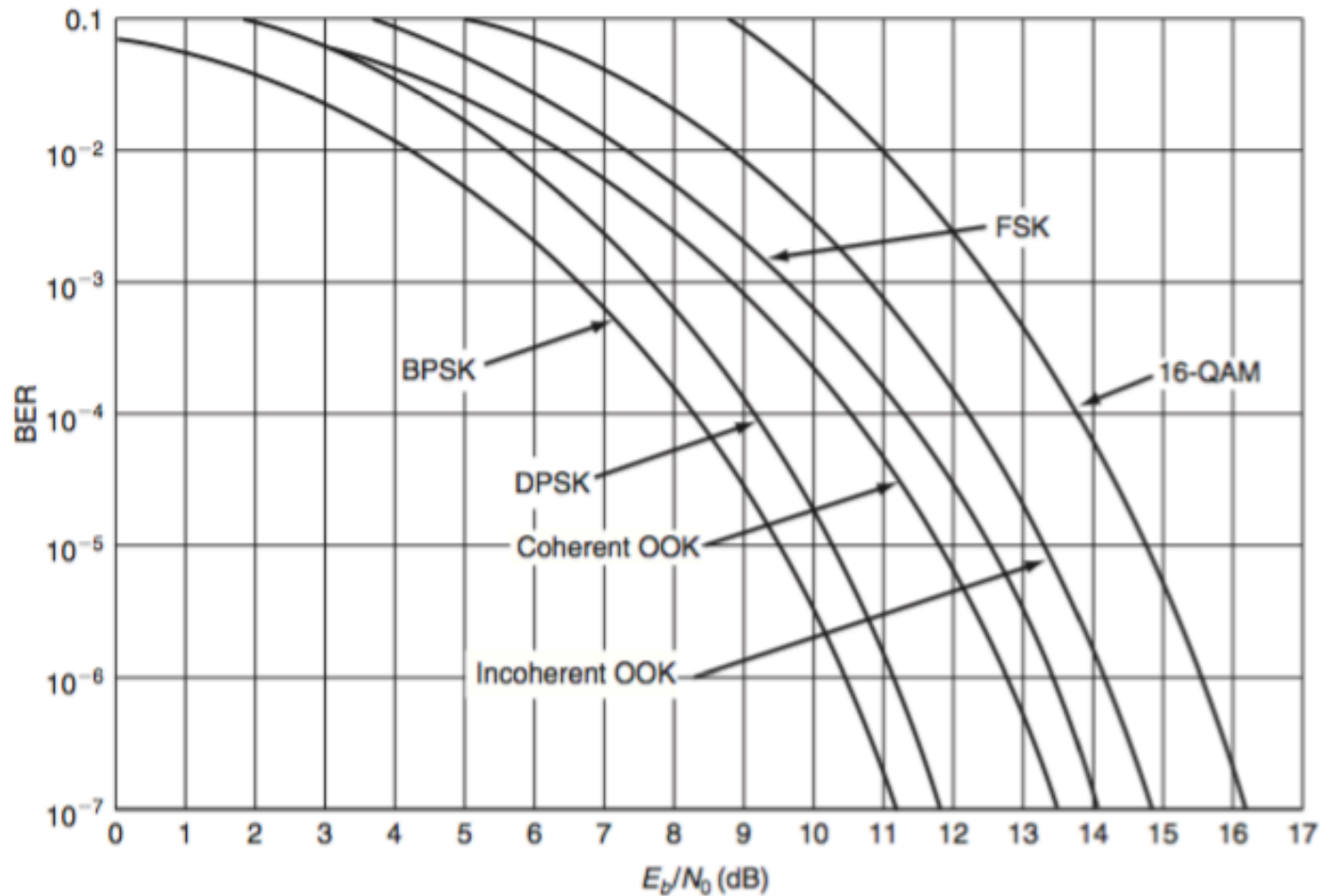
# $E_b/N_0$ vs. Signal-to-Noise Ratio

- A better measure of signal-to-noise ratio for digital data is the ratio of energy per bit transmitted ( $E_b$ ) to the noise power density ( $N_0$ ).
- SNR (a quantity which can be measured) is related to  $E_b/N_0$  (an artificial quantity used in comparisons) by

$$SNR = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{E_b}{N_0} \frac{R_b}{B}$$

$\frac{R_b}{B}$  is the spectral density (bitrate / bandwidth).

# $BER$ vs. $E_b/N_0$ for different modulations



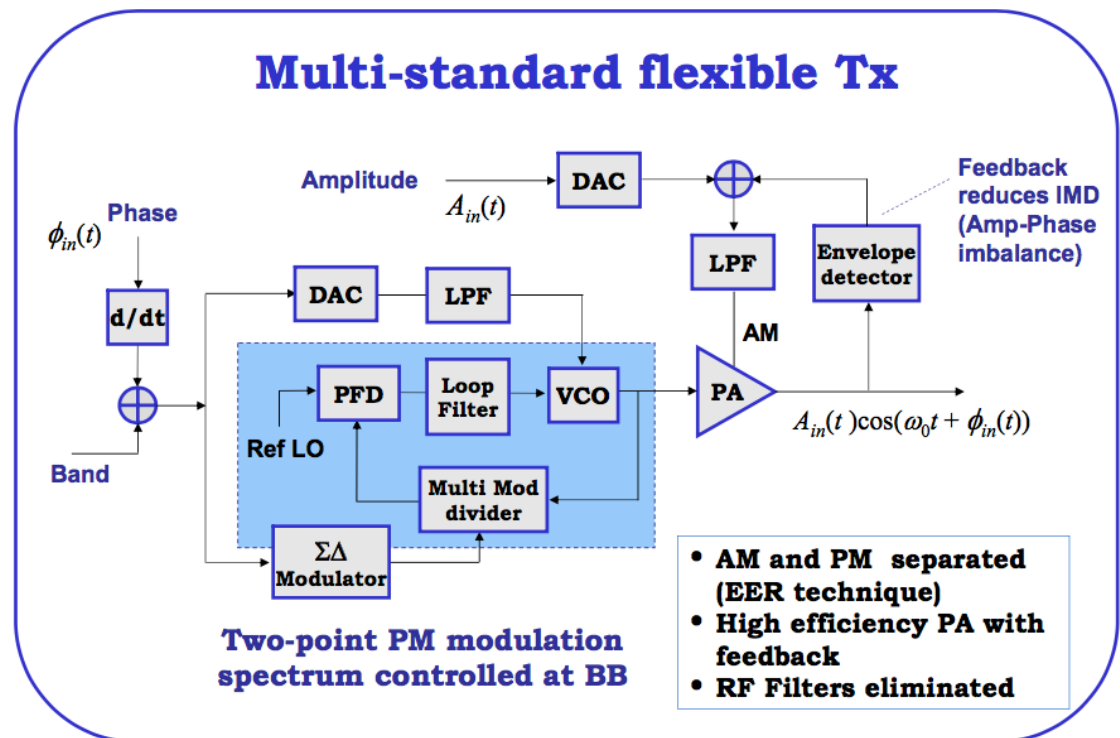


A film strip with the text "THE END" in the center. The film strip is yellowed and has several sprocket holes visible on the left and right edges. The text is in a bold, black, serif font.

**THE END**

# TSEK38: Radio Frequency Transceiver Design VT1 2019

- Advanced continuation of TSEK02 Radio Electronics.
- Learn design methods and techniques for RF front-end design at the system level.
- Work with professional design tools (Keysight ADS).
- Lectures, lab, project work (no exam).



# TSEK03: Radio Frequency Integrated Circuits (2019 HT1)

- Learn the details of the RF blocks used in CMOS digital transceivers:
  - Low-noise amplifiers (LNAs),
  - Mixers,
  - Oscillators,
  - Frequency synthesizers (PLLs),
  - Power amplifiers (PAs).
- Analysis on schematic level, calculation of circuit parameters.
- Labs: circuit simulations in cadence, LNA measurements in lab.
- Lectures, tutorials, lab, exam.

[www.liu.se](http://www.liu.se)